MEASUREMENT AND DATA ANALYSIS FOR ENGINEERING AND SCIENCE

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Laboratory Exercises Manual

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Contents

Preface

This document is the Laboratory Exercises Manual, which presents 12 laboratory exercises that were designed to supplement the material in the accompanying text. Each section describes a particular laboratory exercise. A companion Laboratory Exercise Solutions Manual provides the answers to all of the questions posed in the laboratory exercise handout and the data acquired for each exercise. The laboratory exercises can be performed as written. All have been tested many times by students over the past several years and have been refined. One intent in offering these descriptions is to provide a base for instructors to extrapolate from and generate new exercises.

Typically 6 to 10 exercises are conducted during a one-semester, three-credit-hour undergraduate measurements course. The purpose of these exercises is to introduce the student to the process of conducting experiments and analyzing their results. Some exercises are oriented towards learning about instrumentation and measurement system hardware; others towards examining an actual physical process. The overall objective is to provide students with a variety of measurement and data analysis experiences such that they are fully prepared for subsequent laboratory courses that focus on investigating physical processes, such as those in fluid mechanics, aerodynamics or heat transfer laboratory courses.

Some of the exercises were designed to be performed in series, although each exercise stands alone. In particular, Exercises 2 through 6 progressively introduce the student to the foundational concepts and use of strain gages for both static and dynamic force measurements. Exercises 1, 7 and 10 involve the comparison of measurements with theory within the context of uncertainty. Exercises 8 through 11 introduce the student to various instrumentation and measurement systems. Finally, the Exercise 12 focuses on post-experiment data analysis using files of provided data.

Table 1 lists the instrumentation used for each exercise.

	Laboratory Exercise Number											
Instrumentation	1	2	3	4	5	6	7	8	9	10	11	12
multimeter		√		√	√							
dial indicator		√										
Wheatstone bridge			√		√	√						
cantilever beam			√			√						
oscilloscope			√			√		√	√	√	√	
strain gage			√		√	√						
manometer				√								
barometer				√								
dynamometer					√							
DC power supply					√							
calibration weights					√	√						
stroboscope					√							
function generator								√	√	√		
data acquisition system			√			√		√				
thermocouples										√		
RLC circuit										√		
Helium-Neon laser											√	
diode/detector pair											√	
optics											√	

Table 1: Laboratory Exercise Instrumentation

Chapter 1

Measurement, Modeling and Uncertainty

1.1 Introduction and Objectives

This laboratory exercise demonstrates the roles that modeling and empirical uncertainties play in determining the outcome of a supposedly simple experiment. The experiment involves launching a ball from a pendulum apparatus and measuring the vertical and horizontal distances that the ball travels. The experiment is repeated at different pendulum head release angles a specified number of times for each angle. The average results for each angle are compared with the theoretical predictions within the context of the uncertainties involved in the model and in the experiment.

As part of this exercise, you must develop a model that predicts the horizontal distance, x, ± its uncertainty (estimated at 95 % confidence) where the ball will land based upon the release angle of the pendulum head with respect to the vertical top position, θ_{rel}. The values of some of the model's variables, that is, the coefficient of restitution of the ball, in turn, rely upon other empirical information that may need be gathered by performing subsidiary experiments. A subsidiary experiment is any experiment other than the actual one that needs to be performed to obtain input information for the model.

A schematic of the pendulum apparatus is shown in Figure 1.1. There is a large pendulum that consists of an Al 2024 shaft (46.60 cm long ±0.05 cm; 0.95 cm diameter ± 0.01 cm; mass = 89.8 g ± 0.5 g) that extends into a rectangular yellow brass strike head (length = 6.36 cm ± 0.01 cm; width = 3.18 cm ± 0.01 cm; height = 3.18 cm ± 0.01 cm; mass = 528.8 g ± 0.5 g). The pendulum is swung about a top pivot point, which contains an angle indicator (resolution = 1°). The ball having mass m_2 is located on a tee and placed such that contact with the strike head is made at the bottom of the swing. The distance between the center of the strike head at the top of the swing and the bottom of the swing, h_1, is 90.0 cm ± 0.2 cm. The center of mass of the system consisting of the rod and the strike head lies at 42.00 cm ± 0.01 cm from the center bearing. Figure 1.2 shows schematically the pendulum at the top and bottom of its swing with its nomenclature.

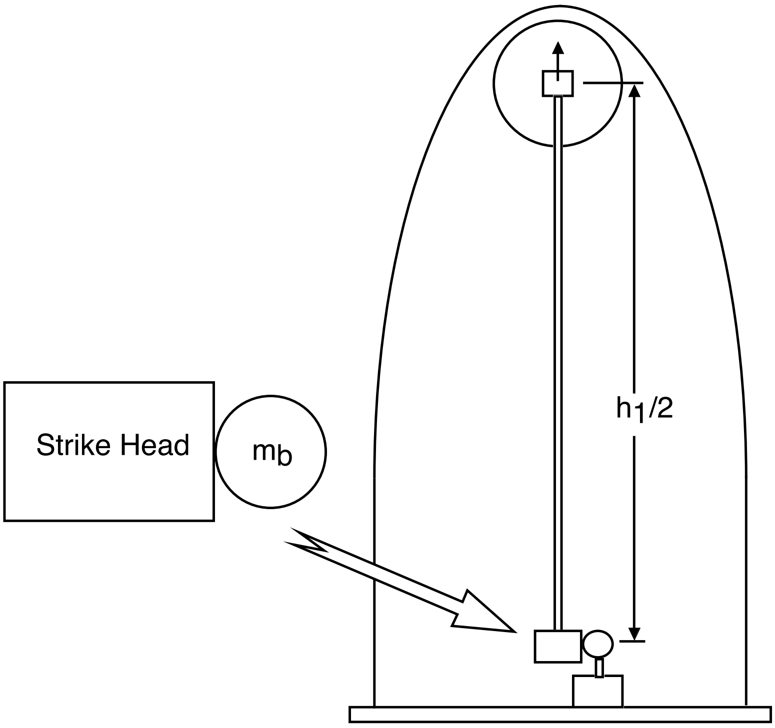

Figure 1.1: The pendulum apparatus

1.2 Instrumentation

- Pendulum apparatus to launch the ball
- a standard racquet ball with a mass of 40.60 g ± 0.05 g and a diameter of 5.61 cm ± 0.05 cm or a standard golf ball with a mass of 45.30 g ± 0.05 g and a diameter of 4.27 cm ± 0.05 cm
- English/metric tape measure

1.3 Measurements

Your objective is to obtain data relating the horizontal distance that a golf ball travels to the pendulum head release angle, which then can be compared to your theoretical predictions. You should repeat the experiment at each release angle 5 times to obtain an average horizontal distance. You must examine a minimum of 4 release angles (more will give you make a better comparison between experiment and theory).

The pendulum launching apparatus should be placed in a stable position on a table top such that the pendulum can travel freely in a complete circle and that there are no obstructions for approximately 20 ft in front of it for the ball to travel. Start by performing a trial experiment to identify where the ball will land on the lab floor. Have one partner launch the ball and the other note where it lands on the

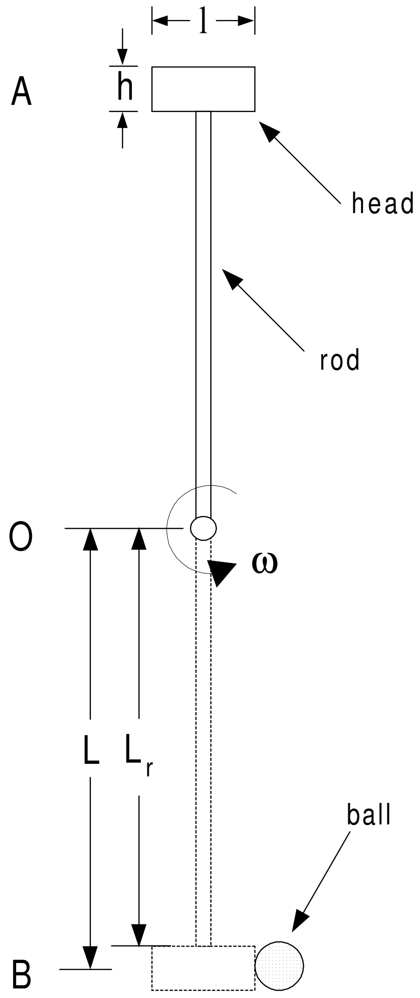

Figure 1.2: Pendulum nomenclature

floor. Repeat the trial several times to identify the approximate impact point. Tape a piece of carbon paper on the floor to mark on the paper where the ball lands each time. Now perform the experiment 5 times, always noting the impact point. When complete, repeat for another pendulum head release angle. Measure your vertical distance and all horizontal distances using the provided tape measure. Before leaving the lab, make sure that you have performed all the necessary subsidiary experiments and measurements.

1.4 What to Report

The results of this effort must be presented in the form of a technical memo according to the format described in the text. The memo at a *minimum* must include the following: [1] a statement that summarizes the agreement or disagreement between the measured and predicted distances and plausible, scientifically-based reasons for any disagreement; [2] a table of the predicted and the measured (average) horizontal distances from the launch point (in cm) to the center of the landing impact point and their associated distance uncertainties. The uncertainty estimates must be supported by detailed calculations (present these in an attached appendix) using standard uncertainty analysis at 95 % confidence (a plot of the same would also be helpful); [3] a brief description of the model developed, stating all parameters and assumptions (any detailed calculations can be put in an appendix); and [4] a table that presents the symbols and lists the values in both SI and Technical English units of all of your model inputs and outputs.

Chapter 2

Resistance and Strain

2.1 Introduction and Objectives

The primary purpose of this exercise is to determine the relationship between the relative change in resistance of a fine wire and the relative change in its length. This concept is the fundamental principle by which nearly all strain gages operate. A strain gage (see Figure 2.1) basically consists of a metallic pattern bonded to an insulating backing. This can be attached to a surface to provide a method to measure strains induced by loading. The concepts of stress, strain, and the way in which structures are loaded will be explained further in your solid mechanics course. The important thing to remember in this exercise is that when a wire is stretched (strained), its resistance changes.

These objectives will be accomplished by stretching a wire and measuring its resistance at various lengths. In the process of performing this experiment you will learn to use a digital multimeter to measure resistance in a wire. You will examine your experimental results by plotting the relative change in resistance versus the relative change in length. From this information you will determine the "gage factor" for this wire. You will also determine the uncertainties in your measurements and relate them to your results.

As stated above, the goal of this lab is to relate a change in resistance to a change in length. This is done through a quantity known as the *local* gage factor, G_l, which is defined as the ratio of the relative resistance change to the relative length change,

$$G_l = \frac{dR/R}{dL/L}. \tag{2.1}$$

The denominator in the above equation is quickly recognized from solid mechanics to be the longitudinal (axial) strain, or ϵ_L. You will notice that the above expression relates *differential* changes in resistance and length. That is, it describes a local gage factor, only valid over a very small (local) range of strain in the neighborhood of interest. We shall see later that local gage factor is very much a function of how much the wire is strained.

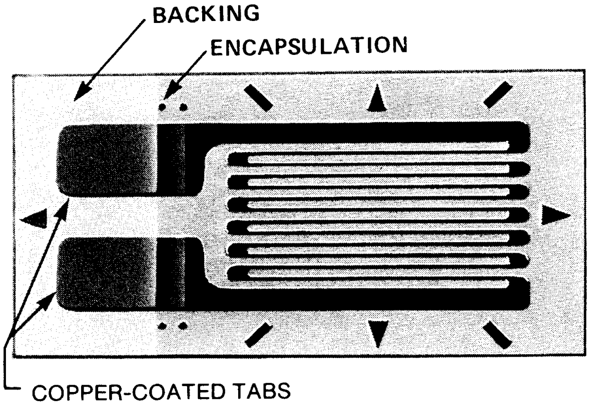

Figure 2.1: A strain gage (from Measurements Group Bulletin 309D).

Of much greater use is a quantity known as the *engineering* gage factor, defined to be

$$G_e = \frac{\Delta R/R}{\Delta L/L}. \tag{2.2}$$

It can be seen that this expression is based on small, but finite changes in resistance and length. The *local* gage factor can be thought of as the instantaneous slope of a plot of $\Delta R/R$ vs. $\Delta L/L$, whereas the *engineering* gage factor would be the slope based on the total resistance change through out the region of interest. It is nearly impossible to measure local changes in length and resistance. Thus, the engineering gage factor is typically the quantity of interest in strain applications, and is what we will measure in this exercise. For a more detailed explanation, see the Supplemental Information section at the end of this handout.

2.2 Instrumentation

In this exercise you will use the following instruments:

- Hewlett Packard 3468A Multimeter (resolution: 1 $\mu\Omega$ in the ohm range)
- Starrett dial indicator (resolution: 0.0005 in.)
- A metal meter stick (resolution: 0.5 mm)
- A wire stretcher for wires approximately 1 m in length

2.3 Measurements

First and foremost - a safety note. It is imperative that you wear safety (or your own) glasses when stretching the wire. It can break and snap back into your face.

In this lab, you are to mount a length of wire between the two clamps of the tension device and load it using the screw mechanism. First, cut a piece of wire approximately 4 feet long from the spool. Figure 2.2 shows a schematic of the clamping mechanism.

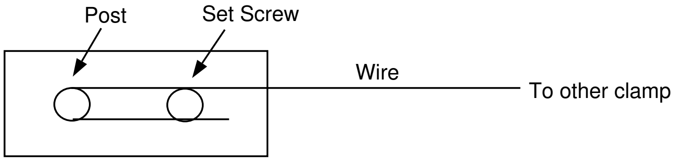

Figure 2.2: Clamping of wire in mechanism.

With the top of the clamp removed, loop the wire once about the end post before directing it to the other clamp. Then replace the top of the clamp and tighten the set screw to prevent slipping. Do likewise with the other clamp. Wire slippage can occur when tension is applied, resulting in an indication of displacement without an expected increase in resistance. When the wire is mounted correctly, it should be straight, not sagging, and the tension end with the brass thumb wheel should have about 1/2 inch of travel available.

Connect the multimeter to the wire for 4-wire resistance measurement as follows:

1. Using one pair of banana-alligator test leads, connect the HI and LO pair of terminals of the multimeter under INPUT to the wire under tension near the clamps, one at each end.

2. Using a second pair of test leads, connect the HI and LO terminals under Ω-Sense to the wire under tension just inside of the two leads of Step (1).

3. Place the multimeter into the 4-wire resistance mode by pressing the "4 WIRE" button (4 Ω should be appear on the LCD display). Also depress with the AUTO/MAN button to put the meter in the manual mode (M RNG should be appear on the display). This will yield a resistance reading resolution of 10 $m\Omega$. The KOHM range should be displayed. If not, depress the up arrow button to obtain its indication. Finally, depress the blue button, then the INT TRIG button to set the multimeter in the auto zero mode. If the auto zero mode is NOT set, then AZ OFF will appear on the display (no indication means it's set correctly). The indicated reading should be approximately 150 Ω to 200 Ω. If a negative resistance is indicated, you can switch the two inner wires to make it positive. Allow a couple of minutes for the meter to warm up before you start taking actual data.

For a brief explanation of resistance measurement methods, see the Supplemental Information section.

Slowly tension the wire by turning the brass thumb wheel until the resistance starts to increase (watch the least and second least significant digits for some consistent increase). This shall be your zero point. Record the reading on the dial gage and the resistance. (The dial indicator scale goes from 0 to 50, corresponding to 0.000 in. to 0.050 in. of travel). Measure the initial length of wire between the measuring points, the leads of Step (2), using the metal meter stick.

At increments of approximately 0.01 inches (increments of 10 on the dial indicator), record the elongation (in.) and the resistance (Ω) until the wire has been stretched about 0.20 in. The resistance changes approximately 0.1 Ω for every 0.01 in. of stretch. Return to a couple of data points and repeat those measurements to see if they have changed at all. Now, try repeating the experiment all the way out to failure of the wire. This should take on the order of 0.50 in. of travel. Try taking around 20 data points, with larger intervals at the beginning, becoming smaller as you get closer to the wire snapping.

When you are finished, turn off the multimeter, bring the dial indicator back to the zero starting point, disconnect the test leads from the wire and the multimeter, and remove your wire.

2.4 What to Report

Outside the lab after all of your data is collected, plot the relative resistance change vs. the relative length change, for both the first case and the case when you stretched the wire to failure. Estimate the uncertainties of $\Delta R/R$ and of $\Delta L/L$, following the procedures that you learned in freshman physics or those detailed in the class notes. Calculate the engineering gage factors for both cases. Are the values the same? Explain this in the context of your measurement uncertainties. Try approximating some local gage factors by calculating slopes over a few data points in your data sets, especially at lower strains. What can you say about the relation between these local gage factors and the extent to which the wire was strained at that point? Plot the local gage factor versus the strain to illustrate this. Compare the local with the engineering gage factors from the two cases, always being aware of the uncertainties involved.

Perform a least-squares linear regression analysis of the relative resistance change versus strain. Determine the correlation coefficient and the percent confidence associated with that correlation coefficient. How does the slope of the best fit line compare with some of the gage factors you calculated earlier?

All of your important experimental results and answers to the posed questions must be presented as a technical memo. Your answers to the posed questions should be contained in the explanation of your results and *not* listed item-for-item.

2.5 Supplemental Information

2.5.1 The Strain Gage

So how exactly does straining a wire change its resistance? From your elementary physics class, you might remember that the resistance of a conductor depends on its resistivity, ρ, its length, L, and its cross sectional area, A. For a wire of circular cross section, the resistance R can be written as

$$R = \rho \frac{L}{\pi r^2}, \tag{2.3}$$

where r is the radius of the wire. Differentiating the above expression and cleaning up some terms, the following result for the relative resistance change can be determined by

$$\frac{dR}{R} = (1 + 2\nu)\epsilon_L + \frac{d\rho}{\rho}. \tag{2.4}$$

In the above relation ϵ_L is the longitudinal strain and ν is Poisson's ratio (the ratio of transverse to longitudinal strains), both of which you'll learn more about in solid mechanics. Poisson's ratio is a material property that relates an axial strain to a radial strain (in the case of a wire). In other words, as you strain a wire by pulling it, its diameter will decrease. The extent to which it will decrease is found from Poisson's ratio. In the above expression, it can be seen that the relative resistance change is clearly dependent on the strain in the wire.

Equation 2.4 can be rewritten in terms of the engineering gage factor

$$G_e = 1 + 2\nu + [\frac{\Delta\rho}{\rho} \cdot \frac{1}{\epsilon_L}]. \tag{2.5}$$

For most metals $\nu \approx 0.3$ and the value of the last term in brackets representing the strain-induced changes in the resistivity (a piezoresistive effect) is around 0.4. Thus, the value of the engineering gage factor is typically around 2 or higher (sometimes up to 3 or 4).

Now, electrical currents and resistance are concepts related to free electrons moving about within a conductor. By making some arguments using atomic physics and materials science, the relative resistance change can be rewritten to eliminate Poisson's ratio from the above expression as

$$\frac{dR}{R} = 2\epsilon_L + \frac{dv_0}{v_0} - \frac{d\lambda}{\lambda} - \frac{dN_0}{N_0}. \tag{2.6}$$

In this equation, v_0 is the average number of electrons in the material in motion between ions, λ is the average distance traveled by an electron between collisions, and N_0 is the total number of conduction electrons. It would appear that things have just taken a turn for the worse. However, getting rid of Poisson's ratio has a nice result: it means that the differential resistance change (and thus the gage factor) is not a function of the material properties of the conductor. This is good, because material properties in a metal will change with strain as the material transitions from the elastic to the plastic regime on a stress-strain curve.

All that is left is to recognize that gage factor is dependent on the strain in the wire (ϵ_L) and any strain-induced changes at the atomic level of the number of free electrons present, their distance between collisions, and their velocity. Unfortunately, this means that gage factor will only be constant when the sum of these changes is either zero or is directly proportional to the strain producing the changes, which is seldom the case.

All is not lost, however. There are some materials in which the gage factor does not vary too much with strain, and therefore can be used for strain gage applications. Hopefully, this supplement provided you just a small understanding of why resistance changes with length change (the fundamental operating principle of strain gages), and also gave some clue as to why the gage factor is not constant over all levels of strain.

2.5.2 Resistance Measurement Methods

There are two ways to measure the resistance using a multimeter: the 2-wire method and the 4-wire method. The 2-wire method is straightforward. Simply connect two test leads to two points on the wire, between which is the desired resistance. The multimeter outputs a known current through the test leads and then measures the total voltage drop across the resistor *and* the test leads. This is no problem provided that the desired resistance is much larger than the resistances of the test leads. However, for this laboratory exercise, this is *not* the case .

The 4-wire method requires the use of two additional test leads. Two of the leads carry a known current to the resistance to be measured and then back to the meter, while the other two leads measure the resulting voltage drop across the resistance. Internally the meter determines (using Ohm's law) and then displays the measured resistance. This method obviously is more accurate and is the one that you will we use in this exercise.

Chapter 3

The Strain-Gage-Instrumented Beam: Calibration and Use

3.1 Introduction and Objectives

This laboratory exercise involves the static calibration of a system consisting of four strain gages mounted on a cantilever beam. Once calibrated, this system can be used in either a static or a dynamic configuration to determine the weight of an object, mass flow rate of a material and the frequency of a vibrating beam. In this exercise you will see how uncertainties enter into the calibration process and how they subsequently enter into the uncertainty in determining such quantities weight, mass flow rate, and frequency.

We will build on the concept that the change of a wire's resistance with strain can be utilized in a practical measurement system. It is possible to take small changes in resistance and, using an electrical circuit, transform the signal into a change in voltage. By making a strain gage one of the resistors in the Wheatstone bridge circuit, you are able to measure a voltage that is proportional to strain. Specifically, in this lab exercise, you will use four strain gages bonded to a cantilever beam. Each of the four gages will serve as one resistor in a leg of a Wheatstone bridge. By using four gages instead of one, the output from the gage effectively is quadrupled.

You will perform a static calibration to obtain a mathematical relationship between bridge output voltage and force. Once you know this expression, the measurement system can be used for all sorts of applications for both static and dynamic situations.

3.2 Instrumentation

The following is a list of equipment for this lab:

- Wheatstone bridge and operational amplifier measurement system
- Cantilever beam load cell

- Tektronix TDS 210 two-channel digital real-time oscilloscope
- Four Micro-Measurement 120 Ω CEA-13-125UW-120 strain gages (bonded to the beam)
- Four unknown materials: circular pipe, rectangular pipe, cylinder and hexagonal cylinder
- Plastic bottle with fine-grained sand
- 1000 mL plastic beaker

3.3 Measurements

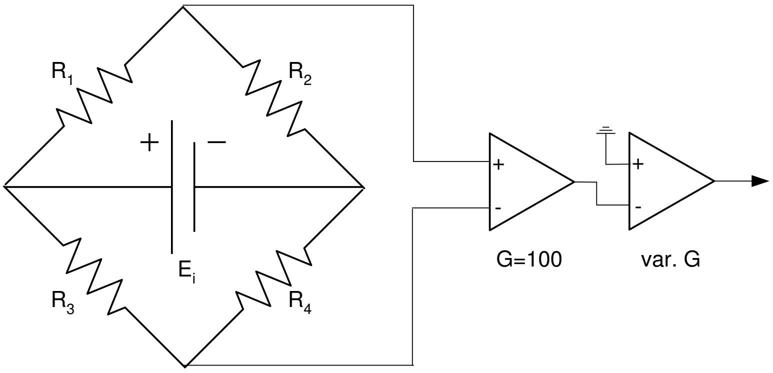

Figure 3.1: Wheatstone bridge and op amp configuration.

Follow this procedure step by step:

Part 1: Static Calibration

1. Record the lab set-up number. The set-up number is located on the top of the cantilever beam.
2. Make sure that the power is ON (the warm-up period for stable readings is about 1 hour).
3. Connect the two strain gages on the tension side of the beam to the Wheatstone bridge as follows: connect one between Bridge Excitation and + Bridge Out, and then other between - Bridge Out and Ground. The tension side gages have yellow end connectors.
4. Connect the two strain gages on the compression side of the beam to the Wheatstone bridge as follows: connect one between Bridge Excitation and - Bridge Out, and then other between + Bridge Out and Ground. The compression side gages have blue end connectors.

5. Set the Bridge Excitation voltage to 4.50 ± 0.01V by adjusting the Bridge Excitation dial and observing the output on the panel meter (be sure that the selector switch is set to Bridge Excite).

6. Set the Amplifier Gain dial position to 4.

7. With no weights attached to the beam end, adjust the Balance dial to obtain an Amplifier Out signal of 0.00 ± 0.02V (be sure that the selector switch is set to Amplifier Out). Wait for about 30 s to 60 s after you adjust the dial to be sure that the voltage stays where you set it. Reset the dial if necessary.

8. Add 450 g of weights to the hanger at the end of the beam. Note that the hanger itself "weighs" 50 g, so the total mass is 500 kg. Make sure not to apply any total mass greater than 1 kg to avoid damaging the strain gages.

9. Adjust the Amplifier Gain dial to achieve 4.00 ± 0.02 V. Wait for about 30 s to 60 s after you adjust the dial to be sure that the voltage stays where you set it. Reset the dial if necessary.

10. Take the hanger and weights off of the end of the beam. Check the Amplifier Output. It should be 0.00 ± 0.02 V. If not, repeat steps 6 through 8.

11. You are now ready to begin a full calibration. First record the actual zero weight reading. Then, start with just the hanger attached. Record the Amplifier Out reading after waiting about 30 s to 60 s. Make sure that the hanger is not moving (small movements will cause variations in your readings). Now proceed to add weights progressively up to 450 g, recording the mass and the Amplifier Out reading each time. Next, progressively take off the weights and record the values, ending up with no weight.

12. Then repeat a few of your measurements to determine the repeatability of your measurements.

13. Remove the weights and hanger and record the voltage. It should be at 0 V (your initial bridge-balanced no-weight condition). If it is not within an acceptable range about 0 V (± 0.02 V), you may want to re-balance the bridge and take your measurements over again.

Part 2: Unknown Object Measurements

1. Once you have taken all of your calibration data, select one of the objects of unknown weight. Using the can at the end of the beam to hold the object (make sure the object is close to the center of the can), determine and record the Amplifier Output voltage. Repeat for each object.

2. Obtain a plastic beaker from the TA. Fill the beaker with 600 mL of water. Place the object in the water and measure the displacement of the water. Repeat for EVERY ONE of the objects.

3. Measure and record the dimensions of each of the unknown objects with calipers. This, in conjunction with the previous water displacement measurements, will give you two separate measurements of the volume for each of the four objects.

Part 3: Measurement of Mass Flow Rate and Oscilloscope Set-up

1. Obtain a squeeze bottle full of sand and a plastic bag from your TA.
2. Turn on the oscilloscope and connect CH. 1 to the Amplifier Output from the bridge using a cable. Now press the Autoset button. Next, Press the CH. 1 button under the vertical section twice. A menu should appear in the display window. Under the menu heading "Coupling", choose "DC". This is done by pressing menu keys next to the display to the immediate right of the display.
3. Using the Volts/Div knob for CH. 1, change the voltage per division to 200 mV/div. The volts per division is displayed in the lower left hand corner of the display.
4. Now change the seconds per division to 5 s/div using the SEC/DIV knob under the Horizontal section of the oscilloscope. Again, the seconds per division is displayed at the bottom of the screen.
5. Move the vertical position of CH.1 down to one division above the bottom of the screen using the vertical position knob for CH. 1.
6. Place the plastic bag in the can.
7. Hold but do not invert the squeeze bottle filled with sand over the can at the end of the beam. Wait until the trace on the oscilloscope reaches the end of the first time division. Remove the end cover of the spout from the squeeze bottle, invert the bottle and let the sand flow freely (do NOT squeeze the bottle) into the bag lining the can.
8. When the signal has reached the end of the oscilloscope's display area or the sand begins to run out, press the Run/Stop button located at the top right of the oscilloscope. The data should be frozen on the screen. Press the Cursor button. Set the "type" to time using the top menu button. Two vertical lines should appear on the screen. Using the vertical position knobs for CH. 1 and CH. 2 to move the lines, measure the horizontal displacement (Time difference) of your data. Measure only the linear region. The time difference is displayed on the screen under the heading "Delta Record".
9. On the cursor menu, select "Voltage" under the menu Heading "Type". Using the vertical knobs for CH. 1 and CH. 2 to move the horizontal lines, measure and record the horizontal displacement (Voltage difference) of your data.
10. Carefully remove the bag from the beam, ensuring that sand does not spill out of the bag. Empty all sand back into the squeeze bottle.

11. Using a dry plastic beaker, measure 100 mL of sand.
12. Place the plastic bag in the can.
13. Place the 100 mL of sand in the plastic bag lining the can and record the Amplifier Out Voltage.
14. Carefully remove the bag from the beam, ensuring that sand does not spill out of the plastic bag. Empty all sand back into the squeeze bottle and return the supplies to your TA.

Part 4: Dynamical Measurement

1. Press the Run/Stop button if necessary to view the signal again. Move the vertical position of the signal to the center of the screen using the vertical position knob for CH. 1.
2. Now change the SEC/DIV under the Horizontal section of the oscilloscope to 50 ms/Div. Gently tap the end of the beam. Wait until the entire signal is visible within the oscilloscope display, then press the Run/Stop button on the oscilloscope to freeze the signal.
3. Press the Cursor button and select "Time" under the "Type" menu. Using the vertical knobs, measure and record the time for several periods of the signal. The frequency of vibration (in cycles per second) is the inverse of ONE period (in seconds) of the signal.

3.4 What to Report

The following must be included in your technical memo (use SI units and 95 % confidence throughout):

1. Plot up the Amplifier Out (V) versus Weight (N), where Amplifier Out is on the ordinate and Weight is on the abscissa. Use different symbols for your *up* calibration sequence (when you *added* weights), and for your *down* sequence (when you *removed* weights). Note that your masses (in g) must be converted into force units (N). Note in your memo any differences between the *up* and *down* calibration sequences.
2. Plot a linear least-squares regression analysis of the data.
3. Based upon the information provided by your least-squares regression analysis, determine the uncertainty in the voltage that is related to the standard error of the fit (the "y estimate").
4. Find the uncertainty in determining weight from a voltage measurement (the "x-from-y estimate"). To determine this uncertainty use a voltage near the

mid-point of your calibration voltages and project it back through the calibration curve to the x-axis to determine the expected x value. Also project back through the appropriate confidence limits to the x-axis to find the minimum and maximum possible x values. Find the differences between these values and the expected x value. Use the greater of the two as the uncertainty.

5. Determine the volume of each unknown material using both the water displacement and caliper measurements.
6. Use both water-displacement and caliper-determined volumes and the weights determined using the calibration, determine the density of each material. For three of the objects you will have two density values (one from each measurement method).
7. In a table report the volume determined by each method and the density for each object. Compare your density values with those found from the literature or on the web. Be sure to cite your source(s).
8. Determine the uncertainty in the volume and density values. This should be done for both types of volume measurement. Which method has the least uncertainty?
9. Assume that you do not have access to calipers and you need a low cost experiment to determine the volume and density of the unknown objects. Design a new water-displacement based experiment that costs less than $100.00 and has a volume uncertainty less than 0.061 in^3. Be specific and cite your sources.
10. Determine the mass flow rate of the sand using your calibration information and the recorded time difference.
11. Determine the uncertainty in the mass flow rate.
12. Present the weight, volume, density and mass flow rate uncertainties all in one table.
13. Determine the density of the dry sand and calculate the uncertainty of the dry sand density.
14. Assume that you want to build an hourglass based on the collected data. How much sand would be needed to measure an hour?
15. Determine the natural frequency of the beam using the data collected from the lab (see Supplemental Information). Calculate a theoretical natural frequency for the beam. Compare the experimental and theoretical values and give reasons for any possible differences.

All of your important experimental results and answers to the posed questions must be presented as a technical memo. Your answers should be contained in the explanation of your results and *not* listed item-for-item.

3.5 Supplemental Information

All solid objects vibrate to some extent when they are hit. Determining the frequency of vibration is very important in design. Vibration can cause wear, reliability problems, and induce unwanted noise. In some instances, vibration is desired and even necessary. A vibrating conveyor belt is a good example of this. The frequency at which an object vibrates depends on both its shape and material properties. We can determine an object's unforced (natural) frequency experimentally by giving the object a short impulse and measuring its frequency of vibration, as is done in this lab. For simple configurations, it is also possible to determine the natural frequency using theoretical approximations. For the cantilever beam configuration used in this lab, the frequency of vibration can be determined by

$$\omega_r = \frac{A_r}{l^2}\left(\frac{EI}{\rho A}\right)^{1/2}, \tag{3.1}$$

where ω_r is the natural frequency, E is the modulus of elasticity, I is the moment of inertia, A is the cross-sectional area, A_r is the mode coefficient, l is length, and ρ is the density. The cantilever beams used in this lab are made of 6061-T65 aluminum with a modulus of elasticity of approximately 10^7 psi and a density of 0.00305 slugs/in.^{3}. The length, l, is the distance for the fixed point to the free end of the beam. For the lab set-up, l is

$$l = (L - D), \tag{3.2}$$

where L and D are the distances shown in Figure 3.2.

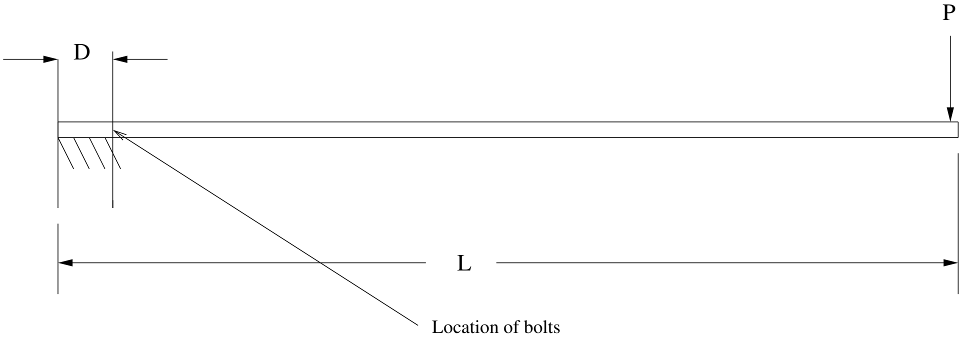

Figure 3.2: Cantilever Beam Diagram

Recall, that the moment of inertia for a rectangular cross-section is

$$I = \frac{bh^3}{12}, \tag{3.3}$$

where b is the width of the beam and h is its height. Figure 3.3 shows a diagram of relevant dimensions. Every beam in this lab has slightly different dimensions; this is true with any engineered object. Table 3.1 contains the measurements for each experimental set-up. When you calculate theoretical values for this lab, make sure

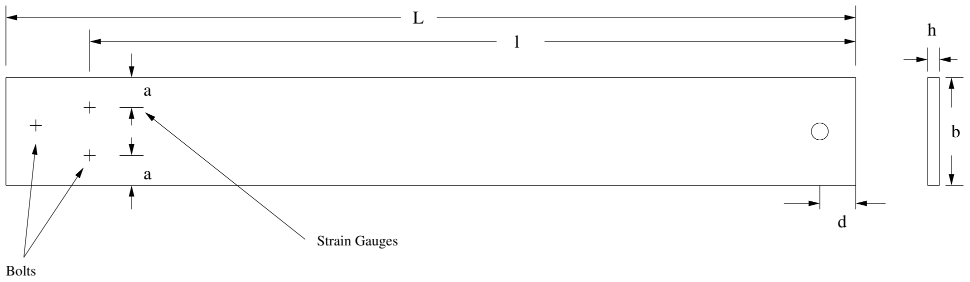

Figure 3.3: Cantilever beam with four strain gages

Beam	**L** (±0.002)	**l** (±0.020)	**d** (±0.0015)	**b** (±0.001)	**h** (±0.0005)
1	11.913	9.930	0.491	1.504	0.1275
2	11.933	9.910	0.492	1.504	0.1275
3	11.919	9.920	0.495	1.503	0.1275
4	11.924	9.920	0.492	1.504	0.1275
5	11.943	9.950	0.488	1.503	0.1275
6	11.944	9.940	0.491	1.503	0.1275

Table 3.1: Beam Dimensions (all units are in inches)

you use the appropriate dimensions for your particular station. The accuracy of each measurement is also given. Notice that the dimensions for a and d are not listed in the table. These dimensions are not critical to the calculations and were not measured.

The mode coefficient, A_r depends upon the mode of vibration, r. The natural frequency is the first mode (when r = 1). As the mode of vibration increases so does the frequency of vibration and the mode coefficient. Table 3.2 gives the mode coefficients for the first three vibration modes.

Mode 1	A_1	1.875
Mode 2	A_2	4.694
Mode 3	A_3	7.854

Table 3.2: Mode Coefficients for the first three vibration modes.

You can find more information on vibration modes in any vibrations or dynamics text.

Chapter 4

The Propeller Dynamometer: Static Thrust, Torque and RPM Measurement

4.1 Introduction and Objectives

In this lab exercise you will build on experience gained using a strain gage-based measurement system. Now you will use two strain gage - Wheatstone bridge amplifier systems to determine the thrust and torque generated by a radio-controlled aircraft propeller under **static** operating conditions. More specifically, in this exercise you will measure the power into the motor, the thrust and torque output of the system, and the RPM of the propeller.

The data taken from a thrust stand such as this can be used to gather very valuable data on different propellers. This in turn can help engineers make more informed design choices when selecting a propeller for a given airframe and propulsion system. If we were to take this experiment a step further, the measurement system as is could be easily modified to be placed in a wind tunnel in order to gather **dynamic** propeller data. Also, such a measurement system could be used to examine the performance of a fan in a heating, ventilating and air-conditioning system. More information regarding propellers and how they work is included in the supplemental information section of this handout.

4.1.1 Instrumentation

- Zinger 11-7 propeller ($d = 11$ in.)
- Two Wheatstone bridge and operational amplifier instrument systems
- propeller dynamometer
- Motor power supply
- Voltmeter/ammeter readout box

- Calibration weights and hanger
- Stroboscope

4.2 Measurements

Before you get started on any of the measurements, a word about safety; this is the first exercise that has the potential of being extremely dangerous. The propeller turning at several thousand RPM will not hesitate to remove your fingers if you are careless enough to place them in the plane of the blade. Therefore, the rule of operation for the dynamometer is: **NEVER REMOVE THE SAFETY CAGE WITHOUT FIRST DISCONNECTING THE POWER SUPPLY TO THE MOTOR**. You do this by disconnecting the red and black banana plug power leads from the motor power supply. Having said that, lets get to the procedure.

1. Connect and adjust the bridges and amplifiers.
2. Calibration:
    - First check to make sure that the motor power supply is disconnected. Once this is done, remove the safety cage from the propeller. Attach the hanging wire to the screw at the center and immediately in front of the propeller, and hang it over the pulley at the front of the test stand.
    - Zero the bridges for both the thrust and torque readouts by adjusting the "BRIDGE BALANCE ADJUST" knobs on the appropriate panels.
    - Perform both thrust and torque calibrations, one at a time, by adding weights to the respective hanger and recording the corresponding voltage output. This data will be used later to convert the voltage measurements taken during the actual running of the propeller to thrust and torque readings. For the thrust calibration, add weights in 50 g increments up to around 1.0 kg. For torque calibration, increase by about 5 g to 10 g up to around 100 g. The little blue basket should be used as the initial torque calibration weight. Its mass is 7 g. The length of the torque calibration moment arm is 8.50 ± 0.05 in.
    - Once you are through calibrating, remove the wire hanger from the front of the propeller shaft and replace the safety cage.
3. Thrust, Torque, and RPM measurements:
    - Reconnect the motor power supply. Turn on the supply. If the red overload light is flashing, depress the button on the voltmeter/ammeter box. Turn the voltage increase dial up just enough to get the prop spinning. Now, depress the voltmeter/ammeter button again. You should hear the propeller increase its RPM.

Motor Voltage (V)	Motor Current (A)	Thrust (V)	Torque (V)	RPM
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

- Going in 1 V increments from 1 V to 12 V, record the following data: voltage into the motor, current into the motor, thrust voltage, torque voltage, and propeller RPM. Record this information in the data table provided. Repeat your measurements at several motor voltages after you have completed the first set. The voltages and currents in are read from the voltmeter/ammeter box panel meters. Thrust and torque voltages are the output voltages from the appropriate bridge/amplifier system. The RPM is measured using the strobe. It is best to turn the room lights out during these measurements so that you can see the standing image of the propeller better.

  The stroboscope has three scales: LOW (100 RPM to 700 RPM), MEDIUM (600 RPM to 4200 RPM) and HIGH (3600 RPM to 25 000 RPM). During the course of your measurements you will use all three scales. You can shift from one to another by depressing the appropriate button on the back of the stroboscope. The propeller is marked near its tip with distinct black lines. Basically, you will adjust the strobe light until you see a standing image (that is, not moving) of the propeller, with the marks on the ends of the propeller appearing identical to what they would if the propeller were not moving. This is a little tricky because standing images with the correct marks (two horizontal lines on one side; two vertical lines on the other) occur at even integer *fractions* of the correct RPM for a two-blade propeller as well as at the correct RPM. However, a standing image obtained at a strobe frequency of twice the correct propeller RPM will not show the correct marks. So, once you get a correct image, keep doubling the strobe frequency until you identify the correct RPM. For our system, at a motor voltage of 1V you should see standing images at approximately 170, 340 and 680 RPM, with correct images at 170 and 340. So, the correct propeller RPM is approximately 340. The correct RPM at 2V should be approximately 1000.

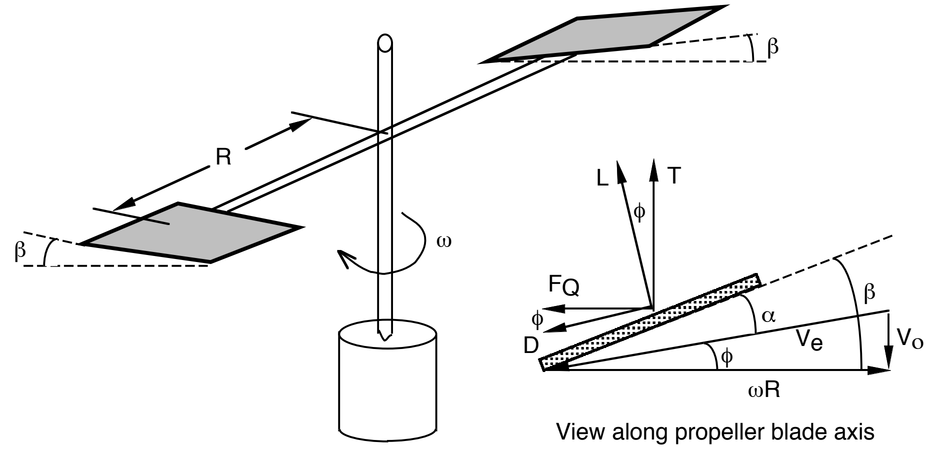

Figure 4.1: Velocities and forces acting on a propeller blade element.

- Disconnect the motor power supply once are you are finished with all your measurements.

4.3 What to Report

Submit your information in the form of a technical memo. Be sure to include at the very least the following (with some discussion of each): [1] plots of your thrust and torque calibrations (T and Q versus the voltage outputs of the measurement system), [2] plots of T, Q and $P_{prop,in}$ versus N_{prop}, [3] a plot of η_m versus N_{prop}, and [4] plots of C_T, C_Q and C_P versus N_{prop}. You decide which plots are the important ones to put in the body of the memo. The other plots you can put in an appendix. Remember to construct all plots according to the format presented in the text. Be very careful with the units when calculating the values of all these parameters. Perhaps you can include as an appendix a sample calculation of each parameter showing how you did the proper unit conversion.

4.4 Supplemental Information

An understanding of how a propeller produces thrust rests in a knowledge of how an airfoil generates lift, L, and drag, D. An airfoil in motion generates lift and drag. If we consider the propeller as a rotating airfoil, we can understand how it generates a forward thrust, T, and a torque, Q, where Q results from a force, F_Q, acting perpendicular to the forward direction.

Let's examine this in more detail. Refer to Figure 4.1, which shows a propeller consisting of two blade elements of pitch angle, β, each located at distance R from the axis of rotation. The velocity V_o is that of the air through which the propeller advances. Because the blade element also is rotating with an angular velocity ω, it will have a rotational velocity of ωR. The velocities V_o and ωr are vectors that combine to yield the relative velocity V_e. This is the velocity of the air relative to the rotating blade element that the propeller "sees". Its approach angle is ϕ. This implies that the actual angle of attack α equals β - ϕ. We have

$$\phi = tan^{-1}(\frac{V_o}{\omega R}) \tag{4.1}$$

and

$$V_e = \sqrt{V_o^2 + (\omega R)^2}. \tag{4.2}$$

Now examine the view along the propeller blade axis. Using trigonometry, you can easily determine that

$$T = 2[Lcos\phi - Dsin\phi] \tag{4.3}$$

and

$$Q = 2RF_Q = 2R[Lsin\phi + Dcos\phi]. \tag{4.4}$$

Further, the power required to turn the propeller is given by

$$P_{req} = Q\omega. \tag{4.5}$$

When $V_o = 0$, we have $\phi = 0$, giving $\alpha = \beta$. This leads to

$$T_{static} = 2L_{static}, \tag{4.6}$$

$$Q_{static} = 2RD_{static}, \tag{4.7}$$

and

$$P_{req,static} = Q_o\omega = 2\omega RD_{static}. \tag{4.8}$$

Thus, for both static ($V_o = 0$) and dynamic ($V_o \neq 0$) conditions, a rotating propeller generates thrust and torque from its lift and drag. Further, the power required to turn the propeller is related to its torque and rotational velocity and hence to its lift and drag.

Now let's examine the power required to turn the propeller in our experimental set-up. We also would like to develop some expressions that relate our measured variables to those that characterize the performance of some of its components. Refer to Figure 4.2.

The power into the motor is simply the product of its input current, i, and voltage, V, both of which we measure. That is,

$$P_{motor,in} = i \cdot V. \tag{4.9}$$

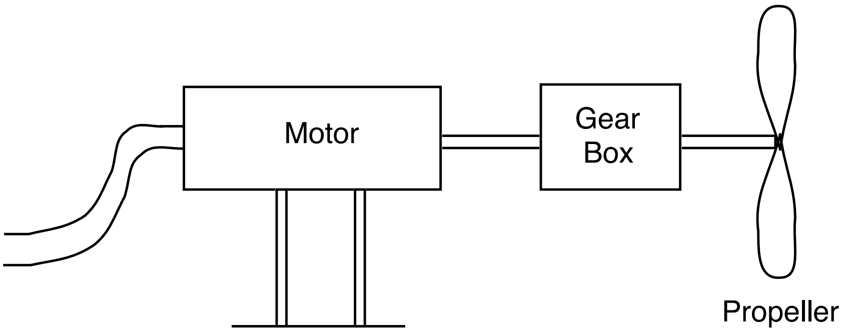

Figure 4.2: Schematic of the motor - gear box - propeller system.

Some of this power will be lost inside the motor and eventually dissipated as heat. We can quantify this by the motor efficiency, η_m, where

$$\eta_m = P_{motor,out}/P_{motor,in}. \tag{4.10}$$

Now, the power out of the motor equals the power into the gear box, $P_{gearbox,in}$. In a similar manner, we can define a gear box efficiency, η_g, where

$$\eta_g = \frac{P_{gearbox,out}}{P_{gearbox,in}}. \tag{4.11}$$

For our experiments, we will assume that $\eta_{gearbox} = 0.95$. We will determine η_m through experiment.

The purpose of the gear box is to reduce (or gear down) the RPM of the motor to one that allows the propeller to operate in its most efficient range. In this experiment, the gear box reduces the motor RPM, N_{motor} to the propeller RPM, N_{prop}, by a factor known as the gear ratio, GR. This is given by

$$N_{motor} = GR \cdot N_{prop}, \tag{4.12}$$

where for our experiments $GR = 2.21$.

From the above equations we can determine that

$$P_{motor,out} = \frac{Q_{prop,in} \cdot N_{prop}}{\eta_g}. \tag{4.13}$$

This yields

$$\eta_m = \frac{Q_{prop,in} \cdot N_{prop}}{\eta_g \cdot i \cdot V}. \tag{4.14}$$

If you examine this equation, you'll see that every term on the right hand side is known or determined from measurements. Hence, we can determine the motor efficiency using our data and Equation 4.14.

Finally, we can use our data to determine the values of three coefficients that are commonly used to characterize propeller performance. These are the thrust coefficient, C_T, the torque coefficient, C_Q, and the power coefficient, C_P. These are defined by the following equations:

$$C_T = \frac{T}{\rho \cdot n^2 \cdot d^4}, \tag{4.15}$$

$$C_Q = \frac{Q_{in}}{\rho \cdot n^2 \cdot d^5}, \tag{4.16}$$

and

$$C_P = \frac{P_{in}}{\rho \cdot n^3 \cdot d^4}, \tag{4.17}$$

where T, Q_{in} and P_{in} are for the propeller, d is its diameter, ρ is the density of air, and n is the propeller's revolutions per second.

Chapter 5

The Solid Rocket Motor: Transient Thrust Measurement

5.1 Introduction and Objectives

In previous exercises, we have used a load cell consisting of four strain gages mounted on a cantilever beam to make various measurements. At first the load cell was calibrated and used as a means of making various static force measurements. Then it was used to examine the dynamic response of the cantilever system to an impact loading (dropping of a golf ball). Finally, in this lab we will build upon prior experience to make dynamic thrust measurements of a solid rocket motor. These measurements in turn will be used in a future exercise to make altitude predictions of a model rocket during ascent.

As always, the first step in this process will be a calibration of the measurement system. Then, the digital oscilloscope will be used in conjunction with a Wheatstone bridge setup to record the thrust time history for the model rocket engine.

5.2 Instrumentation

The following instrumentation will be used in this lab:

- Wheatstone bridge and operational amplifier measurement system
- Cantilever load cell
- Calibration weights and hanger
- Fluke PM3380-A Combiscope (Analog and Digital Oscilloscope)
- Estes A-83 Solid Propellant Rocket Motor
- Estes launch controller and ignitors

5.3 Measurements

Unlike any of the previous lab exercises, these thrust measurements will be made in teams. Before the start of the lab, all groups will be given their rocket motor, and will take turns in acquiring their thrust data. We will assist you in the operation of the firing equipment. For safety, it is important that everyone pay attention and not be near the rocket motors when they are installed in the thrust stand and configured to be lit. If in doubt, by all means ask!

1. Before your turn to fire, weight the motor and record its initial mass.
2. The first step will be a calibration of the load cell. We have done this before and should go by quickly. Calibrate by placing the calibration masses in the cup at the end of the beam and recording the output voltage of the system. We will need to calibrate up to a mass of 1 kg in order to cover the full range of the rocket motor thrust output.
3. Once the system is calibrated, make sure that the rocket motor sleeve is in the can at the end of the beam. Now configure the scope to make the appropriate measurements. The appropriate settings are 0.1 V DC, with a time base of 100 ms. As with the dynamic beam response, set the trigger settings to trigger off of the beam response by adjusting the trigger level to around one division above ground. Give yourself a delay of about one division as well. Depress the “SINGLE” button to arm the scope. Tap on the beam to convince yourself the system is connected properly and triggering as planned. Then arm the scope again.
4. Now we are ready to set up the firing. Insert the motor into the sleeve at the end of the beam. Insert an igniter into the engine and cap it off with an engine plug. Make sure the safety key is removed from the launch controller, and then attach the launch leads to the igniter wires.
5. Once everything is connected, arm the scope and clear out of the firing area. Insert the safety key into the launch controller. The light bulb on the front of the controller should be lit. Fire the motor by depressing the button on the controller.
6. Check the scope to see that you get a reasonable thrust trace. Save the signal into one of the scope’s memory locations. Download this information onto the laboratory computer to save your data for subsequent analysis.
7. Finally, take the empty motor casing and weigh it.

Chapter 6

Model Rocket Launch: Altitude Prediction and Measurement

6.1 Introduction and Objectives

Now that we have information on the thrust curve for the solid rocket motors in a previous exercise, we are able to make predictions as to how high the rockets that we have built will go. This can be accomplished by deriving the appropriate equations of motion, developing appropriate models for all of the force terms, and solving this differential equation for the altitude.

6.2 Equations of Motion

We will simplify our analysis by assuming that the rocket travels only along a straight line in the vertical direction. Drawing an appropriate free body diagram for the rocket and applying Newton's Second Law, we have

$$\sum F = T - mg - D = m\frac{d^2y}{dt^2}. \tag{6.1}$$

In the above equation, T is thrust, m the mass, g gravity, D the drag, and y is the vertical displacement. If we write the drag in terms of a drag coefficient, C_D, we get

$$T - mg - \frac{1}{2}\rho S C_D \left(\frac{dy}{dt}\right)^2 = m\frac{d^2y}{dt^2}. \tag{6.2}$$

The quantity S is the appropriate reference area, which in our case is the body tube cross sectional area. The drag coefficient can be obtained from the handouts provided to you on the drag of model rockets. Note that during the burn phase of the rocket motor, the thrust varies with time, as you found in the previous lab exercise. After the burn phase, the thrust becomes zero. The rocket, however, still continues to travel vertically upward until it reaches its maximum altitude where its velocity equals zero. Hence, there are two phases in the rocket's ascent, the burn phase and the coast phase.

Both phases can be described by the same equation of motion by specifying the value of the thrust to become zero at the end of the burn phase.

Equation 6.2 is a non-linear, second-order differential equation. This equation is complicated by the fact that both the thrust and mass are quantities that are changing with time. This equation cannot be solved analytically unless we make some simplifying assumptions. Alternatively, through the use of numerical techniques, this equation can be integrated directly to give us displacement as a function of time.

6.3 Solution Approaches

6.3.1 Analytical Solution

We can obtain an analytical solution to the above equation of motion if we make the simplifying assumptions that the mass and thrust are constant in time during the burn phase of the rocket motor. These constant values can be calculated from the data that was acquired in your previous solid rocket motor firing lab exercise. The average thrust value should be obtained using the MATLAB® `trapz` function and the thrust data file. The following equations result from integrating the equation of motion by parts.

The altitude, h_b, at the end of the burn phase will be

$$h_b = \frac{\beta_o}{g} ln[cosh(gt_b\sqrt{a_o/\beta_o})], \tag{6.3}$$

where a_o denotes the drag free acceleration in g's as given by

$$a_o = \frac{T}{W} - 1, \tag{6.4}$$

with T being the thrust and W the weight. Also β_o, known as the density ballistic coefficient, is defined as

$$\beta_o = \frac{W}{0.5\rho C_D S}. \tag{6.5}$$

Further, the velocity, V_b, at the end of the burn phase is

$$V_b = \sqrt{a_o \cdot \beta_o} tanh(gt_b\sqrt{a_o/\beta_o}). \tag{6.6}$$

The altitude gained (up to the maximum altitude) during the coast phase, h_c, will be

$$h_c = \frac{\beta_o}{2g} ln(1 + \frac{{V_b}^2}{\beta_o}). \tag{6.7}$$

Thus, the maximum altitude, h_{max}, is

$$h_{max} = h_b + h_c. \tag{6.8}$$

6.3.2 Numerical Solution

To start off, we will rewrite the second-order differential equation as a system of two first-order equations. If we define y_2 to be vertical displacement and y_1 to be vertical velocity, the above second order equation reduces to

$$\begin{aligned} \dot{y_1} &= \frac{1}{m}\left(T - mg - \frac{1}{2}\rho S C_D y_1^2\right) \text{ and} \qquad (6.9) \\ \dot{y_2} &= y_1 \qquad (6.10) \end{aligned}$$

These equations can then be integrated using some numerical integration algorithm to obtain solutions.

We have yet to address the problem of the changing mass and thrust values. The mass can be approximated by assuming that the motor burns at a constant rate. Thus, the mass will decrease linearly from the initial mass of the rocket to the final mass after the motor is done thrusting (the mass will not change once the motor is spent). Thus, if we set m_o to be the initial rocket mass, and m_f its final mass, we have

$$m(t) = m_o - \frac{(m_o - m_f)}{t_b}t, \qquad (6.11)$$

where t_b is the burn time of the rocket motor. This expression is only valid up until the motor stops firing. After the thrust stops, the mass is constant at m_f. This expression is easily incorporated into any solution algorithm. The thrust presents a unique problem in that we have thrust data at discrete values of time. There are several ways to approach this problem. The simplest method would be to obtain some average thrust value and assume that the thrust assumes this constant average value for the duration of the burn time. More involved methods might be to curve fit a polynomial to the thrust data to come up with some continuous representation of the thrust over the time period of interest. Finally, an interpolation algorithm could be setup to find an approximation for the thrust value at any time during the calculation. How you choose to model this term is up to you.

Included below are some MATLAB® commands that will help demonstrate how to make these sorts of calculations. The command `ode23` is a numerical integration algorithm that allow you to solve systems of differential equations. Additional information (as well as additional examples) can be found in the MATLAB® manual.

First create an M-file called `launch.m`. In that M-file, type the command $[t, y] = ode23('alt', t0, tf, y0)$. Before that command you must specify the values for t0 (the initial time, here set equal to zero), tf (the final computation time, on the order of several seconds based upon the type of rocket motor used), and y0 (the initial altitude, here set equal to zero). That command calls another M-file (name it `alt.m`) that will numerically integrate the equations that are set up in `alt.m` and then pass the results back to `launch.m` for subsequent plotting. You can plot altitude versus time in `launch.m` by the command plot(t,y(:,2)). The maximum altitude is given by the command max(y(:,2)).

Also remember that you must create `alt.m`. The essential lines in `alt.m` are the first line: function ydot = alt(y,t) and the last line: $ydot = [(1/m) * (T - m * g - (0.5 * \rho * C_D * S) * y(1).^2; y(1)]$. In between the first and last lines you must define T, m, g, ρ, C_D and S. Here you can use average values for T and m or compute them. I would suggest that you use the average values first until you get your M-files running. Then you can make things more detailed.

Finally, to obtain the solution plot simply type "launch".

6.4 What to Report

The results of your rocket motor firing and rocket launch laboratory exercises are to be submitted together in the form of a full technical report by **each team**. Your report should highlight your altitude predictions and how the data that we acquired relates to these calculations. A comparison between your predicted and actual measured altitudes must be made. Rational, scientifically based explanations (supported by additional calculations) must be presented to explain any differences between your predicted and measured altitudes.

Chapter 7

The Cylinder in a Wind Tunnel: Pressure and Velocity Measurement

7.1 Introduction and Objectives

The main objective of this lab is to become familiar with the techniques and equipment for making pressure measurements on a circular cylinder placed in a cross-flow in a subsonic in-draft wind tunnel in conjunction with velocity measurements. In addition, concepts of uncertainty will be addressed both in the taking of the measurements and the propagation of these uncertainties to obtain estimates for the lift and drag coefficients and the drag of the cylinder.

7.2 Instrumentation

- Dwyer Model 246 0 to 6 in.H_2O inclined manometer (resolution: 0.02 in.H_2O)
- Microswitch Model 163PC01D36 -5 H_2O to +5 in.H_2O differential pressure transducer
- Tenma Model 72-4025 digital multimeter (resolution: 0.01 V on 20 V FS; 0.001 V on 2 V FS)
- Princo barometer (resolutions: 0.01 in.Hg and 1 °C)
- wind tunnel RPM indicator (resolution: 20 RPM)
- cylinder rotating position indicator (resolution: 1° angle)

The test section for this exercise contains a pitot-static tube and a cylinder fitted with pressure taps. The pitot-static probe is located in the front of the test section and will be used to determine the free-stream centerline velocity of the wind tunnel. The cylinder is 1.675 ± 0.005 inches in diameter, 16.750 ± 0.005 inches in length, and has several pressure taps located in a line along its span. For this experiment, we will be using the tap in the middle of the cylinder to minimize any possible wind tunnel wall effects. The cylinder (and more importantly, the pressure tap) can be rotated through 360° using the position indicator on the side of the test section.

In this exercise, we will be using (and comparing) both an inclined manometer and differential pressure transducers for measuring pressure. Each pressure transducer is connected to a voltmeter to measure its output. The transducers used have a *linear* 1.01 V to 6.05 V DC range (corresponding to a range of -5.0 in.H_2O to +5.0 in.H_2O). Therefore, the transducer output will be 3.52 V when the pressure difference is 0 in.H_2O. Any negative differential pressure will be less than 3.52 V. It is important to remember that both the inclined manometer and the transducers measure the difference in the pressure between the two lines connected to them. It is your responsibility to ensure that all of the pressure lines are connected in an appropriate manner.

7.3 Measurements

At the start of the lab, check to see that all of the lines are set up properly (or connect them if they are disconnected). We will use one of the pressure transducers connected hydraulically in parallel with the inclined manometer to measure the pressure difference from the pitot-static probe. The other transducer will measure the pressure difference between the pressure tap on the cylinder and an adjacent static port. The static port for the cylinder can be located on the side wall of the test section, just above the cylinder.

When you are confident that everything is connected correctly, check that the power strip at the back of the test section is turned on as well as the two voltmeters and the pressure transducer power supply. Adjust the voltmeters to the appropriate scales. With the tunnel off, what should the voltmeters be displaying? If the output is not what you expect, be sure to make a note of it so that you can account for the bias later when reducing the data. Now, check the level at the top of the inclined manometer and adjust the manometer until you are satisfied that it is indeed level. If necessary, zero the manometer by loosening and sliding the scale until the bottom of the meniscus is set at zero.

Before you start taking data, you must record the room temperature and pressure using the Princo barometer. Record the room temperature in °C and the pressure in in.Hg. Also record the % correction factor. This factor corrects for the thermal expansion of the metal scale that is used to determine the pressure. For example, the correction factor is 0.385 05 % at 22 °C. So, the actual pressure equals the recorded pressure times (1 - 0.003 850 5). You will use the actual pressure and temperature values later to compute the density of the air in the lab assuming ideal gas behavior. Subsequently, you will need the density value to compute velocities and the Reynolds number.

First you will calibrate the wind tunnel RPM indicator with respect to the wind tunnel velocity. You will do this by setting the tunnel fan at various RPM and record

Tunnel RPM	Manometer ΔP (in.H_2O)	$Transducer_{pstube}$ (V)
100		
200		
300		
400		
500		
600		
700		
800		
900		

Table 7.1: Tunnel velocity calibration

the pressure difference measured using both the inclined manometer and the pressure transducer connected to the pitot-static tube. First start the wind tunnel fan. To do this, follow the directions on the control panel stand. Make sure the circuit breaker is turned to on and push the start button. Set the RPM indicator to 100 and wait a minute for the tunnel to come to steady state. Record the pressure difference indicated on the inclined manometer and the voltage from the voltmeter connected to the output of the pressure transducer that is connected to the pitot-static tube in Table 7.1. Proceed through all the RPM settings on the table, recording the data. Repeat several RPM measurements to assure reproducibility. As a check while you are taking data, convert a recorded pressure transducer voltage to in.H_2O. Are the inclined manometer and pressure transducer readings in agreement? They should be.

Now you will perform pressure measurements on the cylinder. Check that the cylinder's pressure tap orientation is at 0 degrees as indicated on the rotating position indicator. Now set the tunnel RPM to that RPM specified for your group. Record the dynamic pressure from the pitot-static tube from both the voltmeter and the inclined manometer in Table 7.2. Then, in increments of 10°, rotate the cylinder and record the output from the pressure transducer connected to the cylinder. Also record the voltage from the pressure transducer connected to the pitot-static tube. This reading is a good indication of how constant the wind tunnel velocity is during your measurements. When you have circled through the entire 360°, go back and make a couple of spot checks at various angles to check repeatability.

When you are finished collecting the data, turn the dial indicator on the wind tunnel fan control panel back to zero and stop the tunnel. Again, follow the instructions on the control panel. Finally, make sure that all equipment is turned off and that everything is as you found it when you began.

Theta (°)	$Transducer_{cylinder}$ (V)	$Transducer_{pstube}$ (V)
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		
110		
120		
130		
140		
150		
160		
170		
180		
190		
200		
210		
220		
230		
240		
250		
260		
270		
280		
290		
300		
310		
320		
330		
340		
350		

Table 7.2: Cylinder pressure measurements

Measurand	**Units**	**Uncertainty**
Room Air Temperature		
Room Air Pressure		
Manometer Differential Pressure		
Pitot-static Tube Transducer Voltage		
Cylinder Transducer Voltage		
Pitot-static Tube Differential Pressure		
Cylinder Transducer Differential Pressure		
Cylinder Pressure Tap Angle θ		
Tunnel RPM		

Table 7.3: Measurand uncertainties

Result	**Units**	**Uncertainty**
Room Air Density		
Tunnel Velocity		
Re		
C_p		
C_L		
C_D		
D		

Table 7.4: Result uncertainties

7.4 What to Report

Once again, you are to submit your report in the form of a technical memo. Be sure to include (as a minimum) the following information:

- Calculations of the air density, operating tunnel velocity (both in SI units) and the Reynolds number.
- Calculation of the temporal precision error ($= S_x/\sqrt{N}$) of the pitot-static tube pressure transducer voltage taken at the various θ during your cylinder measurements. Compare this value to the mean value (that is, determine the percentage of the precision error with respect to the mean value). Ideally, this should be zero if the wind tunnel velocity remained constant during your measurements period.
- Two plots of the wind tunnel RPM calibration, one with the velocity (in ft/s) calculated from the measured in.H_2O from the inclined manometer and the other with that calculated from the pressure transducer voltage. Plot each manometer measurement or voltage along the y-axis versus the tunnel RPM along the x-axis. Perform the necessary regression analysis and display the proper error bars. In performing your regression analysis, keep in mind the actual relationship between differential pressure and velocity.
- Plot of the pressure coefficient, C_p, on the y-axis as a function of azimuthal angle, θ, on the x-axis (include on this plot the analytical, inviscid solution for comparison - see the Supplemental Information section)
- Calculations of the lift and drag coefficient of the cylinder, C_L and C_D, and the drag force on the cylinder, D, in units of N. Does this calculated drag force appear reasonable?
- Uncertainty estimates presented in the form of tables supported by example calculations. Two tables are required. Their format is presented in Tables 7.3 and 7.4. The first table concerns the measurand uncertainties and the second the result uncertainties. Supporting calculations of all of your uncertainty estimates should be contained in an appendix.

Calculations of the lift and drag coefficients will require you to perform some numerical integrations of your C_p data. The appropriate equations are included in the next section. You can use a spreadsheet to perform a simple trapezoidal rule integration. What should be the C_L value of the cylinder? If you know the answer, you can use your C_L calculation to check your calculations.

Some possible areas of discussion for this lab might include: What does the analytical solution predict for a drag coefficient? Does your experimental value confirm this? What might some possible reasons be for this? Try to think of the assumptions made in the analytical solution. Are these assumptions valid?

7.5 Supplemental Information

7.5.1 Velocity Calculation

If one makes the assumption of incompressible, inviscid, irrotational flow, then the complete form of the momentum equation reduces to what is commonly referred to as Bernoulli's equation,

$$P - P_\infty = \frac{1}{2}\rho u_\infty^2, \tag{7.1}$$

where $P - P_\infty$ is the pressure difference measured by the pitot-static tube, ρ is the density, and u_∞ is the free stream velocity. Solving for u_∞ gives

$$u_\infty = \sqrt{\frac{2\Delta P}{\rho}}. \tag{7.2}$$

This relation can be used to calculate the wind tunnel velocity for a given RPM setting. This relation is also used to estimate the uncertainty in the calculated velocity based on your experimental uncertainties in both the density and pressure difference. Using the root-sum-square method for combining uncertainties, the appropriate formula for this velocity calculation would be

$$\partial u_\infty = \sqrt{\left(\frac{\partial u_\infty}{\partial \Delta P} u_{\Delta P}\right)^2 + \left(\frac{\partial u_\infty}{\partial \rho} u_\rho\right)^2}. \tag{7.3}$$

7.5.2 Reynolds Number

By now the definition of the Reynolds number should be well known to all of you, it is repeated here simply as a reminder:

$$Re = \frac{\rho u_\infty D}{\mu}, \tag{7.4}$$

This is a Reynolds number based on the cylinder diameter, D, the free stream velocity, u_∞, the density, ρ, and the absolute (dynamic) viscosity, μ. For air, the absolute viscosity is given by the equation

$$\mu = \frac{b \cdot T^{3/2}}{S + T}, \tag{7.5}$$

where μ is in units of N·s/m^2, T in K, $S = 110.4$ K, and $b = 1.458 \times 10^{-6}$ kg/(m·s·K$^{1/2}$).

7.5.3 Pressure Coefficient

The pressure coefficient (C_p) is defined as

$$C_p = \frac{P_\theta - P_\infty}{\frac{1}{2}\rho u_\infty^2}. \tag{7.6}$$

The pressure difference in this equation is the ΔP measured for each individual rotation angle, θ. Thus, for every angle, you will be able to calculate a C_p value. At $\theta = 0$, a stagnation point exists, for which $C_p = 1$. If you were to calculate C_p based on the dynamic pressure measured by the pitot-static tube upstream of the cylinder, you might find that at $\theta = 0$ your C_p value was slightly less than one. This mainly is due to a small pressure drop down through the tunnel between the pitot-static tube and the cylinder. In calculating the C_p values from your data, it is often easiest to simply assume $C_p = 1$ at $\theta = 0$, and calculate the rest of your C_p values based on your $\theta = 0$ pressure measurement. That is, the corrected C_p value is given by

$$C_{p,corrected} = \frac{C_{p,\theta}}{C_{p,\theta=0}} = \frac{V_{trans,\theta}}{V_{trans,\theta=0}}, \tag{7.7}$$

where V_{trans} denotes the voltage of the pressure transducer after being corrected for its offset voltage at zero velocity. Equation 7.7 is valid because the differential pressure is related linearly to the transducer voltage after the offset correction.

The derivation of the analytic solution for C_p for this situation can be found in a standard aerodynamics text book. The final result is

$$C_p = 1 - 4\sin^2\theta. \tag{7.8}$$

7.5.4 Lift and Drag Coefficients

The formulas for the lift and drag coefficients of a circular cylinder in cross-flow are derived in detail in Anderson. The results are presented here:

$$\begin{aligned} C_D &= -\frac{1}{2}\int_0^{2\pi} C_p(\theta)cos(\theta)d\theta \text{ and} && (7.9) \\ C_L &= -\frac{1}{2}\int_0^{2\pi} C_p(\theta)sin(\theta)d\theta. && (7.10) \end{aligned}$$

As mentioned above, these can be calculated for your data using a numerical integration algorithm. You can use the `trapz` function in MATLAB® (the command z=trapz(x,y) computes the integral of y with respect to x using trapezoidal integration, where x and y are vectors of the same length). Once C_D is known, the actual drag force on the cylinder can be found using C_D, the dynamic pressure and the frontal area of the cylinder.

Chapter 8

The Digital Oscilloscope and Function Generator

8.1 Objectives

The objective of this laboratory exercise is to introduce you to the capabilities of a function generator and digital oscilloscope, and their use in basic measurements.

8.2 Instrumentation

Only two instruments will be used in this exercise:

- Hewlett Packard HP 33120A Function Generator
- Fluke PM3380-A CombiScope [Analog and Digital Oscilloscope]

8.3 Measurements

In this laboratory exercise you will begin to learn about the capabilities of a function generator (FG) and a digital oscilloscope (DO). You will use the DO to observe and analyze various signals produced by the FG. You will also study the triggering capabilities of the DO.

The FG is an electronic instrument that generates waveforms of preset shape, amplitude and frequency. Often it is used in a laboratory setting to provide a known input to data acquisition devices such as the DO or a computer. This helps the investigator debug and calibrate measurement systems. The DO is perhaps the most used piece of electronic equipment in a laboratory. It is the experimenter's electronic "eye". It has the basic capability to acquire, store, display and analyze signals, and download them to other devices. The typical DO has at two amplifiers (with variable gains), a sample and hold circuit and an analog-to-digital (A/D) converter. The digitized signal is stored in its random access memory (RAM), and the output is sent to the video display. Most digital scopes use a single CCD (charge coupled device) array per channel to sample the signal and hold that value until the A/D has time to convert the signal. Once data has been acquired by the DO, it can be overwritten to display a new signal or it can be saved and analyzed using on-board software programs. Also, it can be downloaded to another device such as a plotter or computer for further analysis.

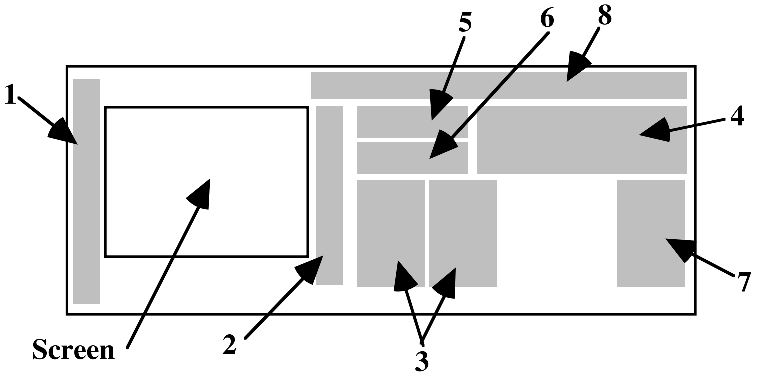

Figure 8.1: Schematic of the front panel of the PM3380A CombiScope.

The front panel of the DO is divided into seven functional areas, as shown in Figure 8.1. First, examine these areas, referring to their functions listed below:

Area 1: Basic screen and power controls with self-explanatory labels.

Area 2: Screen text control buttons and menu buttons.

Area 3: Basic controls for input Channels 1 and 2. There are controls for amplitude scaling (volts per division), positioning the signal, establishing the signal coupling (AC, DC or GND), turning the channel ON or OFF, scaling the signal, AUTO RANGE, and determining whether the signal will be increasing or decreasing for triggering. Each area also has two additional keys that have special application. The first is the VERT MENU key (which you will not use at this time) and an AVERAGE key which averages the signals on both channels simultaneously. The last key is the INV key which is applicable to Channel 2 only (it inverts the signal).

Area 4: The time and trigger control section for the main time base (to be presented later). Again, there is an AUTO RANGE control plus controls for time scaling (per division), trace position, magnification and several trigger controls that will be discussed later.

Area 5: The cursor control section. The TRACK control knob has a dual purpose. If it is used for measurements of a voltage versus time trace, it sets the reference x-cursor, while the delta control knob (the one with the Δ above it) positions the measurement x-cursor. A reading of the value of the reference x-cursor or the difference between the x-cursors (in volts, time, or both) is provided in text on the screen if chosen. The TRACK control also is used for selection in the menu items and the requirement for their use is indicated by a small ‘T’ inside of a circle.

Area 6: The delayed time base control area has a special application that will not be covered in this exercise.

Area 7: External trigger section which allows one to use a signal of choice as the control for initiating acquisition of data other than the signals of either Channels 1 or 2 or the line signal.

Area 8: This is the extended function area. The simplest function is the AUTOSET button. This automatically finds your signal and adjusts the settings to produce a properly-proportioned signal on the screen. The other buttons provide the user with a host of powerful built-in functions of math, measurement and presentation.

Lastly, there are the hard wired inputs along the bottom that are clearly labeled.

Now let us begin to get some experience using the FG and DO.

Part 1: Viewing a periodic signal on the DO.

Connect the OUTPUT of the FG to Channel 1 of the DO. Set the FG to deliver a square wave having a frequency of 150 Hz and VPP (peak-to-peak) amplitude of 4 volts as observed on the DSO (which is 2 VPP on the FG). (NOTE: The VPP amplitude set on the FG appears as twice that amplitude on the DSO. This is because of an impedance mismatch which we won’t deal with at this point). Make sure that the DO is in digital mode by pressing the yellow ANALOG button which toggles between the two modes and indicates briefly the mode you’re in on the screen. The trace should show one, or at most two, complete cycles of the signal while maximizing the vertical display. Do NOT change the FG setting from what you initially set. Use the DO’s vertical gain control, specified in units of volts (implying volts per division), and horizontal time control, specified in seconds (meaning seconds per division). Center the trace vertically using the position control for that channel. Place the start of the trace on the left edge of the screen grid by adjusting the X POS knob. Record the following using visual observation (not using the cursors).

1. Vertical scaling per division (volts peak-to-peak):

2. Time scaling per division (s):

3. Frequency (Hz=cycles/s=1/time for one cycle):

Did the display show an actual square wave, top and bottom parallel with the horizontal grid with little connection between the two (very faint compared to the horizontal lines)? If not, correct it by changing the signal coupling on the DO. Immediately to the right of the vertical scaling value displayed on the screen is a = sign for DC, a $\sim$ sign for AC and a $\perp$ sign for ground. Observe the signal first with DC coupling and then with AC coupling. Sketch each of the two traces.

Now repeat the measurements using the cursors. To do so, simply press the CURSORS key. Select the second of the =, ‖, #, or 'auto' choices. Then select and follow the READOUT menu, selecting reading of V1 (voltage measured using first cursor) and 1/ΔT (frequency). Use the cursors to determine the following:

4. Minimum voltage (volts):

5. Maximum voltage (volts):

6. Maximum - Minimum voltage (volts):

7. Frequency (Hz):

Now we will examine more closely the function of the types of couplings, AC, DC and GND. First set the coupling on the DO to GND. This grounds the input to the channel, resulting in a horizontal line trace at 0 volts. Next set the coupling to DC. Note the shape and position of the trace. Now add a DC offset of +0.5 volt on the FG to the generated signal. Adjust the scaling or position of the signal on the DO as necessary to keep it in view. What has happened to the trace?

Now, change the coupling to AC. What change in the display occurred? Sketch the DC and AC coupled traces below.

Often the signal being observed has a midpoint, zero voltage level, that is not as obvious. Then, it's necessary that you establish a zero reference line. On the DO, this reference is arbitrary. To establish a zero reference, simply ground the input signal. Establish a zero reference on the center horizontal graticule from the bottom by rotating the vertical position knob and then unground the signal.

Now go back and observe your original signal with DC coupling. Is it centered on your reference line? Yes or No? Now set the FG offset back to 0 volts. Is it centered on the reference line? Yes or No? Try this also with AC coupling with and without offset. How are these different than with DC coupling?

Congratulations! You have just become familiar with the most basic measurement function of the DO, which is viewing a periodic signal and performing a basic quantitative analysis of the signal. A simpler approach available is to use the AUTOSET button, which, based up some preset criteria, will select the appropriate time and amplitude scaling and coupling. Another function is the magnification function, MAGNIFY. Before using this, change the time base to get approximately 20 cycles on the screen without changing the FG settings. Now press the right directional arrow MAGNIFY button. Notice that a horizontal bar temporarily indicates what portion of the trace in memory is currently being displayed. Now keep pressing the right arrow MAGNIFY button until you get only one to two cycles on the screen. The amount of time amplification is indicated temporarily on the screen by a * followed by a number indicating the amount. Note that the signal does not look like a perfect square wave any more. This is because the signal stored in the RAM consists of a fixed number of digital points. Magnifying it produces a signal constructed by drawing lines in between the points. Finally, using the left directional arrow MAGNIFY button, bring the time amplification back to 1.

Part 2: Using the trigger on the DO

In most situations, the user of an oscilloscope just wants to see what a continuous signal looks like. Therefore, the DO is normally operated in a continuous sweep mode. However, there is something happening in the background that is not readily apparent to the user. In most instances, if the user has selected a time scale that allows viewing of several cycles of a standing wave, it appears that the signal is starting at the same point in its cycle. This is because the DO, in its default mode, is being triggered by the signal itself. The trigger on a DO determines when the trace will begin. On most DOs, there are two conditions that must be met for a trigger to occur: a specified voltage relative to ground and a direction of change. In addition, there are several choices for the source of the trigger, be it the signal into Channel 1 or 2, an external signal, or the (power) line signal (in this country, a 60 Hz signal). In all cases, the trigger initiates the trace which then continues until it is completed. Then, based upon the trigger options selected, the trace will start over on the next trigger received, stop all together, or execute some other user selected option.

To explore the trigger, let's start with something already familiar. Establish a square wave as done before with the same characteristics (150 Hz, 4 volts peak-to-peak on the DO, no DC offset). Check your settings (press the STATUS button).

The first thing is to observe the effect of direction of change, or slope, on the trace output. In the control box of the channel selected for input, press the TRIG button and observe the change. The slope direction is indicated on the far right, bottom corner of the screen. If you wish to see what happens in the setup as you do this, keep the STATUS text on screen during the process.

Next, in the time and trigger control section, press the TRIGGER button. Scan through the menu to see its options. Note that when you change the trigger source (the second item down), an important thing happens. If the trigger source and the input signal do not have a common frequency, the trace is no longer stable. For each trigger source, observe the stability of the trace. Now connect the SYNC of the FG to the EXT TRIG INPUT of the DO. Go back and put the trigger source on 'ext.trig'. The square wave should be stable now on the screen. This is because the FG sends out a sharply rising pulse at the instant the square wave begins, which is an excellent source for an external trigger.

Now, set the trigger for edge, ch1, level-pp off, noise off, and ac coupling. Then press the TB MODE button (Time Base) and toggle through the selectable trigger options for the top item of the menu. Note that when on 'single', the trace freezes. As the name indicates, it will select a trigger only *once* in this mode. To obtain a new trace, press the SINGLE button in the time and trigger section. As you do this, note that a red light to the right of the button briefly comes on. When it is lit, a trigger is armed (indicating that a trigger signal has not been received). After the trigger occurs, the light goes off.

As stated earlier, the DO is normally used to observe periodic signals and the trigger not as much. But, with the trigger option, the DO can be a valuable data acquisition and analysis tool. Consider the following situation. Assume that a signal from a single-event test (e.g., the firing of a rocket motor) is expected to have an amplitude of 1.5 to 5 volts and a duration of 1.5 to 6 ms. To see the complete event from start to finish you would like to view the trace starting 1 ms before the event occurs and ending 1 ms after it ends.

To do this, we will set up a known signal having an initially rapid amplitude change, a square wave. Change the settings on the FG to obtain a peak-to-peak amplitude of 2.5 VPP on the FG display with a DC offset of 1.0 volts. Set the FG frequency to 250 Hz to obtain a period of one square wave equal to 4 ms. Be sure that the TB MODE is in 'auto'. Verify these requirements using the cursors and record the information below.

8. Maximum voltage (volts):

9. Minimum voltage (volts):

10. Period of one cycle (ms):

Note that if the trace is moving in an apparent random fashion, you can stop the action by pressing the RUN/STOP button in the time and trigger section. If any changes are made to the generated signal, you must allow the DO to run to allow the changes to be displayed. Another method is to use the MEASURE function with MEAS 1 set to measure 'pkpk' and MEAS 2 set to 'freq'. Both must be turned on. Once everything is set, press the MEASURE button again to turn the menu display off.

Choose a time scale to obtain at least one period of the signal plus 1 ms before and after. Record this below.

11. Time scale chosen (ms per division):

Remember, the DO provides ranges in the 1, 2, 5 format. Now set the pretrigger (the time-record length prior to receiving the trigger). To do this, turn the TRIGGER POSITION knob until the appropriate reading (this will be indicated by a '-dv' on the screen). The position will be shown by a small triangle only for '-dv's. All data gathered to the left of this symbol occurs prior to the trigger; all the data to the right occurs after the trigger. The time to the left of the symbol should be 1 ms. Now the trigger level must be set. First, make sure that the conditions of the trigger used above are set, namely edge, ch1, level-pp off, noise off, and ac coupling. Set 'auto' in the TB MODE. Determine the lowest trigger level that will produce an active, stable trace (constantly being refreshed and remaining steady) by turning the knob labeled TRIGGER LEVEL. The value will be indicated by 'Level=' on the screen. Record the value below. Now find the highest trigger level and record its value below.

12. Lowest trigger level (volts):

13. Highest trigger level (volts):

14. Difference between trigger levels (volts):

Now repeat the process using a DC coupling on the trigger. Record the values below.

15. Lowest trigger level (volts):

16. Highest trigger level (volts):

17. Difference between trigger levels (volts):

Now repeat the process again for DC coupling but with no DC offset on the FG. Record the values below. (The AC coupling case would be the same as before, so we will not repeat it).

18. Lowest trigger level (volts):

19. Highest trigger level (volts):

20. Difference between trigger levels (volts):

How do the *ranges* of the trigger levels compare for the three cases? How do the absolute trigger levels compare for the three cases?

Next, with a properly triggered signal, press the SINGLE button in the time and trigger section of the front panel. If all was done correctly, you will get a stationary trace of the square wave signal. If you do not, go back and repeat before proceeding to the next step.

Now return the DO to 'auto' under the TB MODE. Set the FG to display an amplitude setting of 2 VPP, a frequency of 1 Hz and no DC offset. The low frequency will allow you to see the actual trigger occur. Adjust the amplitude setting to 1 volt DC and the time base setting to 100 ms. Center the trace on the screen by establishing a zero reference. View the signal. It should be repeating over and over in time on the screen. Now set 'single' in under TB MODE and 'ch1', 'level-pp off' and 'dc' under TRIGGER. A 'T-' should be on the screen, indicating the amplitude level of the trigger. Move it up and down by rotating the TRIGGER LEVEL knob, then finally position it about one division above the top level of the square wave. Press the SINGLE button in the time and trigger area. You should see only a horizontal line on the screen, indicating that the signal has not triggered. The red 'ARM'D' light should be on. Now gradually rotate the knob slowly to bring down the trigger level while watching the 'Level=' value on the screen. Observe and record below the value indicated when the signal triggers. If you want to, you can repeat the triggering process by pressing the SINGLE button to arm the trigger and then moving the knob in smaller increments to get a better estimate of the trigger level. Now move the indicator below the bottom of the signal and find the minimum value (by moving the indicator up) where the signal will trigger. Record the value below. Thus, determine the range of the trigger level that will properly trigger the DO for this square wave.

21. Lowest trigger level (volts):

22. Highest trigger level (volts):

23. Difference between trigger levels (volts):

8.4 What to Report

Turn in this document with all questions answered. No technical memo is required for this exercise.

Chapter 9

Digital Data Acquisition

9.1 Objectives

The objective of this laboratory exercise is to investigate several aspects of digital data acquisition using both a digital oscilloscope and a computer data acquisition system. Specifically, you will learn several ways to acquire, store and analyze various waveforms and also examine some of the limitations of digital data acquisition.

9.2 Instrumentation

The following instruments will be used in this exercise:

- Hewlett Packard HP 33120A Function Generator
- Fluke PM3380-A CombiScope [Analog and Digital Oscilloscope]
- Compaq 386 computer with a United Electronics Incorporated (UEI) 12-bit, 16 channel A/D board for -5 V to +5 V input, and associated software

9.3 Measurements

This laboratory exercise is divided into three sections. The first considers the use of a computer data acquisition system to digitally sample a simple periodic wave of known frequency. The second and third involve the use of the digital oscilloscope to capture, store and subsequently analyze various waveforms.

9.3.1 Sampling a Periodic Waveform Using a Computer Data Acquisition System

For this section you will use the function generator (FG) to generate a simple sine wave. The output of the FG will be sent to both the digital oscilloscope (DO) and the computer data acquisition system (DAS). You will use the FG and DO to ascertain the amplitude and frequency of the wave. You will view the waveform acquired by the DAS using the graphical display of a software package. Then, you will vary the sample rate of the DAS and record the waveform frequency. This will allow you to investigate the relation between the sampling frequency and the frequency of the input wave (and thus to observe the effects of signal aliasing).

To begin, check to see that the BNC cable from the FG to the DAS is DISCONNECTED (input signal amplitudes greater than 7V will destroy the input circuitry of the A/D board). Also check to see that the FG output is CONNECTED to channel 2 of the DO.

Using the FG, generate a 500Hz sine wave (that is, $f_1 = 500Hz$) with 100mv peak-to-peak with zero DC offset. Press the light green AUTOSET button on the DO to view the waveform. Acquire a single trace by pressing the SINGLE button in the time base settings area. This freezes a single trace on the display. Using the cursors (press the CURSORS button), record the observed signal frequency and peak-to-peak amplitude. Record this below.

1. DO frequency (Hz):
2. DO peak-to-peak amplitude (mV):

Once you are confident that the signal's amplitude does not exceed 1V, connect the DAS BNC cable to the T-connector at the FG output. Now you are ready to use the DAS to acquire and view the input signal. From this point on in this section the FG or DO settings remain the same. All you will do is vary the sampling frequency of the DAS and record its output.

Table 9.1 is provided for you to record and analyze the data acquired in this section. You will set the sampling rate on the configuration page of the software package and then measure Δt from the software package's graph. The second and third columns will be recorded during the lab. The fourth and fifth columns should be filled in **before** coming to lab. $f_{\Delta t}$ is computed directly from Δt, f_N is one-half of f_{sample} and f_{calc} from f_1 and f_{sample} (see our class notes on how to use the folding diagram to calculate the f_{calc} values). f_{calc} is the aliased frequency that you would expect to occur at f_{sample}.

Now you are ready to take data using the DAS. Open the data acquisition software by double-clicking on the UEI icon, then on the UEI Status for Windows icon. Under File select Load configuration and then lab8.cfg. Then under Analog select Configure. This brings up the data acquisition parameter display. You can set the sampling rate by typing it in or selecting it (if available). The samples per channel should be set (and remain at) 128. The duration is simply the samples per channel divided by the sampling rate (when only one channel is used, as in this case). After the desired sampling rate is set, press enter, then the F7 key to start the acquisition process. When completed a graph will come up on the screen displaying the acquired signal. Remember that this is the digital representation of the signal, so it will not always look exactly like the input signal.

Observe the graph. You will see that you have acquired many periods of the wave. Now examine only a few of the periods (between 2 and 5) by using the expansion icon shown immediately under the word Graph. Select a region of interest by dragging the mouse and then clicking it. The selected region should now occupy the entire graph.

$\mathbf{f_{sample}}$ (Hz)	$\mathbf{\Delta t}$ (ms)	$\mathbf{f_{\Delta t}}$ (Hz)	$\mathbf{f_N}$ (Hz)	$\mathbf{f_{calc}}$ (Hz)
20,000				
1500				
1200				
1000				
800				
600				
400				
200				
120				

Table 9.1: DAS Sampling Data

Next, position the two cursors by single clicking the mouse at each of two points on the graph. Try to position the cursors at the same position on the wave (e.g., its top) over several periods such that you can obtain and record a more accurate value of the Δt for ONE period. Record the Δt for $f_{sample} = 20000$ Hz in the table and then enter its peak-to-peak amplitude immediately below.

3. DAS peak-to-peak amplitude (in mV) at $f_{sample} = 20,000$ Hz:

4. How does this value compare to the DO peak-to-peak amplitude recorded above?

Now go back to the parameter display and change the sampling rate. Repeat this process until you have investigated all of the listed sampling rates and have filled in the raw data values in the table. When you are done, exit the UEI software. Also, **disconnect** the DAS BNC cable from the T-connector. 5. When you have completed all of the columns in Table 9.1 after lab, state immediately below how f_{calc} and $f_{\Delta t}$ compare for each f_{sample}:

9.3.2 Examining the Frequency Spectra of Several Waveforms Using a Digital Oscilloscope

Now set the FG to deliver a 1 kHz sine wave with a 100 mV peak-to-peak amplitude with zero DC offset. Press the AUTOSET button on the DO and then adjust the settings on the DO to display around 10 cycles on the screen. Acquire a single trace by pressing the SINGLE button in the time base settings area. This provides a “frozen” signal to perform our FFT on.

Enter the MATH menu by pressing the MATH button along the top of the scope. Then under the MATH 1 feature select “fft” and “ch2”, then press ENTER. Then

Wave Form	**Frequency** (kHz)	**Amplitude** (dB)
Sine Wave	–	–
Peak 1		
Square Wave	–	–
Peak 1		
2		
3		
4		
5		

Table 9.2: Amplitude-Frequency Data for Sine and Square Waveforms

select "on" (the Fast Fourier Transform [FFT] of the sine wave should appear on the screen) and "no" for DISPLAY source (this will remove the sine wave trace from the display). You are now (hopefully) looking at the FFT of the input sine wave. Press the CURSORS button and then use the cursors to measure the frequency and amplitude (in dB) of the largest peak in the frequency spectrum. Note that the scope displays the amplitudes by referencing all of the values to the largest peak present. In essence, the amplitude at each peak corresponds to the Fourier coefficient (the A_n or B_n) at that frequency (see our class notes). If we define the amplitude of the largest peak as A_1 and that of a subsequent i-th peak as A_i, then the value reported by the scope (in dB) is found from the definition of the decibel

$$dB = 20 \log \left(\frac{A_i}{A_1} \right). \tag{9.1}$$

Thus, it can be seen that the largest peak will be reported by the scope as having a value of 0 dB because, for this case, $A_i = A_1$. Further, if we know that actual value of A_1, we can compute the values of each of the A_i's using the above equation. Also remember that the DO displays digital information, so there may be two adjacent frequencies having maximum amplitudes. In this case, the actual frequency at maximum amplitude lies in between the two frequencies. Record the single frequency value at maximum amplitude (or the average value if there are two local maxima) in Table 9.2. Is this frequency what you expected?

Press the AUTOSET button to start over and then repeat the above procedure for a square wave (the second-from-the-left button on the FG) for the same frequency, peak-to-peak amplitude and zero DC offset. Record both the frequencies and the amplitudes of the first five major peaks in the spectra in Table 9.2. After lab, compute the amplitudes (in dB) of the first five major peaks in the spectra of a 1 kHz square wave. The amplitudes (in dB) can be found in the same manner as done in the class notes for a step function. Compare these calculated amplitudes with those obtained above. If there are any differences, explain what could be the cause(s).

9.3.3 Sampling an Aperiodic Waveform Using a Digital Oscilloscope

In this part of the exercise, you will use the digital oscilloscope to capture a transient waveform. The event that we wish to record is the oscillatory response of the cantilever load cell to an impact loading. This will be accomplished by dropping a golf ball into the can at the end of the cantilever beam.

To start off, make sure that the load cell is connected to the bridge circuit correctly. The panel meter wire with end-connector should be connected to the output of the second op amp on the bridge circuit box. As you have done many times before, balance the bridge to zero by adjusting the ZERO ADJUST knob on the panel. Test to make sure the load cell is connected properly by lightly depressing the beam and ensuring the panel meter is responding (you should get approximately 0.1 to 0.2 V indication).

Check that the BNC output of the bridge circuit is connected to Channel 1 of the DO. Set the scope to the following settings: DC coupling, 0.2 V, 100 ms. This will ensure that you will capture the full signal. Now, go into the TRIGGER menu and set "edge", "ch1","level-pp" to off, and "dc". If "ch2" appears instead of "ch1", simply press the TRIG 1 button in the Ch1 area on the panel. Then "ch1" should appear in the TRIGGER menu. The trigger level should now be marked on the scope with a "T". Set it at about one division above the centerline of the display using the TRIGGER LEVEL knob (at the far right of the DO panel). Now set a delay for the trigger such that a part of the signal prior to the trigger event will be displayed. Do this by turning the TRIGGER POSITION knob counterclockwise. A small Δ should appear on the screen. Set it at approximately -1.00 dv. Finally, press the TB MODE button and select "single". Then press the TB MODE button again to exit that menu. Depress the SINGLE button on the scope such that the red arming light comes on and the scope is "waiting" for the event to trigger. You may have to adjust the trigger level knob slightly higher such that the scope does not trigger off of electronic noise.

Once you are sure that the scope settings are correct and that the trigger level is set properly, arm the scope again (if needed) by pressing SINGLE. Now, take the golf ball and drop it into the can from a height just above the top of the can. Did the scope trigger? Were you able to capture the signal as you wanted? If you aren't happy with the signal you got, adjust the scope settings until you are confident that you have a good oscillatory response from the beam.

The response of the beam should be an oscillation damped in time. When hit, the beam vibrates at its natural frequency, which can be measured using the strain gauge and Wheatstone bridge configuration. As you did in the previous section, use the math function to calculate the FFT of the trace you just acquired. What is the dominant frequency in this signal?

Set the MATH PLUS menu to "off" and then press the MATH button to exit that menu. The stored trace of the signal should be the only item remaining on the screen.

Now download the data to the laboratory computer to save the information in a text file. Then, using the text file, plot the data to reproduce the trace as you saw it on the scope screen. Also determine and plot the amplitude-frequency spectrum of the signal.

9.4 What to Report

Turn in this document, being sure to include the plots requested, answers to all questions posed, and the calculations of the square wave amplitudes and frequencies for comparison with the measured values.

Chapter 10

Dynamic Response of Measurement Systems

10.1 Introduction and Objectives

The main objective of this laboratory exercise is to investigate the dynamic response characteristics of first-order and second-order measurement systems. First, the dynamic responses of two different-size thermocouples (first-order systems) to step input changes in temperature will be studied. Then, the second-order system dynamic response characteristics of a RLC circuit to a sinusoidal input will be investigated. All data will be acquired, stored and analyzed using a digital oscilloscope.

10.2 Instrumentation

A schematic of the set-up for part 1 is shown in Figure 10.1. The instrumentation consists of a thermocouple (TC), an ice bath (IB), a thermocouple reference junction and amplifier (TRJA) and a digital oscilloscope (DO).

- Digital Oscilloscope
- Two type-K (chromel-alumel) thermocouples of different size
- Analog Devices AD595AQ type-K thermocouple reference junction, linearizer and amplifier chip in a box
- Ice bath (beaker filled with crushed ice and water)

A schematic of the set-up for part 2 is shown in Figure 10.2. The instrumentation consists of a function generator (FG), a RLC circuit box (RLC) and a digital oscilloscope (DO).

- Function Generator
- Digital Oscilloscope
- RLC circuit box

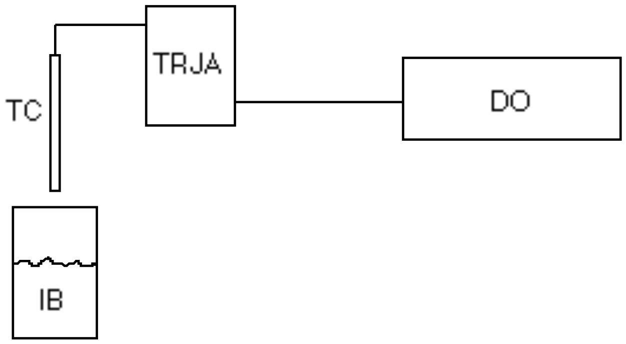

Figure 10.1: First-order system response experimental set-up

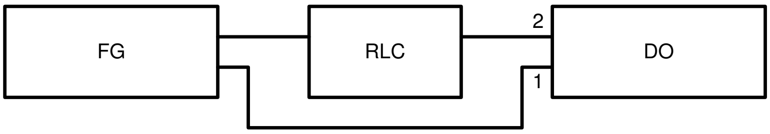

Figure 10.2: Second-order system response experimental set-up

10.3 Measurements

10.3.1 First-Order System Response

In this part of the lab exercise you will use the DO to determine the time constants of two thermocouples. A thermocouple is a passive temperature sensor. It consists of two dissimilar wires connected together at two junctions, namely the hot junction and the cold junction. When one junction is hotter than the other, an emf (electro-motive-force: a voltage difference) is developed between the two junctions. This voltage difference is proportional to the temperature difference between the two junctions and is generally in the millivolt range. The TRJA simulates a cold junction, amplifies the thermocouple output and linearizes it with the temperature at the hot junction at a 10 mv/C.

In fact, the thermocouple behaves as a first-order system. The equation that describes the energy exchange between the thermocouple's tip and the environment is

$$mC_v\frac{dT}{dt} = hA_s\left[T_\infty - T(t)\right], \tag{10.1}$$

where m is the mass of the tip, C_v is the specific heat at constant volume of the tip, A_s is the surface area of the tip, h the heat transfer coefficient, $T(t)$ the temperature of the tip with respect to time, and T_∞ the temperature of the liquid at "infinity". The solution to this first-order, linear differential equation is

$$T(t) = T_\infty + (T_o - T_\infty)\exp\left(-t/\tau\right), \tag{10.2}$$

where τ is the time constant of the thermocouple, equal to mC_v/hA_s. This equation can be rearranged to yield

$$ln\frac{(T(t) - T_\infty)}{(T_o - T_\infty)} = -t/\tau. \tag{10.3}$$

A plot of the ln term versus time, t, will yield a line with the decreasing slope equal to $1/\tau$. The time constant is the characteristic measure of the thermocouple's rate of response. The time constant τ can be determined readily by exposing the thermocouple to a step input in temperature. In this section, you will expose two thermocouples (A and B) each having different-sized tips to an "instantaneous" (step) decrease in temperature (from room temperature to 0 $°C$).

To start, turn on the DO and the TRJA. Connect thermocouple A to the TRJA and the TRJA's output to channel 1 on the DO. Note that the TRJA gives a linear output of 10 mv/°C referenced from 0 mv at 0 °C. Thus, it's output voltage will decrease ∼ 200 mv for a ∼20 °C decrease in temperature. press auto set on the DO. You should see a steady line with some noise that reads the current temperature in the lab. Adjust the voltage scale on channel 1 to be 50 mv/division and adjust the time scale to be 250 ms for the thin thermocouple and 2.5 s for the thick one. This assures that you can capture the signal with good details. Adjust the ch1 vertical position knob such that the signal baseline is displayed just approximately one division *below* the top of the DO display.

Now set up the DO to trigger correctly in response to a step input forcing: Press the trigger menu button. Make sure to read edge, slope = falling (that is, the DO will trigger when the slope falls), source = ch1 (that is, the trigger function is looking for signal from channel 1), mode = single (that is, the trigger is waiting for a single event to occur), coupling = DC.

Move the horizontal position button to point on one division from the left side of the screen. This assures that you see the original level of the signal on one division on the left and the triggered signal on the remaining part of the screen. Move the trigger level knob to point at about 0.4 division below the signal level. This assures the the DO will not trigger until the signal slope falls to this level. (decreasing the 0.4 to a smaller value has the risk that the DO can be triggered with noise). Make sure that the thermocouple is away from the ice bath to avoid triggering the DO.

Immerse the thermocouple into the ice bath. After ∼30 s the screen should display the triggered signal starting from the room temperature to the ice temperature. Press the cursor button, adjust type = voltage (i.e., the cursors will be horizontal to measure voltage), source = ch1 and use the voltage cursors and the screen grid lines to take 10 readings of voltage versus time. Put these values in Table 10.1. The time constant is the time needed by the thermocouple to reach 63.2 % of the final voltage, which can be read directly from a plot of voltage versus time.

Another method to obtain the time constant uses a least-squares regression fit of the data. Because the thermocouple can be represented by a first order system, the voltage changes with time is governed by

$$V(t) = V_\infty + (V_o - V_\infty)\exp\left(-t/\tau\right). \tag{10.4}$$

No.	V_{tcA} (mV)	$Time_{tcA}$ (ms)	V_{tcB} (mV)	$Time_{tcB}$ (ms)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Table 10.1: Thermocouple response data

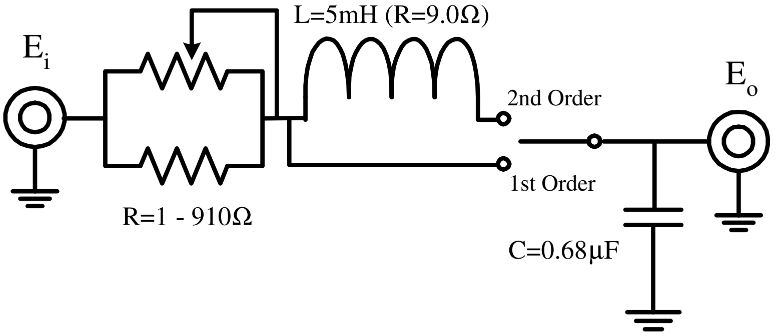

Figure 10.3: RLC Circuit Diagram

By taking the logarithm of both sides of Equation 10.4, the variables can be transformed such that a least-squares linear regression analysis can be performed. From this information the time constant can be determined.

Note any obvious physical differences between thermocouples A and B here:

Finally, please turn OFF the TRJA's power and disconnect the TRJA BNC from Ch1 of the DO when finished with this part of the exercise.

10.3.2 Second-Order System Response

In this part you will use a FG and a DO to determine the response characteristics (the magnitude ratio and the phase lag as functions of the input frequency) of an electrical "RLC" circuit. This circuit consists of a resistor (R), an inductor (L), and a capacitor (C) and has the response characteristics of a second-order system. The circuit will be characterized by providing an input sinusoidal wave of known amplitude and frequency from the FG to the RLC circuit and measuring the circuit's output amplitude and time delay using the DO, as depicted schematically in Figure 10.2.

The electrical diagram of the RLC circuit is shown in Figure 10.3. The input to the circuit is between the resistor and ground and the output is measured across the capacitor connected to ground. The resistor is the parallel combination of a 1 Ω-10kΩ variable resistor and a fixed 1kΩ resistor, yielding an effective variable resistance between approximately 1 and 910 Ω using a knob. The capacitance is fixed at 0.68 μF and the inductance at 5 mH. The passive resistance of the inductor is 9.0 Ω, so the lowest effective resistance that the circuit can have is approximately 10 Ω (9.0Ω+1Ω).

The voltage drops, V, across each component in an AC circuit are $V = RI$ for the resistor, $V = L\frac{dI}{dt}$ for the inductor and $V = Q/C$ for the capacitor, where $I = \frac{dQ}{dt}$. In this circuit, all three components are in series. Thus, application of Kirchoff's Voltage Law for the circuit gives

$$L(\frac{d^2Q}{dt^2}) + R(\frac{dQ}{dt}) + \frac{Q}{C} = E_i \sin(\omega t). \tag{10.5}$$

This second-order, linear differential equation can be solved for Q to yield the steady state output voltage amplitude

$$E_o = \frac{Q}{C} = \frac{E_i}{C\sqrt{\left[\frac{1}{C} - L\omega^2\right]^2 + [R\omega]^2}}. \tag{10.6}$$

From Equation 10.6 and the solution equation for Q, the magnitude ratio is

$$M(\omega) \equiv \frac{E_o}{E_i} = \frac{1}{\sqrt{\left[1 - (\frac{\omega}{\omega_n})^2\right]^2 + \left[2(\frac{R}{R_c})(\frac{\omega}{\omega_n})\right]^2}} \tag{10.7}$$

and the phase lag is

$$\phi(\omega) \equiv \tan^{-1}\left(\frac{2(\frac{R}{R_c})(\frac{\omega}{\omega_n})}{1 - (\frac{\omega}{\omega_n})^2}\right). \tag{10.8}$$

This equation yields positive values of $\phi(\omega)$. By convention, because $\phi(\omega)$ is a phase *lag*, it is plotted as having negative values. Further, for $\omega > \omega_n$, the phase shift must be referenced correctly. Thus, the conventional plot of $\phi(\omega)$ (in °) versus ω would actually be $-\phi(\omega)$ for $\omega \leq \omega_n$ and $-180° - \phi(\omega)$ for $\omega > \omega_n$. Also note that in Equations 10.7 and 10.8 the resonant frequency is given by $\omega_n = \sqrt{1/LC}$ and the critical resistance by $R_c = 2\sqrt{L/C}$.

To start, make sure that the output cable from the FG is attached to the input of the RLC box and in parallel to channel 1 on the DO. The output of the RLC box should be connected to channel 2 of the DO. In that way, you can view both the input and output signals of the RLC box on the DO. Make sure that the toggle switch is set to 2nd order. Now turn the R knob on the RLC box fully counter-clockwise

Freq. (Hz)	E_i (V)	E_o (V)	Δt (s)	ω (rad/s)	$M(\omega)$	$\phi(\omega)$ (°)
10						
100						
500						
1000						
1200						
1600						
2000						
2200						
2500						
2650						
2800						
3100						
3600						
4000						
5000						
7000						
10 000						

Table 10.2: RLC-high resistance response data

(to MAX). This sets the resistance in the circuit to its highest value, corresponding to a high "damping ratio". Then set the FG and the DO to their initial prescribed settings. These are, for the FG, sine wave with 100 Hz frequency, 4 volt peak-to-peak (V_{pp}) amplitude and no dc offset; for the DO, Chs 1 and 2, both ac with divisional settings of 2 V and 2 ms (change these settings if you need to or press auto set to let the DO select the best settings) .

Your data will be analyzed in the final form of $M(\omega)$ and $\phi(\omega)$, each versus the normalized frequency ratio, ω/ω_n. These values will be determined from the "raw" data that you take in lab. This includes the input and output amplitudes, E_i and E_o, and the phase lag time, Δt, which is the time between the peak of E_i and the corresponding peak of E_o. The phase lag in degrees equals $-(360°)(\Delta t/T_i)$, where T_i is the inverse of the input frequency in Hz and the minus sign indicates a lag in time.

Once you have captured a satisfactory set of signals on the DO display, use the cursors to record the data. Enter all the "raw" data in the first four columns in Table 10.2. The last two columns can be filled in after the lab. When you are done with an input frequency, set the next one on the FG and repeat the process.

Finally, when you are done with all the frequencies, rotate the R knob on the RLC box clockwise such that the mark on the knob points to the top of the "I" in "MIN". This sets the resistance in the circuit to another value, corresponding to a different "damping ratio". The damping ratio is denoted by ζ, which for this circuit equals R/R_c. Then repeat the whole procedure again for all frequencies, recording

Freq. (Hz)	E_i (V)	E_o (V)	Δt (s)	ω (rad/s)	$M(\omega)$	$\phi(\omega)$ (°)
100						
500						
1000						
1200						
1600						
2000						
2200						
2500						
2800						
3100						
3600						
4000						
5000						
7000						
10 000						

Table 10.3: RLC-lower resistance response data

your "raw" data in Table 10.3. When you are all done, using the voltmeter, measure the total resistance of the RLC circuit (between the center pins of the IN and the OUT connectors). Subtract 9 Ω (the resistance of the inductor) from this value and record it. This is the value of R for this case, which you will need to use later in your calculations.

10.4 What to Report

Turn in this document along with answers to the questions posed. Also attach any pertinent plots and m-file listings.

1. Using the data in Table 10.1, plot the data for each thermocouple and determine the time constant directly. Then transform the variables and performed a linear least-squares regression analysis. From that determine the each time constant. Compare the time constants obtained from the two methods. Which method is more accurate and why?

2. Compare the time constant of thermocouple A to that of thermocouple B. How and why are the time constants different? Provide a plausible physical explanation for their difference.

3. Complete the columns for $\omega, M(\omega)$ and $\phi(\omega)$ in both Tables 10.2 and 10.3.

4. Using these results and a program (e.g., m-file) that you write, construct two plots, one of $M(\omega)$ and other of $\phi(\omega)$ versus the normalized frequency ratio,

where each of these two plots contains both of the high R and low R cases. These plots must contain your data along with the "theoretical" curves given by Equations 10.7 and 10.8, substituting the appropriate values for R, L and C for each case. Plot the data for each of the two cases using a different set of symbols for each case.

5. Does your data support the conclusion the RLC circuit behaves as a second-order system in both cases? Finally, compare the values of the "damping ratio", ζ (which equals R/R_c for this exercise) found for each case with each value of R/R_c. You can do this by comparing your data with the corresponding values determined using various values of R/R_c in Equations 10.7 and 10.8. How well do the experimental and theoretical values of ζ compare?

Chapter 11

Fundamentals of Optics: Lenses, Lasers, Detectors

11.1 Introduction and Objectives

The main objective of this exercise is to become familiar with concepts in optical design, and to apply basic design techniques to several model problems. The exercises will involve incoherent- and coherent-light sources, lenses, and optical detectors. Intuition gained from these exercises will prepare you for laboratory exercises involving the application of optical techniques.

11.2 Instrumentation

The following tools will be needed for the lab exercises:

- small flashlight, to be used as a white-light source
- ruler and protractor
- lens, double-convex, $\phi = 65$ mm, unknown f
- Metrologic ML-211, diode-based laser, of unknown λ; with mount
- holographic, diffraction grating, 750 lines/mm, with mount
- diode/detector pair, with amplifier circuit and battery
- digital, mini-tachometer
- digital oscilloscope
- spinning wheel, of unknown rotation rate, N
- dispersing prism, with unknown index, n

The lab exercises will use an optical rail and a full optical bench as convenient platforms to mount various components. Several of the exercises will depend on the careful alignment of the optical elements.

This investigation will be broken into several sections. In the first exercise, a lens will be used to image a source onto a screen. The source will be the bulb of a flashlight. With sufficient magnification, fine details of the filament may be seen. This exercise will demonstrate the mounting and positioning of lenses, based on the thin-lens equation.

In the second exercise, the use of a laser will be explored by passing the laser radiation through a diffraction grating. Two of the most important aspects of laser light are its coherence and its monochromaticity. Both of these features will be utilized in the measurement of laser wavelength, based on the grating equation.

In the third exercise, an LED-photodiode pair will be provided. This circuit, along with an oscilloscope, will allow for the measurement of the rotation rate of a spinning wheel. The measurement can be compared to that of the mini- tachometer, also provided

In the fourth exercise, the properties of a prism will be investigated. The laser will be used to make a measurement that will allow for the calculation of the index of refraction of the glass.

11.3 Laser Safety

The laser to be used is a relatively safe, low-power, laser pointer. As such, special eye protection is not needed. However, direct, prolonged exposure of the eye to this laser can still cause damage, so common sense and caution should be exercised. The following steps can help provide for a safe lab experience.

- The laser should never be aimed directly into a person's eyes.
- The laser beam should be blocked off so that it cannot extend beyond the limits of the individual laboratory section. Dull, non-reflective barriers such as dark-colored paper or stacks of books can be used for this.
- Reflective objects in the area should be covered with cloth or blocked, to prevent secondary reflections in the lab.

11.4 Measurements

The goal of the first exercise is to become proficient in the use of simple lenses. To this end, the flashlight will be imaged onto a viewing screen. In order to properly image the filament, the distances, or conjugates, between the source and lens, and between the lens and screen, must satisfy the thin- lens equation, which is provided in the Supplemental-Information section.

Using the optical rail provided, experiment with the positioning of the lens versus the source and viewing screen. For each choice of conjugates, a magnification of the image can be calculated. Try a variety of conjugate-distance combinations, and adjust the optical system to be sure the image is in focus at the screen for each combination. Record the results in the table provided, and calculate the magnification of each attempt. You should find that the calculation for the focal length of the lens is the same in each case; the focal length is a constant property of the lens, not of the system.

In the second exercise, measure the wavelength, λ, of a laser by utilizing a diffraction grating. By using the grating equation, which is provided in the Supplemental-Information section, and given the spatial frequency of the grating to be 750 lines/mm, the wavelength can be calculated. Carefully measure the position of the first fringe (i.e. for $m = 1$ or $m = -1$), and average several readings. A table has been provided for your convenience.

In the third exercise, calculate the rotation rate of the spinning wheel. The rotation rate, N, of a wheel can be measured by configuring a light source on one side of the wheel, and a detector on the other side, and preparing the wheel so that it periodically obstructs the beam of light. Such a sensor is representative of many simple sensors that are used in industrial areas to anchor process-control loops. In order to calculate N, capture the chopped signal on an oscilloscope. Estimate the chopping frequency, and use the geometry of the wheel to infer the rotations per minute. Compare your result to that of the mini-tachometer. Capture the oscilloscope plot, and import it to your final report.

In the fourth exercise, investigate the properties of a dispersing prism. As detailed in the Supplemental-Information section, a prism can be used to separate, or "disperse" the various frequencies of the signal. Using the equation given, and knowing the wavelength of the laser light from the second exercise, estimate what must be the refractive index of the prism.

11.5 What to Report

Once again, you are to submit your report in the form of a Technical memo. Be sure to include (as a minimum) the following information:

- calculation of the focal length, f, of the double-convex lens
- calculation of laser wavelength, λ
- calculation of wheel-rotation rate, N
- digitized plot of the oscilloscope trace of the rotation-rate sensor
- calculation of refractive index of the prism, n
- Uncertainty estimates presented in the form of tables supported by example calculations

Be sure to include any interesting observations from any of the four sections of the lab.

11.6 Supplemental Information

11.6.1 Imaging an Incoherent Source

The thin-lens equation is given by Smith [1] as

$$\frac{-1}{l} + \frac{1}{l'} = \frac{1}{f} \tag{11.1}$$

according to the following diagram,

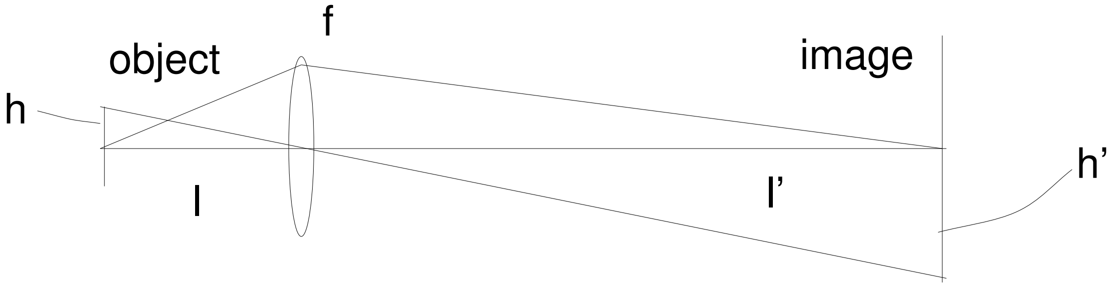

Figure 11.1: Imaging with a Thin Lens

where l is the distance from the lens to the object (typically a negative number, as in the above diagram), l' is the distance from the lens to the image, and f is the focal length of the lens. Note that in this case, for simplicity, the object is the source of radiation, the lamp. Thus, if the system is arranged with an unknown lens such that the image is in sharp focus, the conjugates, l and l' can be measured, and the focal length of the lens can be calculated.

In addition, the magnification of the system can be calculated from the conjugates as

$$m = \frac{l'}{l} = \frac{h'}{h}, \tag{11.2}$$

where h is the object height and h' is the image height. The image height will typically be negative, by convention, where a negative magnification indicates an inverted image. Note that for larger magnifications, the irradiance of the image will decrease.

As an example, if the distance from the lens to the object is -20 mm, and the distance from the lens to the image is 100 mm, then the focal length will be 16.7 mm, and the magnification will be -5. A negative magnification indicates that the image is inverted, compared to the object.

11.6.2 Characterizing a Coherent Source

Laser light in general is both monochromatic and spatially-coherent, and these are the reasons the laser is such an important tool in many fields. The spatial coherence of the laser means that the beam is perfectly collimated, as if it had originated infinitely far away. It is this property that allows for strong interference fringes when the beam is crossed with itself, and this is the basis of interferometry, which will be investigated in another lab. The monochromaticity of the laser is the purity of its wavelength. Because the laser can deliver significant power at such a narrow bandwidth, it is useful in fields ranging from spectroscopy to fiber-optic communications.

The wavelength of the laser diode used in this lab can be calculated by measuring the positions of the fringes in the diffraction pattern beyond a grating. The grating equation is given by Metrologic [2] as

$$m\lambda = dsin\theta, \tag{11.3}$$

according to the following diagram,

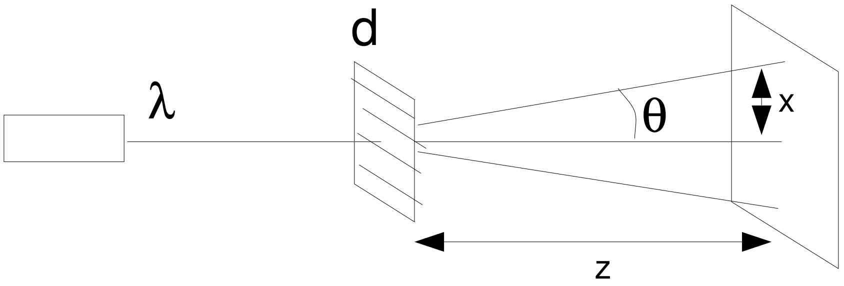

Figure 11.2: Laser-Beam Diffraction

where m is the order of the fringe, λ is the wavelength of radiation, d is the grating spacing, and θ is the angle of deviation off axis of the diffraction fringe.

Now, for the simplified case of $m = 1$, and given that for small angles, $sin\theta$ can be approximated by θ, the wavelength of the laser can be calculated using the relation

$$\lambda = d\frac{x}{z}, \tag{11.4}$$

where x is the deflection of the fringe off of the optical axis, and z is the distance from the grating to the screen.

11.6.3 Optical Measurement of Wheel-Rotation Rate

In this exercise you will use a source-detector pair for the measurement of the rotation rate of a spinning wheel. Some simple source and detector circuits are provided by Mims [3]. Often light-emitting diodes (LED's) are used as the source of a simple sensor. An LED is typically dc-powered, and, when biased, it radiates light at a relatively narrow bandwidth. This type of source can be controlled very precisely, compared to an incandescent lamp. A simple detector will often be a silicon photodiode, followed by an operational amplifier. Such a detector is based on the photoelectric effect, where a small current is generated by the photodiode upon exposure to light, and the current is amplified and converted to voltage by the op-amp.

In this exercise, an LED source will be coupled with a phototransistor for signal detection. The phototransistor output need not be amplified, but rather just fed directly into an oscilloscope for analysis. The circuits are shown as follows:

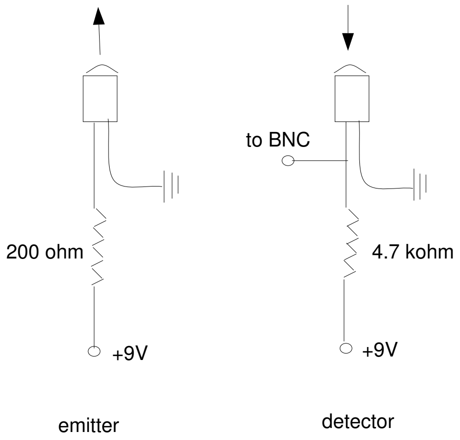

Figure 11.3: Emitter and Detector Circuits

Connect the spinning wheel (a muffin fan) to the wall outlet. Couple the detector circuit to channel 1 of the oscilloscope, and switch the detector circuit "on." When not in use, the detector should be switched off, to conserve the battery. Align the detector circuit, mini-tachometer, and wheel such that both sensors can view the chopping at the same time. The mini-tachometer will need an illuminating source (like a flashlight) on the other side of the fan. Acquire the signal on the oscilloscope; the following notes may be of help to you:

- trigger the oscilloscope off of the detector signal itself; use the buttons, "trig", and "chan 1"
- use ac-coupling, to optimize the signal on the scope
- set the amplitude on the scope to 0.1 V/div
- if at this point you do not see a nice square wave, check the circuit's battery with a voltmeter; a replacement may be needed
- use the cursors to measure the chopping frequency; use the buttons, "cursors", "on", and then the "track" and "arrow" buttons

By measuring the frequency of the chopped signal, estimate the rotation rate of the wheel. Save the scope plot, and also record the measurement of the mini-tachometer. How close are the two measurements? If time allows, try a couple different measurement positions along the wheel, and see if the result is repeatable.

11.6.4 Measurement of Refractive Index of Prism

Because the index of refraction of glass is dependent upon the wavelength of the radiation, prisms have been used as the basis of simple monochrometers. The dispersion of light by a prism is given by Smith [1] as

$$d = i_1 - a + arcsin[(n(\lambda)^2 - sin^2(i_1))^{1/2} sin(a) - cos(a) sin(i_1)], \tag{11.5}$$

according to figure 1.4.

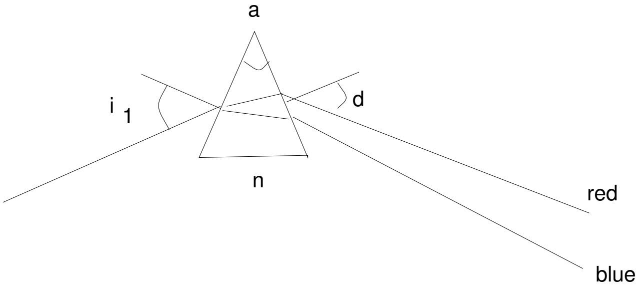

Figure 11.4: Refraction by a Prism

In figure 1.4, d is the final angle of exit of the beam, i_1 is the angle of incidence, a is the characteristic angle of the prism, and $n(\lambda)$ is the wavelength-dependent index of refraction of the glass. Direct the laser through the prism, and carefully measure the various geometric quantities. Then, knowing the wavelength of radiation, calculate the index of refraction of the prism. You might try a couple different orientations of the prism, varying the angle of incidence slightly, to see if the result is repeatable.

reading	l[cm]	l'[cm]	obj height	im height	mag	f[cm]
1						
2						
3						

Table 11.1: Imaging of Flashlight

reading	x[cm]	z[cm]	d[mm/line]	λ[nm]
1			1/750	
2			1/750	
3			1/750	

Table 11.2: Measurement of Laser Wavelength

reading	chopping freq	[rot/sec]	N [RPM]	mini tach [RPM]
1				
2				
3				

Table 11.3: Measurement of Wheel Rotation Rate

reading	i_1 [°]	a [°]	d [°]	index n
1				
2				
3				

Table 11.4: Measurement of Prism Index of Refraction

Measurand	**Units**	**Uncertainty**
conjugates l and l'	cm	
object and image height, h,h'	cm	
diffraction-angle parameters, x,z	cm	
wheel chopping frequency	Hz	
index-of-refraction angles,i_1,a,d	rad	

Table 11.5: Measurand uncertainties

Result	**Units**	**Calculation**
focal length of lens, f	cm	
laser wavelength, λ	nm	
rotation-rate of wheel, N	RPM	
prism index of refraction, n	dimensionless	

Table 11.6: Results

Result	**Units**	**Uncertainty**
focal length of lens, u_f	cm	
laser wavelength, u_λ	nm	
rotation-rate of wheel, u_N	RPM	
prism index of refraction, u_n	dimensionless	

Table 11.7: Result uncertainties

Bibliography

[1] Smith, W.J. Modern Optical Engineering. McGraw-Hill, New York, 1966.

[2] "Laser-Pointer Education Kit," Metrologic Instruments, Bellmawr, NJ, 1996.

[3] Mims, Forrest M. Engineer's Mini-Notebook, Optoelectronic Circuits. Printed by Forrest Mims, 1986.

Chapter 12

Statistical Analysis of Data Using MATLAB®

12.1 Introduction and Objectives

The overall objectives of this exercise are to solidify your understanding of several statistical and probabilistic methods and to provide you with the opportunity to learn how to use MATLAB® to retrieve, analyze and display actual experimental data on a computer system.

There are three different data files that you will use in this exercise. The first file shed.dat consists of two columns of data acquired during an experiment in a subsonic wind tunnel. The first column is the time (in s), t, at which a value of the velocity (the second column, in m/s), U(t), was acquired using a hot-wire sensor located immediately behind a cylinder positioned in the wind tunnel. The second file gball.dat is a single column of the golf ball weights (in N). The third file voltage.dat is a single column of voltage readings from an amplifier/Wheatstone bridge system taken at fixed time increments.

In this exercise you **must** use MATLAB® to analyze this data. You are required to write several M-files. Be sure to provide statements at the beginning of each M-file describing its particular function. The following explains specifically what you must do. A technical memo is NOT required for this exercise; simply turn in what is asked for. Organize professionally in order and label all information that you turn in.

Some helpful information on using MATLAB® for probability and statistical calculations is presented in text. Other information can be obtained using MATLAB® 's "help" command. Note: some of the commands that you need to use are *not* contained in the student version of MATLAB® .

12.2 Temporal Realization of Time Series Data

Using shed.dat, determine the following and put your answers in Table 1 on a single page: [1] the period (in s) of the signal's *third* cycle, [2] the average period (in s) of one cycle, [3] the signal's average cyclic frequency (in Hz), [4] the mean velocity (in m/s), [5] the standard deviation of the velocity (in m/s), [6] the minimum velocity (in m/s) and [7] the maximum velocity (in m/s). Write an M-file to read in the data from shed.dat and then plot (label it Figure 1) the velocity (y:ordinate) versus time (x:abscissa) denoting each data point with a symbol and without any curve fitting in between them. Include in that M-file the commands to automatically identify on your plot the mean velocity, the mean velocity +1 standard deviation and the mean velocity -1 standard deviation. HINTS: The following MATLAB® commands coded into an M-file may be useful: [a] eval(['load info.dat']) loads the data from info.dat into the MATLAB® workspace, [b] col1 = eval(['info(:,1)']) assigns the first column of info.dat the name col1, [c] text(xpos,ypos,'whateveryoulike') places "whateveryoulike" (e.g., a line or an arrow) at the coordinates xpos,ypos on a plot. For this part, you are required to hand in Table 1, Figure 1, the M-file you wrote and proof of your calculations of the seven above quantities (this could be, for example, the printout of your MATLAB® session).

12.3 Distribution Comparisons

Using gball.dat, write an M-file to read in the data and then plot (label it Figure 2) the histogram and frequency distribution side by side on one page (HINT: use MATLAB® 's "subplot" command). Follow the rules for determining the number of bins as described in text. Assume that u_w (the uncertainty in the measurement of the weight, w) equals 0.01 N. Check to make sure that your bin width is greater than this value. Next, by either writing another M-file or adding on to the previous one, determine and then plot (label it Figure 3) on one page the histogram of the data and the expected values as determined for a Normal distribution (use the sample mean and standard deviation for the mean and standard deviation of the Normal pdf). HINTS: The following MATLAB® commands coded into an M-file may be useful: [a] hist(x,k) plots the histogram of x with k bins, [b] [a,b]=hist(x,k) produces the column matrices a and b, where a contains the counts in each bin and b contains the center coordinates for each bin, [c] bar(b,a/N) will plot the frequency distribution, where N is the total number of x values, [d] q=c:dq:e will produce values of q ranging from c to e in increments of dq, and [e] plot(q,sin(q)) will plot q on the abscissa versus sin(q) on the ordinate as a continuous and smooth curve provided that the increment dq is much smaller than the range of q. For this part, you are required to hand in Figures 2 and 3 and the M-files you wrote.

12.4 Finite versus Infinite Samples

Using voltage.dat, write an M-file to plot (label it Figure 4) the running mean of the data from 1 up to all 1000 points (running mean on the ordinate; the number of points on the abscissa). The running mean of N points is simply the mean of those N points. As N gets larger, the running mean should approach a constant value equal to the true mean of the underlying population from which the N points were drawn. On the plot, indicate the running mean of 1000 points and its value using the previously described "text" command. Next, determine the number of measurements, N*, required for the running mean to stay *continually* within 1 % of the running mean of 1000 points. Then, compute the sample mean, the sample standard deviation and the standard deviation of the means for all 1000 points. Write a statement for the estimate of the true mean value of the parent population from which this data was drawn at the 95 % confidence level based upon these values. Put the values of N*, the sample mean, the sample standard deviation, the standard deviation of the means and the true mean estimate statement in Table 2 on one page. For this part, you are required to hand in Figure 4, the M-file you wrote and Table 2.

12.5 χ^2 Analysis

Continue with the analysis that you did to construct the plot of the histogram of the gball.dat weights versus the values expected for a normal distribution. Perform a χ^2 analysis to determine the % confidence that the golf ball weights are Normally distributed. HINT: The following MATLAB® command coded into an M-file may be useful: [a] alpha = 100-100*chi2cdf(chisq,nu). Report the % confidence value and hand in the M-file that you wrote to calculate this.

12.6 What to Report

Turn in the answers to the posed questions, your plots and listings of all of the M-files that you have written. No technical memo is required.