chapter

2

Describing Motion

chapter overview

The main purpose of this chapter is to provide clear definitions and illustrations of the terms used in physics to describe motion, such as the motion of the car described in this chapter's opening example. Speed, velocity, and acceleration are crucial concepts for the analysis of motion in later chapters. Precise description is the first step to understanding. Without it, we remain awash in vague ideas that are not defined well enough to test our explanations.

Each numbered topic in this chapter builds on the previous section, so it is important to obtain a clear understanding of each topic before going on. The distinctions between speed and velocity and velocity and acceleration are particularly important.

chapter outline

1 **Average and instantaneous speed.** How do we describe how fast an object is moving? How does instantaneous speed differ from average speed?

2 **Velocity.** How do we introduce direction into descriptions of motion? What is the distinction between speed and velocity?

3 **Acceleration.** How do we describe changes in motion? What is the relationship between velocity and acceleration?

4 **Graphing motion.** How can graphs be used to describe motion? How can the use of graphs help us gain a clearer understanding of speed, velocity, and acceleration?

5 **Uniform acceleration.** What happens when an object accelerates at a steady rate? How do the velocity and distance traveled vary with time when an object is uniformly accelerating?

unit one

Imagine that you are in your car stopped at an intersection. After waiting for cross traffic, you pull away from the stop sign, accelerating eventually to a speed of 56 kilometers per hour (35 miles per hour). You maintain that speed until a dog runs in front of your car and you hit the brakes, reducing your speed rapidly to 10 km/h (fig. 2.1). Having missed the dog, you speed up again to 56 km/h. After another block, you come to another stop sign and reduce your speed gradually to zero.

We can all relate to this description. Measuring speed in miles per hour (MPH) may be more familiar than the use of kilometers per hour (km/h), but speedometers in cars now show both. The use of the term *acceleration* to describe an increase in speed is also common. In physics, however, these concepts take on more precise and specialized meanings that make them even more useful in describing exactly what is happening. These meanings are sometimes different from those in everyday use. The term *acceleration,* for example, is used by physicists to describe any situation in which velocity is changing, even when the speed may be decreasing or the direction of the motion may be changing.

How would you define the term *speed* if you were explaining the idea to a younger brother or sister? Does *velocity* mean the same thing? What about *acceleration*—is the notion vague or does it have a precise meaning? Is it the same thing as velocity? Clear definitions are essential to developing clear explanations. The language used by physicists differs from our everyday language, even though the ideas are related and the same words are used. What are the exact meanings that physicists attach to these concepts, and how can they help us to understand motion?

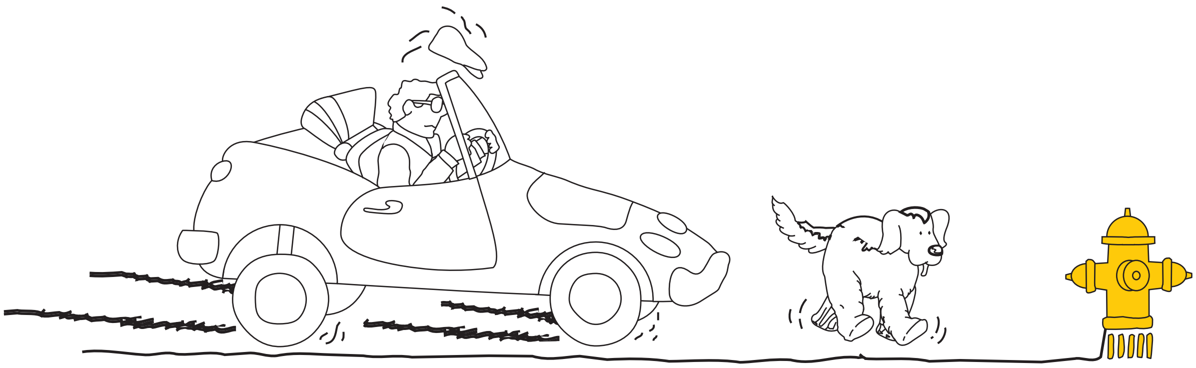


figure 2.1 As the car brakes for the dog, there is a sudden change in speed.

2.1 Average and Instantaneous Speed

Since driving or riding in cars is a common activity in our daily lives, we are familiar with the concept of speed. Most of us have had experience in reading a speedometer (or perhaps failing to read it carefully enough to avoid the attention of law enforcement). If you describe how fast something is moving, as we did in our example in the introduction, you are talking about **speed.**

How is average speed defined?

What does it mean to say that we are traveling at a speed of 55 MPH? It means that we would cover a distance of 55 miles in a time of 1 hour if we traveled steadily at that speed. Carefully note the structure of this description: there is a number, 55, and some units or dimensions, miles per hour. Numbers and units are both essential parts of a description of speed.

The term *miles per hour* implies that miles are divided by hours in arriving at the speed. This is exactly how we would compute the **average speed** for a trip: suppose, for example, that we travel a distance of 260 miles in a time of 5 hours, as shown on the road map of figure 2.2. The average speed is then 260 miles divided by 5 hours, which is equal to 52 MPH. This type of computation is familiar to most of us.

We can also express the definition of average speed in a word equation as

> Average speed equals the distance traveled divided by the time of travel.

or

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time of travel}}.$$

We can represent this same definition with symbols by writing

$$s = \frac{d}{t},$$

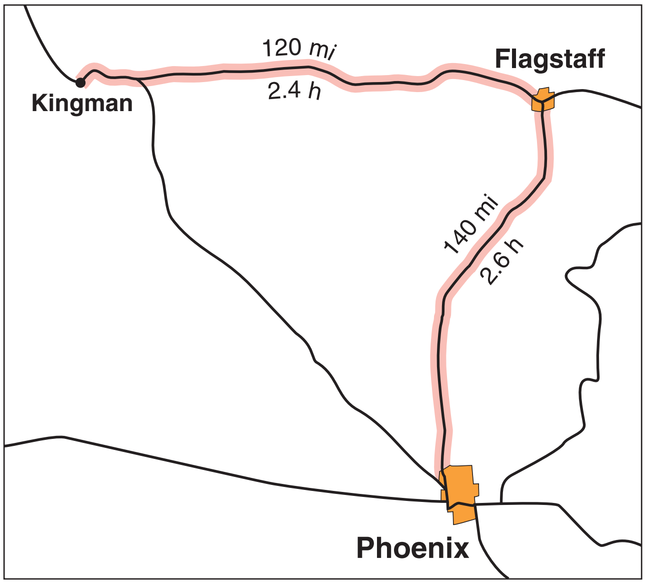


figure **2.2** A road map showing a trip of 260 miles, with driving times for the two legs of the trip.

example box 2.1

Unit Conversions

1 km = 0.6214 miles
1 mile = 1.609 km

Convert 70 kilometers per hour to miles per hour.

$$\left(70\,\frac{\cancel{\text{km}}}{\text{h}}\right)\left(0.6214\,\frac{\text{miles}}{\cancel{\text{km}}}\right) = \mathbf{43.5\ MPH}$$

Convert 70 kilometers per hour to meters per second.

$$\left(70\,\frac{\cancel{\text{km}}}{\text{h}}\right)\left(1000\,\frac{\text{m}}{\cancel{\text{km}}}\right) = 70\,000\ \text{m/h}$$

$$\text{But } 1\ \text{h} = (60\ \cancel{\text{min}})\left(60\,\frac{\text{s}}{\cancel{\text{min}}}\right) = 3600\ \text{s}$$

$$\frac{70\,000\ \text{m}/\cancel{\text{h}}}{3600\ \text{s}/\cancel{\text{h}}} = \mathbf{19.4\ m/s}$$

Lines drawn through the units indicate cancellation.

where the letter *s* represents the speed, *d* represents distance, and *t* represents the time. As noted in chapter 1, letters or symbols are a compact way of saying what could be said with a little more effort and space with words. Judge for yourself which is the more efficient way of expressing this definition of average speed. Most people find the symbolic expression easier to remember and use.

The average speed that we have just defined is the **rate** at which distance is covered over time. Rates always represent one quantity divided by another. Gallons per minute, pesos per dollar, and points per game are all examples of rates. If we are considering time rates, the quantity that we divide by is *time,* which is the case with average speed. Many other quantities that we will be considering involve time rates.

What are the units of speed?

Units are an essential part of the description of speed. Suppose you say that you were doing 70—without stating the units. In the United States, that would probably be understood as 70 MPH, since that is the unit most frequently used. In Europe, on the other hand, people would probably assume that you are talking about the considerably slower speed of 70 km/h. If you do not state the units, you will not communicate effectively.

It is easy to convert from one unit to another if the conversion factors are known. For example, if we want to convert kilometers per hour to miles per hour, we need to know the relationship between miles and kilometers. A kilometer is roughly 6⁄10 of a mile (0.6214, to be more precise). As shown in example box 2.1, 70 km/h is equal to 43.5 MPH. The process involves multiplication or division by the appropriate conversion factor.

Units of speed will always be a distance divided by a time. In the metric system, the fundamental unit of speed is meters per second (m/s). Example box 2.1 also shows the conversion of kilometers per hour to meters per second, done as a two-step process. As you can see, 70 km/h can also be expressed as 19.4 m/s or roughly 20 m/s. This is a convenient size for discussing the speeds of ordinary objects. (As shown in example box 2.2, the convenient unit for measuring the growth of grass has a very different size.)

example box 2.2

Sample Question: Watching Grass Grow

Question: The units km/h or m/s have an appropriate size for moving cars or people. Many other processes move much more slowly, though. What units would have an appropriate size for measuring the average speed with which a blade of grass grows?

Answer: When grass is well fertilized and watered, it is not unusual for it to grow 3 to 6 centimeters in the course of a week. This can be seen by measuring the length of the clippings after mowing. If we measured the speed in m/s, we would obtain an extremely small number that would not provide a good intuitive sense of the rate of growth. The units of cm/week or mm/day would provide a better indication of this speed.

table 2.1

Familiar Speeds in Different Units

20 MPH	=	32 km/h	=	9 m/s
40 MPH	=	64 km/h	=	18 m/s
60 MPH	=	97 km/h	=	27 m/s
80 MPH	=	130 km/h	=	36 m/s
100 MPH	=	160 km/h	=	45 m/s

Table 2.1 shows some familiar speeds expressed in miles per hour, kilometers per hour, and meters per second to give you a sense of their relationships.

What is instantaneous speed?

If we travel a distance of 260 miles in 5 hours, as in our earlier example, is it likely that the entire trip takes place at a speed of 52 MPH? Of course not; the speed goes up and down as the road goes up and down, when we overtake slower vehicles, when rest breaks occur, or when the highway patrol looms on the horizon. If we want to know how fast we are going at a given instant in time, we read the speedometer, which displays the **instantaneous speed** (fig. 2.3).

How does instantaneous speed differ from average speed? The instantaneous speed tells us how fast we are going at a given instant but tells us little about how long it will take to travel several miles, unless the speed is held constant. The average speed, on the other hand, allows us to compute how long a trip might take but says little about the variation in speed during the trip. A more complete description of how the speed of a car varies during a portion of a trip could be provided by a graph such as that shown in figure 2.4. Each point on this graph represents the instantaneous speed at the time indicated on the horizontal axis.

Even though we all have some intuitive sense of what instantaneous speed means from our experience in reading

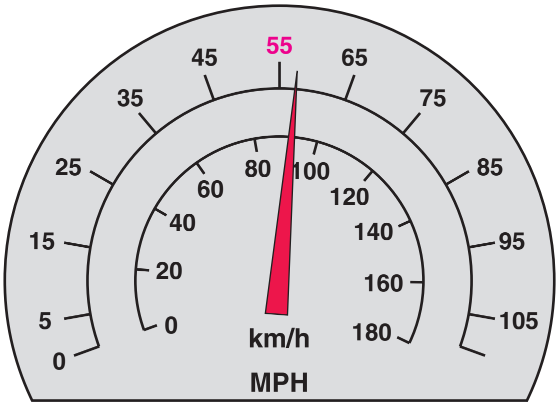


figure 2.3 A speedometer with two scales for measuring instantaneous speed, MPH and km/h.

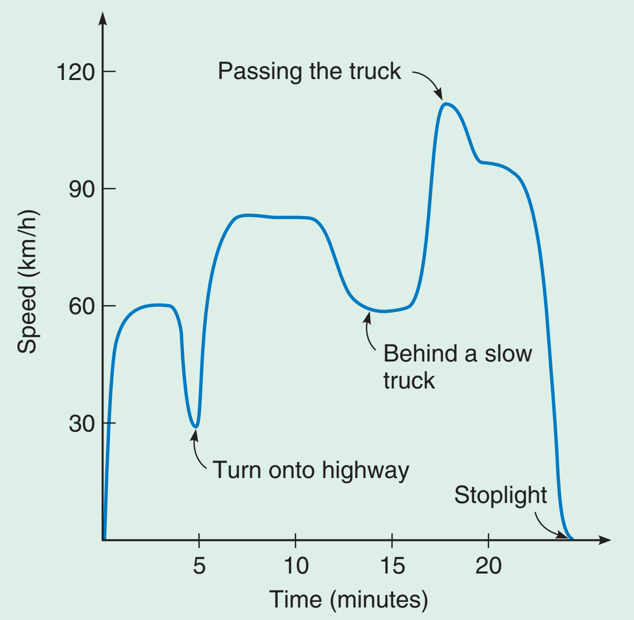


figure 2.4 Variations in instantaneous speed for a portion of a trip on a local highway.

speedometers, computing this quantity presents some problems that we did not encounter in defining average speed. We could say that instantaneous speed is the rate that distance is being covered at a given instant in time, but how do we compute this rate? What time interval should we use? What is an instant in time?

Our solution to this problem is simply to choose a very short interval of time during which a very short distance is covered and the speed does not change drastically. If we know, for example, that in 1 second a distance of 20 meters was covered, dividing 20 meters by 1 second to obtain a speed of 20 m/s would give us a good estimate of the instantaneous speed, provided that the speed did not change much during that single second. If the speed was changing rapidly, we would have to choose an even shorter interval of time. In principle, we can choose time intervals as small as we wish, but in practice, it can be hard to measure such small quantities.

If we put these ideas into a word definition of instantaneous speed, we could state it as

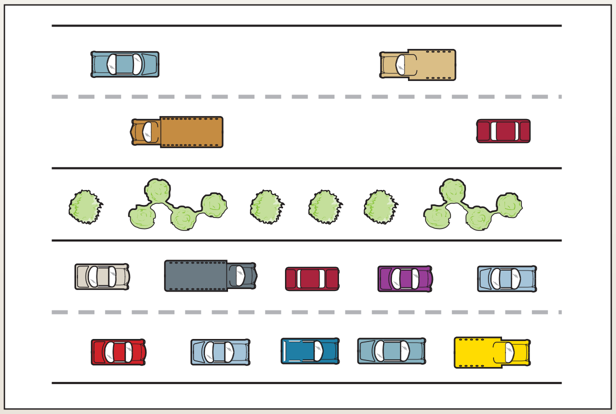
> Instantaneous speed is the rate at which distance is being covered at a given instant in time. It is found by computing the average speed for a very short time interval in which the speed does not change appreciably.

Instantaneous speed is closely related to the concept of average speed but involves very short time intervals. When discussing traffic flow, average speed is the critical issue, as shown in everyday phenomenon box 2.1.

everyday phenomenon box 2.1

Transitions in Traffic Flow

The Situation. Jennifer commutes into the city on a freeway every day for work. As she approaches the city, the same patterns in traffic flow seem to show up in the same places each day. She will be moving with the flow of traffic at a speed of approximately 60 MPH when suddenly things will come to a screeching halt. The traffic will be stop-and-go briefly and then will settle into a wavelike mode with speeds varying between 10 and 30 MPH. Unless there is an accident, this will continue for the rest of the way into the city.



The traffic in the upper lanes is flowing freely with adequate spacing to allow higher speeds. The higher-density traffic in the lower lanes moves much more slowly.

What causes these patterns? Why does the traffic stop when there is no apparent reason such as an accident? Why do ramp traffic lights seem to help the situation? Questions like these are the concern of the growing field of traffic engineering.

The Analysis. Although a full analysis of traffic flow is complex, there are some simple ideas that can explain many of the patterns that Jennifer observes. The density of vehicles, measured in vehicles per mile, is a key factor. Adding vehicles at entrance ramps increases the density. The spacing between vehicles varies with speed so that speed and density are interrelated.

When Jennifer and other commuters are traveling at 60 MPH, they need to keep a spacing of several car lengths between vehicles. Most drivers do this without thinking about it, although there are always some who follow too closely or *tailgate.* Tailgating runs the risk of rear-end collisions when the traffic suddenly slows.

When more vehicles are added at an entrance ramp, the density must increase, reducing the distance between vehicles. As the distance between vehicles decreases, drivers should reduce their speed to maintain a safe stopping distance. If this occurred uniformly, there would be a gradual decrease in the average speed of the traffic to accommodate the greater density. This is not what usually happens, however.

(continued)

We find an average speed by dividing the distance traveled by the time required to cover that distance. Average speed is therefore the average rate at which distance is being covered. Instantaneous speed is the rate that distance is being covered at a given instant in time and is found by considering very small time intervals or by reading a speedometer. Average speed is useful for estimating how long a trip will take, but instantaneous speed is of more interest to the highway patrol.

2.2 Velocity

Do the words *speed* and *velocity* mean the same thing? They are often used interchangeably in everyday language, but physicists make an important distinction between the two terms. The distinction has to do with direction: which way is the object moving? This distinction turns out to be essential to understanding Newton's theory of motion (introduced in chapter 4), so it is not just a matter of whim or jargon.

What is the difference between speed and velocity?

Imagine that you are driving a car around a curve (as illustrated in figure 2.5) and that you maintain a constant speed of 60 km/h. Is your velocity also constant in this case? The answer is no, because **velocity** involves the direction of motion as well as how fast the object is going. The direction of motion is changing as the car goes around the curve.

To simply state this distinction, speed as we have defined it tells us how fast an object is moving but says nothing about the direction of the motion. Velocity includes the idea of direction. To specify a velocity, we must give both

A significant proportion of drivers will attempt to maintain their speed of 50 to 60 MPH even when densities have increased beyond the point where this is advisable. This creates an unstable situation. At some point, usually near an entrance ramp, the density becomes too large to sustain these speeds. At this point there is a sudden drop in average speed and a large increase in the local density. As shown in the drawing, cars can be separated by less than a car length when they are stopped or moving very slowly.

Once the average speed of a few vehicles has slowed to less than 10 MPH, vehicles moving at 50 to 60 MPH begin to pile up behind this slower moving jam. Because this does not happen smoothly, some vehicles must come to a complete stop, further slowing the flow. At the front end of the jam, on the other hand, the density is reduced due to the slower flow behind. Cars can then start moving at a speed consistent with the new density, perhaps around 30 MPH. If every vehicle moved with the appropriate speed, flow would be smooth and the increased density could be safely accommodated. More often, however, overanxious drivers exceed the appropriate speed, causing fluctuations in the average speed as vehicles begin to pile up again.

Notice that we are using average speed with two different meanings in this discussion. One is the average speed of an individual vehicle as its instantaneous speed increases and decreases. The other is the average speed of the overall traffic flow involving many vehicles. When the traffic is flowing freely, the average speed of different vehicles may differ. When the traffic is in a slowly moving jam, the average speeds of different vehicles are essentially the same, at least within a given lane.

Traffic lights at entrance ramps that permit vehicles to enter one-at-a-time at appropriate intervals can help to smoothly integrate the added vehicles to the existing flow. This reduces the sudden changes in speed caused by a rapid increase in density. Once the density increases beyond the certain level, however, a slowing of traffic is inevitable. The abrupt change from low-density, high-speed flow to higher-density, slow flow is analogous to a phase transition from a gas to a liquid. (Phase transitions are discussed in chapter 10.) Traffic engineers have used this analogy to better understand the process.

If we could automatically control and coordinate the speeds of all the vehicles on the highway, the highway might carry a much greater volume of traffic at a smooth rate of flow. Speeds could be adjusted to accommodate changes in density and smaller vehicle separations could be maintained at higher speeds because the vehicles would all be moving in a synchronized fashion. Better technology may someday achieve this dream.

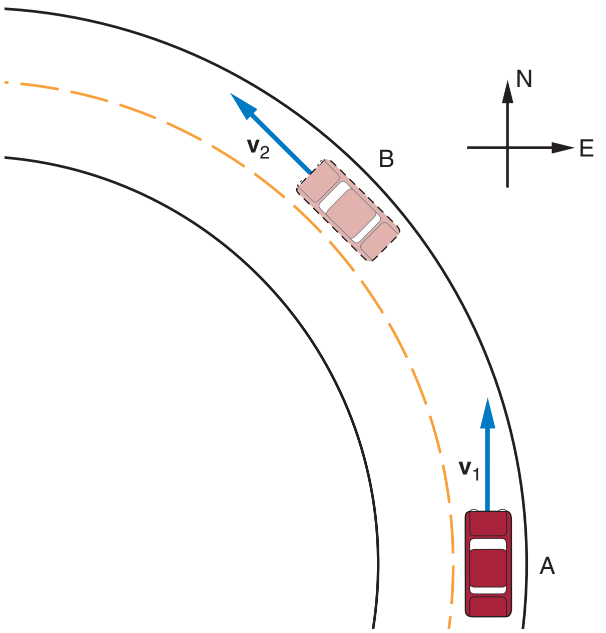


figure 2.5 The direction of the velocity changes as the car moves around the curve, so that the velocity $\mathbf{v}_2$ is not the same as the velocity $\mathbf{v}_1$ even though the speed has not changed.

its size or **magnitude** (how fast) and its direction (north, south, east, up, down, or somewhere in between). If you tell me that an object is moving 15 m/s, you have told me its *speed.* If you tell me that it is moving due west at 15 m/s, you have told me its *velocity.*

At point A on the diagram in figure 2.5, the car is traveling due north at 60 km/h. At point B, because the road curves, the car is traveling northwest at 60 km/h. Its velocity at point B is different from its velocity at point A (because the directions are different). The speeds at point A and B are the same. Direction is irrelevant in specifying the speed of the object. It has no effect on the reading on your speedometer.

Changes in velocity are produced by forces acting upon the car, as we will discuss further in chapter 4. The most important force involved in changing the velocity of a car is the frictional force exerted on the tires of the car by the road surface. A force is required to change either the size or the direction of the velocity. If no net force were acting on the car, it would continue to move at constant speed in a straight line. This happens sometimes when there is ice or oil on the road surface, which can reduce the frictional force to almost zero.

study hint

Science has always relied on pictures and charts to get points across. Throughout the book, a number of concepts will be introduced and illustrated. In the illustrations, the same color will be used for certain phenomena.

- Blue arrows are velocity vectors.
- Green arrows depict acceleration vectors.
- Red arrows depict force vectors.
- Purple arrows show momentum, a concept we will explore in chapter 7.

What is a vector?

Velocity is a quantity for which both the size and direction are important. We call such quantities **vectors.** To describe these quantities fully, we need to state *both the size and the direction.* Velocity is a vector that describes how fast an object is moving and in what direction it is moving. Many of the quantities used in describing motion (and in physics more generally) are **vector quantities.** These include velocity, acceleration, force, and momentum, to name a few.

Think about what happens when you throw a rubber ball against a wall, as shown in figure 2.6. The speed of the ball may be about the same after the collision with the wall as it was before the ball hit the wall. The velocity has clearly changed in the process, though, because the ball is moving in a different direction after the collision. Something has happened to the motion of the ball. A strong force had to be exerted on the ball by the wall to produce this change in velocity.

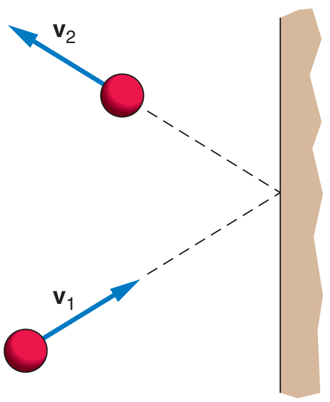


figure **2.6** The direction of the velocity changes when a ball bounces from a wall. The wall exerts a force on the ball in order to produce this change.

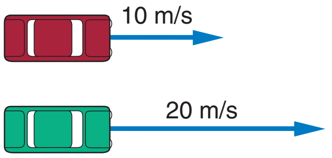


figure **2.7** The length of the arrow shows the size of the velocity vector.

The velocity vectors in figures 2.5 and 2.6 are represented by arrows. This is a natural choice for depicting vectors, since the direction of the arrow clearly shows the direction of the vector, and the length can be drawn proportional to the size. In other words, the larger the velocity, the longer the arrow (fig. 2.7). In the text, we will represent vectors by printing their symbols in boldface and larger than other symbols: $\mathbf{v}$ is thus the symbol for velocity. A fuller description of vectors can be found in appendix C.

How do we define instantaneous velocity?

In considering automobile trips, *average* speed is the most useful quantity. We do not really care about the direction of motion in this case. *Instantaneous* speed is the quantity of interest to the highway patrol. **Instantaneous velocity,** however, is most useful in considering physical theories of motion. We can define instantaneous velocity by drawing on our earlier definition of instantaneous speed.

> Instantaneous velocity is a vector quantity having a size equal to the instantaneous speed at a given instant in time and having a direction corresponding to that of the object's motion at that instant.

Instantaneous velocity and instantaneous speed are closely related, but velocity includes direction as well as size. It is *changes* in instantaneous velocity that require the intervention of forces. These changes will be emphasized when we explore Newton's theory of mechanics in chapter 4. We can also define the concept of average velocity, but that is a much less useful quantity for our purposes than either instantaneous velocity or average speed.

> To specify the velocity of an object, we need to state both how fast and in what direction the object is moving; velocity is a vector quantity. Instantaneous velocity has a magnitude equal to the instantaneous speed and points in the direction that the object is moving. Changes in instantaneous velocity are where the action is, so to speak, and we will consider these in more detail when we discuss acceleration in section 2.3.

2.3 Acceleration

Acceleration is a familiar idea. We use the term in speaking of the acceleration of a car away from a stop sign or the acceleration of a running back in football. We feel the effects of acceleration on our bodies when a car's velocity changes rapidly and even more strikingly when an elevator lurches downward, leaving our stomachs slightly behind (fig. 2.8). These are all accelerations. You can think of your stomach as an acceleration detector—a roller-coaster gives it a real workout!

Understanding acceleration is crucial to our study of motion. **Acceleration** is the rate at which velocity *changes.* (Note that we said velocity, not speed.) It plays a central role in Newton's theory of motion. How do we go about finding a value of an acceleration, though? As with speed, it is convenient to start with a definition of average acceleration and then extend it to the idea of instantaneous acceleration.

How is average acceleration defined?

How would we go about providing a quantitative description of an acceleration? Suppose that your car, pointing due east, starts from a full stop at a stop sign, and its velocity increases from zero to 20 m/s as shown in figure 2.9. The change in velocity is found simply by subtracting the initial velocity from the final velocity (20 m/s − 0 m/s = 20 m/s).

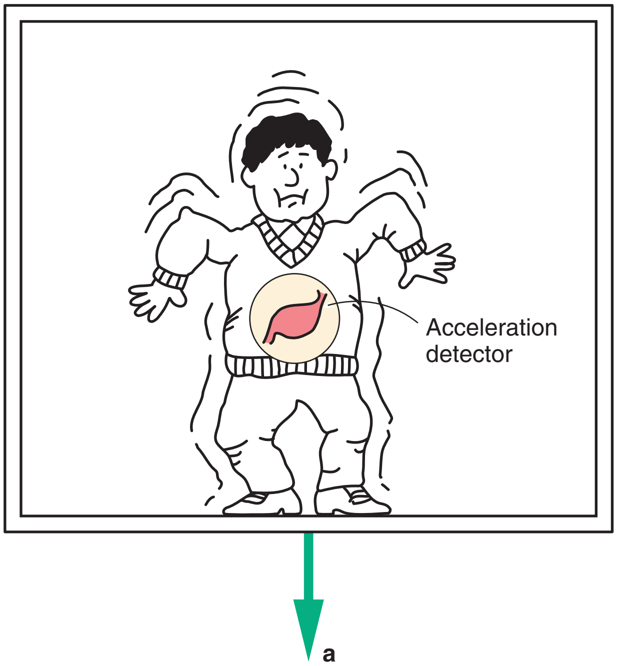


figure 2.8 Your acceleration detector senses the downward acceleration of the elevator.

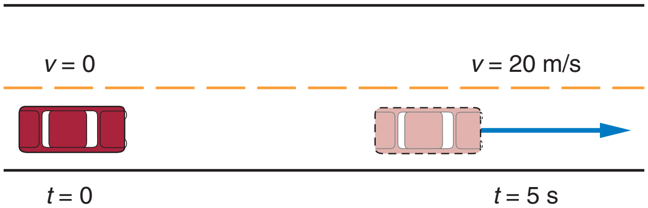


figure 2.9 A car, starting from rest, accelerates to a velocity of 20 m/s due east in a time of 5 s.

To find its *rate of change,* however, we also need to know the time needed to produce this change. If it took just 5 seconds for the velocity to change, the rate of change would be larger than if it took 30 seconds.

Suppose that a time of 5 seconds was required to produce this change in velocity. The rate of change in velocity could then be found by dividing the size of the change in velocity by the time required to produce that change. Thus the size of the **average acceleration,** *a,* is found by dividing the change in velocity of 20 m/s by the time of 5 seconds,

$$a = \frac{20\ \text{m/s}}{5\ \text{s}} = 4\ \text{m/s/s}.$$

The unit m/s/s is usually written m/s^2 and is read as *meters per second squared.* It is easier to understand it, however, as *meters per second per second.* The car's velocity (measured in m/s) is changing at a rate of 4 m/s every second. Other units could be used for acceleration, but they will all have this same form: distance per unit of time per unit of time. In discussing the acceleration of a car on a drag strip, for example, the unit *miles per hour per second* is sometimes used.

The quantity that we have just computed is the size of the average acceleration of the car. The average acceleration is found by dividing the total change in velocity for some time interval by that time interval, ignoring possible differences in the rate of change of velocity that might be occurring within the time interval. Its definition can be stated in words as

> Average acceleration is the change in velocity divided by the time required to produce that change.

We can restate it in symbols as

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{elapsed time}}$$

or

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{t}.$$

Because change is so important in this definition, we have used the special symbol Δ (the Greek letter delta) to mean a change in a quantity. Thus $\Delta\mathbf{v}$ is a compact way of writing *the change in velocity,* which otherwise would be expressed as $\mathbf{v}_2 - \mathbf{v}_1$, since a change is the difference between two quantities. Because the concept of change is critical, this notation will appear often.

The idea of change is all-important. Acceleration is *not* velocity over time. It is the *change* in velocity divided by time. It is common for people to associate large accelerations with large velocities, when in fact the opposite is often true. The acceleration of a car may be largest, for example, when it is just starting up and its velocity is near zero. The rate of change of velocity is greatest then. On the other hand, a car can be traveling at 100 MPH but still have a zero acceleration if its velocity is not changing.

What is instantaneous acceleration?

Instantaneous acceleration is similar to average acceleration with an important exception. Just as with instantaneous speed or velocity, we are now concerned with the rate of change at a given instant in time. It is *instantaneous* acceleration that our stomachs respond to. It can be defined as

> Instantaneous acceleration is the rate at which velocity is changing at a given instant in time. It is computed by finding the average acceleration for a very short time interval during which the acceleration does not change appreciably.

If the acceleration is changing with time, choosing a very short time interval guarantees that the acceleration computed for that time interval will not differ too much from the instantaneous acceleration at any time within the interval. This is the same idea used in finding an instantaneous speed or instantaneous velocity.

What is the direction of an acceleration?

Like velocity, acceleration is a vector quantity. Its direction is important. The direction of the acceleration vector is that of the *change* in velocity $\Delta\mathbf{v}$. If, for example, a car is moving in a straight line and its velocity is increasing, the change in velocity is in the same direction as the velocity itself, as shown in figure 2.10. The change in velocity $\Delta\mathbf{v}$ must be added to the initial velocity $\mathbf{v}_1$ to obtain the final velocity $\mathbf{v}_2$. All three vectors point forward. The process of adding vectors can be readily seen when we represent the vectors as arrows on a graph. (More information on vector addition can be found in appendix C.)

If the velocity is decreasing, however, the change in velocity $\Delta\mathbf{v}$ points in the opposite direction to the two velocity vectors, as shown in figure 2.11. Because the initial velocity $\mathbf{v}_1$ is larger than the final velocity $\mathbf{v}_2$, the change in velocity must point in the opposite direction to produce a shorter $\mathbf{v}_2$ arrow. The acceleration is also in the opposite direction to the velocity, since it is in the direction of the *change* in velocity. In Newton's theory of motion, the force required to produce this acceleration would also be opposite in direction to the velocity. It must push backward on the car to slow it down.

The term *acceleration* describes the rate of *any* change in an object's velocity. The change could be an increase (as in our initial example), a decrease, or a change in direction. The term applies even to decreases in velocity (*decelerations*). To a physicist these are simply accelerations with a direction opposite that of the velocity. If a car is braking while traveling in a straight line, its velocity is decreasing and its acceleration is negative if the velocity is positive. This situation is illustrated in the sample exercise in example box 2.3.

The minus sign is an important part of the result in the example in example box 2.3 because it indicates that the change in velocity is negative. The velocity is getting smaller. We can call it a deceleration if we like, but it is

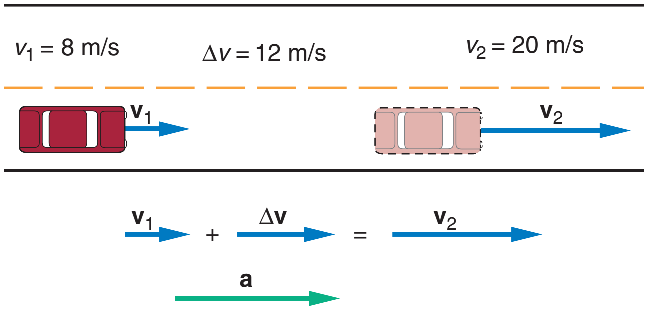


figure 2.10 The acceleration vector is in the same direction as the velocity vectors when the velocity is increasing.

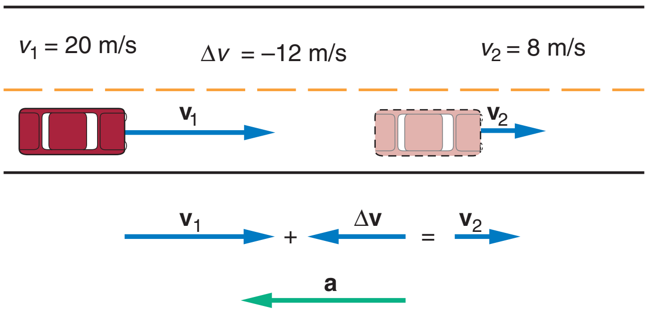


figure 2.11 The velocity and acceleration vectors for decreasing velocity: $\Delta\mathbf{v}$ and $\mathbf{a}$ are now opposite in direction to the velocity. The acceleration $\mathbf{a}$ is proportional to $\Delta\mathbf{v}$.

example box 2.3

Sample Exercise: Negative Accelerations

The driver of a car steps on the brakes, and the velocity drops from 20 m/s due east to 10 m/s due east in a time of 2.0 seconds. What is the acceleration?

$\mathbf{v}_1 = 20$ m/s due east
$\mathbf{v}_2 = 10$ m/s due east
$t = 2.0$ s
$\mathbf{a} = ?$

$$a = \frac{\Delta v}{t} = \frac{v_2 - v_1}{t}$$

$$= \frac{10\text{ m/s} - 20\text{ m/s}}{2.0\text{ s}}$$

$$= \frac{-10\text{ m/s}}{2.0\text{ s}}$$

$$= -5\text{ m/s}^2$$

$$\mathbf{a} = \mathbf{5.0\ m/s^2\ due\ west}$$

Notice that when we are dealing just with the magnitude of a vector quantity, we do not use the boldface notation. The sign can indicate direction, however, in a problem involving straight-line motion.

the same thing as a negative acceleration. One word, *acceleration,* covers all situations in which the velocity is changing.

Can a car be accelerating when its speed is constant?

What happens when a car goes around a curve at constant speed? Is it accelerating? The answer is yes, because the direction of its velocity is changing. If the direction of the velocity vector is changing, the velocity is changing. This means that there must be an acceleration.

This situation is illustrated in figure 2.12. The arrows in this drawing show the direction of the velocity vector at different points in the motion. The change in velocity $\Delta\mathbf{v}$ is the vector that must be added to the initial velocity $\mathbf{v}_1$ to obtain the final velocity $\mathbf{v}_2$. The vector representing the change in velocity points toward the center of the curve, and therefore, the acceleration vector also points in that direction. The size of the change is represented by the length of the arrow $\Delta\mathbf{v}$. From this we can find the acceleration.

Acceleration is involved whenever there is a change in velocity, regardless of the nature of that change. Cases like figure 2.12 will be considered more fully in chapter 5 where circular motion is discussed.

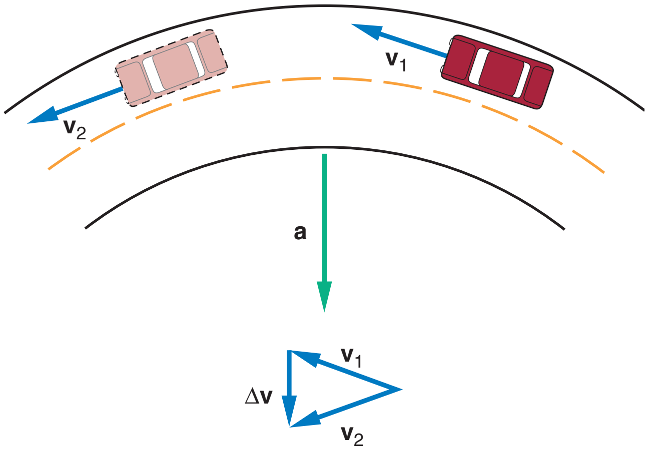


figure 2.12 A change in the direction of the velocity vector also involves an acceleration, even though the speed may be constant.

Acceleration is the rate of change of velocity and is found by dividing the change in the velocity by the time required to produce that change. Any change in velocity involves an acceleration, whether an increase or a decrease in speed, or a change in direction. Acceleration is a vector having a direction corresponding to the direction of the change in velocity, which is not necessarily the same direction as the instantaneous velocity itself. The concept of change is crucial. The graphical representations in section 2.4 will help you visualize changes in velocity as well as in other quantities.

2.4 Graphing Motion

It is often said that a picture is worth a thousand words, and the same can be said of graphs. Imagine trying to describe the motion depicted in figure 2.4 precisely in words and numbers. The graph provides a quick overview of what took place. A description in words would be much less efficient. In this section, we will show how graphs can also help us to understand velocity and acceleration.

What can a graph tell us?

How can we produce and use graphs to help us describe motion? Imagine that you are watching a battery-powered toy car moving along a meter stick (fig. 2.13). If the car is moving slowly enough, you could record the car's position while also recording the elapsed time using a digital watch. At regular time intervals (say, every 5 seconds), you would note the value of the position of the front of the car on the meter stick and write these values down. The results might be something like those shown in table 2.2.

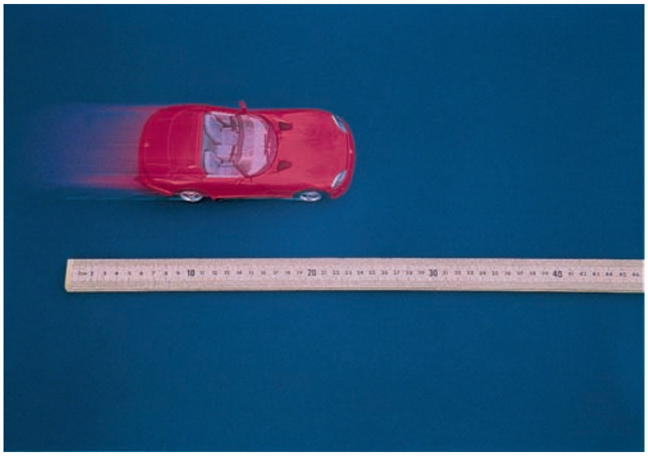


figure **2.13** A toy car moving along a meter stick. Its position can be recorded at different times.

How do we graph these data? First, we create evenly spaced intervals on each of two perpendicular axes, one for distance traveled (or position) and the other for time. To show how distance varies with time, we usually put time on the horizontal axis and distance on the vertical axis. Such a graph is shown in figure 2.14, where each data point from table 2.2 is plotted and a line is drawn through the points. To make sure that you understand this process, choose different points from table 2.2 and find where they are located on the graph. Where would the point go if the car was at 21 centimeters at 25 seconds?

The graph summarizes the information presented in the table in a visual format that makes it easier to grasp at a glance. The graph also contains information on the velocity and acceleration of the car, although that is less obvious. For example, what can we say about the average velocity of the car between 20 and 30 seconds? Is the car moving during this time? A glance at the graph shows us that the distance is not changing during that time interval, so the car is *not* moving. The velocity is zero during that time, which is represented by a horizontal line on our graph of distance versus time.

What about the velocity at other points in the motion? The car is moving more rapidly between 0 and 20 seconds than it is between 30 and 50 seconds. The distance curve is rising more rapidly between 0 and 20 seconds than between 30 and 50 seconds. Since more distance is covered in the same time, the car must be moving faster there. A steeper slope to the curve is associated with a larger speed.

In fact, the **slope** of the distance-versus-time curve at any point on the graph is equal to the *instantaneous velocity* of the car.* The slope indicates how rapidly the distance is changing with time at any instant in time. The rate of change of distance with time is the instantaneous speed according to the definition given in section 2.1. Since the motion takes place along a straight line, we can then represent the direction of the velocity with plus or minus signs. There are only two possibilities, forward or backward. We then have the instantaneous velocity, which includes both the size (speed) and direction of the motion.

When the car travels backward, its distance from the starting point decreases. The curve goes down, as it does between 50 and 60 seconds. We refer to this downward-sloping portion of the curve as having a *negative slope* and also say that the velocity is negative during this portion of the motion. A large upward slope represents a large instantaneous velocity, a zero slope (horizontal line) a zero velocity, and a downward slope a negative (backward) velocity. Looking at the slope of the graph tells us all we need to know about the velocity of the car.

table **2.2**

Position of the Toy Car along the Meter Stick at Different Times

Time	Position
0 s	0 cm
5 s	4.1 cm
10 s	7.9 cm
15 s	12.1 cm
20 s	16.0 cm
25 s	16.0 cm
30 s	16.0 cm
35 s	18.0 cm
40 s	20.1 cm
45 s	21.9 cm
50 s	24.0 cm
55 s	22.1 cm
60 s	20.0 cm

Velocity and acceleration graphs

These ideas about velocity can be best summarized by plotting a graph of velocity against time for the car (fig. 2.15). The velocity is constant wherever the slope of the distance-versus-time graph of figure 2.14 is constant. Any straight-line segment of a graph has a constant slope, so the velocity changes only where the slope of the graph in figure 2.14 changes. If you compare the graph in figure 2.15 to the graph in figure 2.14 carefully, these ideas should become clear.

*Since the mathematical definition of slope is the change in the vertical coordinate Δd divided by the change in the horizontal coordinate Δt, the slope, $\Delta d/\Delta t$, is equal to the instantaneous velocity, provided that Δt is sufficiently small. It is possible to grasp the concept of slope, however, without appealing to the mathematical definition.

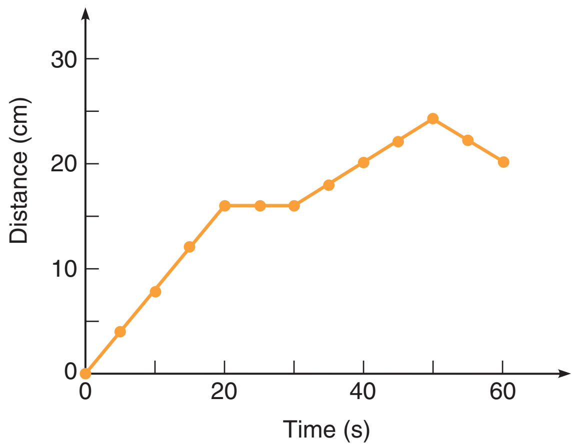


figure 2.14 Distance plotted against time for the motion of the toy car. The data points are those listed in table 2.2.

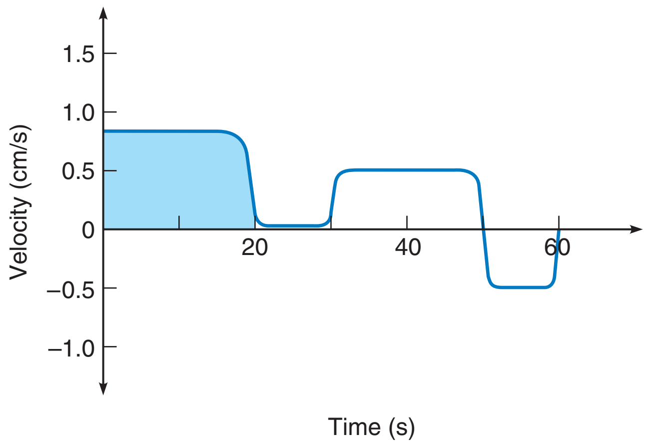


figure 2.15 Instantaneous velocity plotted against time for the motion of the toy car. The velocity is greatest when distance traveled is changing most rapidly.

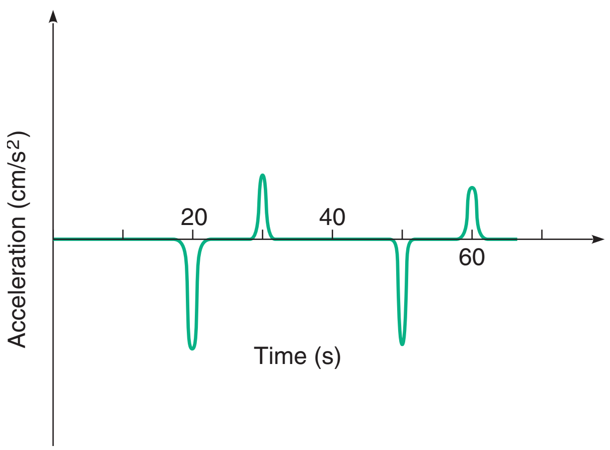


figure 2.16 An approximate sketch of acceleration plotted against time for the toy-car data. The acceleration is non-zero only when the velocity is changing.

What can we say about the acceleration from these graphs? Since acceleration is the rate of change of velocity with time, the velocity graph (fig. 2.15) also provides information about the acceleration. In fact, the instantaneous acceleration is equal to the slope of the velocity-versus-time graph. A steep slope represents a rapid change in velocity and thus a large acceleration. A horizontal line has zero slope and represents zero acceleration. The acceleration turns out to be zero for most of the motion described by our data. The velocity changes at only a few points in the motion. The acceleration would be large at these points and zero everywhere else.

Since our data do not indicate how rapidly the changes in velocity actually occur, we do not have enough information to say just how large the acceleration is at those few points where it is not zero. We would need measurements of distance or velocity every tenth of a second or so to get a clear idea of how rapid these changes are. As we will see in chapter 4, we know that these changes in velocity cannot occur instantly. Some time is required. So we can sketch an approximate graph of acceleration versus time, as shown in figure 2.16.

The spikes in figure 2.16 occur when the velocity is changing. At 20 seconds, there is a rapid decrease in the velocity represented by a downward spike or negative acceleration. At 30 seconds, the velocity increases rapidly from zero to a constant value, and this is represented by an upward spike or positive acceleration. At 50 seconds, there is another negative acceleration as the velocity changes from a positive to a negative value. If you could put yourself inside the toy car, you would definitely feel these accelerations. (Everyday phenomenon box 2.2 provides another example of how a graph is useful for analyzing motion.)

Can we find the distance traveled from the velocity graph?

What other information can be gleaned from the velocity-versus-time graph of figure 2.15? Think for a moment about how you would go about finding the distance traveled if you knew the velocity. For a constant velocity, you can get the distance simply by multiplying the velocity by the time, $d = vt$. In the first 20 seconds of the motion, for example, the velocity is 0.8 cm/s and the distance traveled is 0.8 cm/s times 20 seconds, which is 16 cm. This is just the reverse of what we used in determining the velocity in the first place. We found the velocity by dividing the distance traveled by the time.

How would this distance be represented on the velocity graph? If you recall formulas for computing areas, you may recognize that the distance d is the area of the shaded rectangle on figure 2.15. The area of a rectangle is found by multiplying the height times the width, just what we have done here. The velocity, 0.8 cm/s, is the height and the everyday phenomenon box 2.2

The 100-m Dash

The Situation. A world-class sprinter can run 100 m in a time of a little under 10 s. The race begins with the runners in a crouched position in the starting blocks, waiting for the sound of the starter's pistol. The race ends with the runners lunging across the finish line, where their times are recorded by stopwatches or automatic timers.

Runners in the starting blocks, waiting for the starter's pistol to fire.

What happens between the start and finish of the race? How do the velocity and acceleration of the runners vary during the race? Can we make reasonable assumptions about what the velocity-versus-time graph looks like for a typical runner? Can we estimate the maximum velocity of a good sprinter? Most importantly for improving performance, what factors affect the success of a runner in the dash?

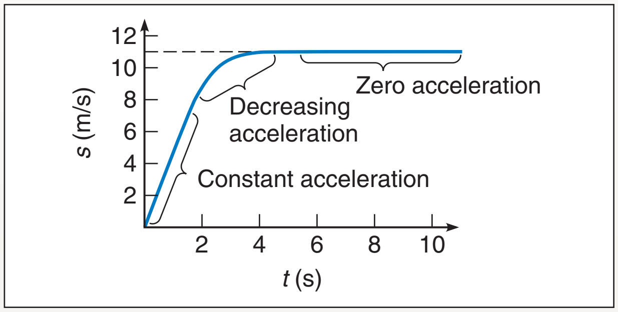
The Analysis. Let's assume that the runner covers the 100-m distance in a time of exactly 10 s. We can compute the average speed of the runner from the definition $s = d/t$:

$$s = \frac{100\text{ m}}{10\text{ s}} = 10\text{ m/s}.$$

Clearly, this is not the runner's instantaneous speed throughout the course of the race, since the runner's speed at the beginning of the race is zero and it takes some time to accelerate to the maximum speed.

The objective in the race is to reach a maximum speed as quickly as possible and to sustain that speed for the rest of the race. Success is determined by two things: how quickly the runner can accelerate to this maximum speed and the value of this maximum speed. A smaller runner often has better acceleration but a smaller maximum speed, while a larger runner sometimes takes longer to reach top speed but has a larger maximum speed.

The typical runner does not reach top speed before traveling at least 10 to 20 m. If the average speed is 10 m/s, the runner's maximum speed must be somewhat larger than this value, since we know that the instantaneous speed will be less than 10 m/s while the runner is accelerating. These ideas are easiest to visualize by sketching a graph of speed plotted against time, as shown. Since the runner travels in a straight line, the magnitude of the instantaneous velocity is equal to the instantaneous speed. The runner reaches top speed at approximately 2 to 3 s into the race.



A graph of speed versus time for a hypothetical runner in the 100-m dash.

The average speed (or velocity) during the time that the runner is accelerating is approximately half of its maximum value if the runner's acceleration is more or less constant during the first 2 s. If we assume that the runner's average speed during this time is about 5.5 m/s (half of 11 m/s), then the speed through the remainder of the race would have to be about 11.1 m/s to give an average speed of 10 m/s for the entire race. This can be seen by computing the distance from these values:

$$\begin{aligned} d &= (5.5\text{ m/s})(2\text{ s}) + (11.1\text{ m/s})(8\text{ s}) \\ &= 11\text{ m} + 89\text{ m} = 100\text{ m}. \end{aligned}$$

What we have done here is to make some reasonable guesses for these values that will make the average speed come out to 10 m/s; we then checked these guesses by computing the total distance. This suggests that the maximum speed of a good sprinter must be about 11 m/s (25 MPH). For sake of comparison, a distance runner who can run a 4-min mile has an average speed of about 15 MPH, or 6.7 m/s.

The runner's strategy should be to get a good jump out of the blocks, keeping the body low initially and leaning forward to minimize air resistance and maximize leg drive. To maintain top speed during the remainder of the race, the runner needs good endurance. A runner who fades near the end needs more conditioning drills. For a given runner with a fixed maximum speed, the average speed depends on how quickly the runner can reach top speed. This ability to accelerate rapidly depends upon leg strength (which can be improved by working with weights and other training exercises) and natural quickness.

time, 20 seconds, is the width of this rectangle on the graph.

It turns out that we can find the distance this way even when the areas involved on the graph are not rectangles, although the process is more difficult when the curves are more complicated. The general rule is that the distance traveled is equal to the area under the velocity-versus-time curve. When the velocity is negative (below the time axis on the graph), the object is traveling backward and its distance from the starting point is decreasing.

Even without computing the area precisely, it is possible to get a rough idea of the distance traveled by studying the velocity graph. A large area represents a large distance. Quick visual comparisons give a good picture of what is happening without the need for lengthy calculations. This is the beauty of a graph.

A good graph can present a picture of motion that is rich in insight. Distance traveled plotted against time tells us not only where the object is at any time, but its slope also indicates how fast it was moving. The graph of velocity plotted against time also contains information on acceleration and on the distance traveled. Producing and studying such graphs can give us a more general picture of the motion and the relationships between distance, velocity, and acceleration.

2.5 Uniform Acceleration

If you drop a rock, it falls toward the ground with a constant acceleration, as we will see in the next chapter. An unchanging or **uniform acceleration** is the simplest form of accelerated motion. It occurs whenever there is a constant force acting on an object, which is the case for a falling rock as well as for many other situations.

How do we describe the resulting motion? The importance of this question was first recognized by Galileo, who studied the motion of balls rolling down inclined planes as well as objects in free fall. In his famous work, *Dialogues Concerning Two New Sciences,* published in 1638 near the end of his life, Galileo developed the graphs and formulas that are introduced in this section and that have been studied by students of physics ever since. His work provided the foundation for much of Newton's thinking a few decades later.

How does velocity vary in uniform acceleration?

Suppose a car is moving along a straight road and accelerating at a constant rate. We have plotted the acceleration against time for this situation in figure 2.17. The graph is very simple, but it illustrates what we mean by uniform acceleration. A uniform acceleration is one that does not change as the motion proceeds. It has the same value at any time, which produces a horizontal-line graph.

The graph of velocity plotted against time for this same situation tells a more interesting story. From our discussion in section 2.4, we know that the slope of a velocity-versus-time graph is equal to the acceleration. For a uniform positive acceleration, the velocity graph should have a constant upward slope; the velocity increases at a steady rate. A constant slope produces a straight line, which slopes upward if the acceleration is positive as shown in figure 2.18. In plotting this graph, we assumed that the initial velocity is zero.

This graph can also be represented by a formula. The velocity at any time t is equal to the original velocity *plus* the velocity that has been gained because the car is accelerating. The change in velocity Δv is equal to the acceleration times the time, $\Delta v = at$ since acceleration is defined as $\Delta v/t$. These ideas result in the relationship

$$v = v_0 + at.$$

The first term on the right, v_0, is the original velocity (assumed to be zero in figure 2.18), and the second term,

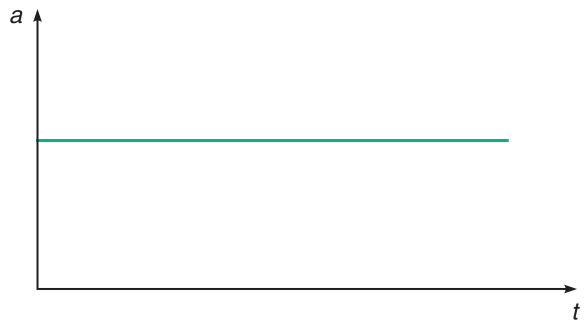


figure 2.17 The acceleration graph for uniform acceleration is a horizontal line. The acceleration does not change with time.

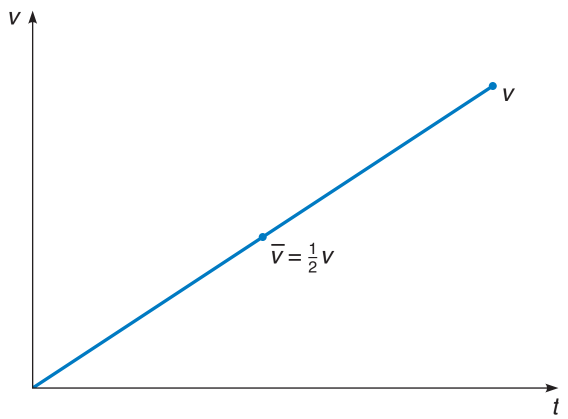


figure 2.18 Velocity plotted against time for uniform acceleration, starting from rest. For this special case, the average velocity is equal to one-half the final velocity.

at, represents the change in velocity due to the acceleration. Adding these two terms together yields the velocity at any later time *t.*

A numerical example applying these ideas to an accelerating car is found in part a of example box 2.4. The car could not keep on accelerating indefinitely at a constant rate because the velocity would soon reach incredible values. Not only is this dangerous, but physical limits imposed by air resistance and other factors prevent this from happening.

What happens if the acceleration is negative? Velocity would decrease rather than increase, and the slope of the velocity graph would slope downward rather than upward. Because the acceleration is then negative, the second term in the formula for v would subtract from the first term, causing the velocity to decrease from its initial value. The velocity then decreases at a steady rate.

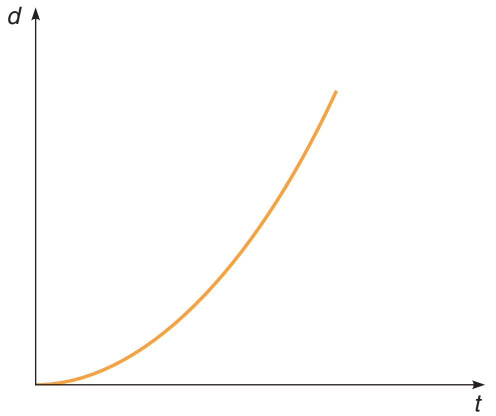


figure 2.19 As the car accelerates uniformly, the distance covered grows more and more rapidly with time because the velocity is increasing.

How does distance traveled vary with time?

If the velocity is increasing at a steady rate, what effect does this have on the distance traveled? As the car moves faster and faster, the distance covered grows more and more rapidly. Galileo showed how to find the distance for this situation.

We find distance by multiplying velocity by time, but in this case we must use an average velocity since the velocity is changing. By appealing to the graph in figure 2.18, we can see that the average velocity should be just half the final velocity, v. If the initial velocity is zero, the final velocity is *at,* so multiplying the average velocity by the time yields

$$d = \frac{1}{2}at^2.$$

The time t enters twice, once in finding the average velocity and then again when we multiply the velocity by time to find the distance.*

The graph in figure 2.19 illustrates this relationship; the distance curve slopes upward at an ever-increasing rate as the velocity increases. This formula and graph are only valid if the object starts from rest as shown in figure 2.18. Since distance traveled is equal to the area under the velocity-versus-time curve (as discussed in section 2.4), this expression for distance can also be thought of as the area under the triangle in figure 2.18. The area of a triangle is equal to one-half its base times its height, which produces the same result.

If the car is already moving before it begins to accelerate, the velocity graph can be redrawn as pictured in figure 2.20. The total area under the velocity curve can then be split in two pieces, a triangle and a rectangle, as shown. The total distance traveled is the sum of these two areas,

$$d = v_0 t + \frac{1}{2}at^2.$$

The first term in this formula represents the distance the object would travel if it moved with constant velocity v_0, and the second term is the additional distance traveled because the object is accelerating (the area of the triangle in figure 2.20). If the acceleration is negative, meaning that the object is slowing down, this second term will subtract from the first.

example box 2.4

Sample Exercise: Uniform Acceleration

A car traveling due east with an initial velocity of 10 m/s accelerates for 6 seconds at a constant rate of 4 m/s^2.

a. What is its velocity at the end of this time?
b. How far does it travel during this time?

a. $v_0 = 10$ m/s $\quad v = v_0 + at$

$a = 4$ m/s^2 $\quad = 10$ m/s $+ (4$ m/s$^2)(6$ s$)$

$t = 6$ s $\quad = 10$ m/s $+ 24$ m/s

$v = ?$ $\quad =$ **34 m/s**

v = 34 m/s due east

b.
$$\begin{aligned} d &= v_0 t + \tfrac{1}{2}at^2 \\ &= (10\text{ m/s})(6\text{ s}) + \tfrac{1}{2}(4\text{ m/s}^2)(6\text{ s})^2 \\ &= 60\text{ m} + (2\text{ m/s}^2)(36\text{ s}^2) \\ &= 60\text{ m} + 72\text{ m} = \mathbf{132\text{ m}} \end{aligned}$$

*Expressing this argument in symbolic form, it becomes

The average velocity $\bar{v} = \frac{1}{2}v = \frac{1}{2}at$

$$d = \bar{v}t = \left(\frac{1}{2}at\right)t = \frac{1}{2}at^2.$$

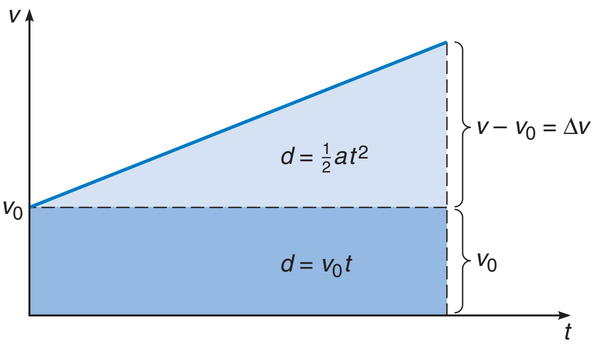


figure 2.20 The velocity-versus-time graph redrawn for an initial velocity different from zero. The area under the curve is divided into two portions, a rectangle and a triangle.

This more general expression for distance may seem complex, but the trick to understanding it is to break it down into its parts, as just suggested. We are merely adding two terms representing different contributions to the total distance. Each one can be computed in a straightforward manner, and it is not difficult to add them together. The two portions of the graph in figure 2.20 represent these two contributions.

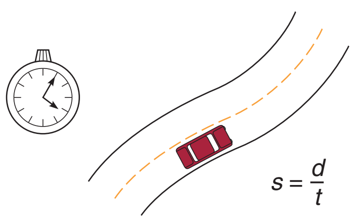
The sample exercise in example box 2.4 provides a numerical example of these ideas. The car in this example accelerates uniformly from an initial velocity of 10 m/s due east to a final velocity of 34 m/s due east and covers a distance of 132 meters while this acceleration is taking place. Had it not been accelerating, it would have gone only 60 meters in the same time. The additional 72 meters comes from the acceleration of the car.

Acceleration involves change, and uniform acceleration involves a steady rate of change. It therefore represents the simplest kind of accelerated motion that we can imagine. Uniform acceleration is essential to an understanding of free fall, discussed in chapter 3, as well as to many other phenomena. Such motion can be represented by either the graphs or the formulas introduced in this section. Looking at both and seeing how they are related will reinforce these ideas.

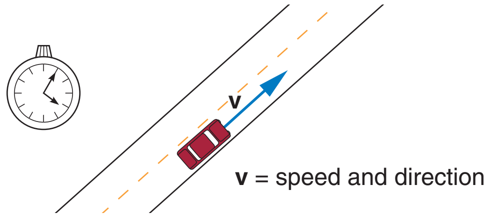
summary

The main purpose of this chapter is to introduce concepts that are crucial to a precise description of motion. To understand acceleration, you must first grasp the concept of velocity, which in turn builds on the idea of speed. The distinctions between speed and velocity, and between velocity and acceleration, are particularly important.

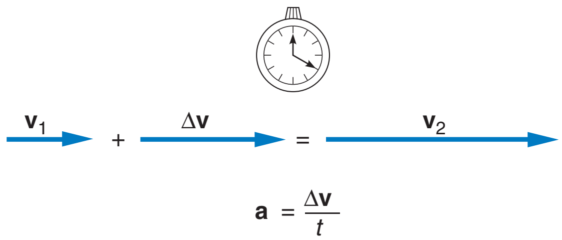
1 **Average and instantaneous speed.** Average speed is defined as the distance traveled divided by the time. It is the average rate at which distance is covered. Instantaneous speed is the rate at which distance is being covered at a given instant in time and requires that we use very short time intervals for computation.



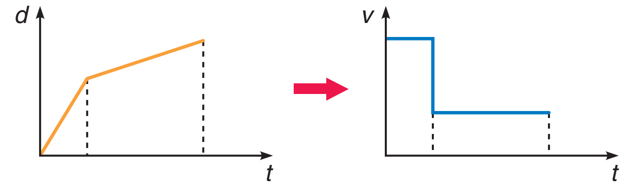
2 **Velocity.** The instantaneous velocity of an object is a vector quantity that includes both direction and size. The size of the velocity vector is equal to the instantaneous speed, and the direction is that of the object's motion.



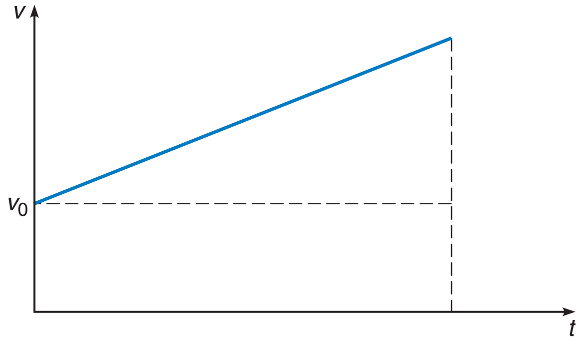
3 **Acceleration.** Acceleration is defined as the time rate of change of velocity and is found by dividing the *change* in velocity by the time. Acceleration is also a vector quantity. It can be computed as either an average or an instantaneous value. A change in the direction of the velocity can be as important as a change in magnitude. Both involve acceleration.



4 Graphing motion. Graphs of distance, speed, velocity, and acceleration plotted against time can illustrate relationships between these quantities. Instantaneous velocity is equal to the slope of the distance-time graph. Instantaneous acceleration is equal to the slope of the velocity-time graph. The distance traveled is equal to the area under the velocity-time graph.



5 Uniform acceleration. When an object accelerates at a constant rate producing a constant-slope graph of velocity versus time, we say that it is uniformly accelerated. Graphs help us to understand the two formulas describing how velocity and distance traveled vary with time for this important special case.



$$v = v_0 + at$$

$$d = v_0 t + \tfrac{1}{2}at^2$$

key terms

Speed, 19
Average speed, 19
Rate, 20
Instantaneous speed, 21
Velocity, 22
Magnitude, 23
Vector, 24
Vector quantity, 24
Instantaneous velocity, 24
Acceleration, 25
Average acceleration, 25
Instantaneous acceleration, 26
Slope, 28
Uniform acceleration, 31

questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available on the Online Learning Center

Q1. Suppose that critters are discovered on Mars who measure distance in *boogles* and time in *bops*.
  a. What would the units of speed be in this system? Explain.
  b. What would the units of velocity be? Explain.
  c. What would the units of acceleration be? Explain.

Q2. Suppose that we choose inches as our basic unit of distance and days as our basic unit of time.
  a. What would the units of velocity and acceleration be in this system? Explain.
  b. Would this be a good choice of units for measuring the acceleration of an automobile? Explain.

Q3. What units would have an appropriate size for measuring the rate at which fingernails grow? Explain.

Q4. A tortoise and a hare cover the same distance in a race. The hare goes very fast for brief intervals, but stops frequently, whereas the tortoise plods along steadily and finishes the race ahead of the hare.
  a. Which of the two racers has the greater average speed over the duration of the race? Explain.
  b. Which of the two racers is likely to reach the greatest instantaneous speed during the race? Explain.

Q5. A driver states that she was doing 80 when stopped by the police. Is that a clear statement? Would this be interpreted differently in England than it would be in the United States? Explain.

Q6. Does the speedometer on a car measure average speed or instantaneous speed? Explain.

Q7. Is the average speed over several minutes more likely to be close to the instantaneous speed at anytime for a car traveling in freely flowing, low-density traffic or for one traveling in high-density traffic? Explain.

*Q8. The highway patrol sometimes uses radar guns to identify possible speeders and at other times uses associates in airplanes who note the time taken for a car to pass between two marks some distance apart on the highway. What do each of these methods measure, average speed or instantaneous speed? Can you think of situations in which either one of these methods might unfairly penalize a driver? Explain.

Q9. A ball is thrown against a wall and bounces back toward the thrower with the same speed as it had before hitting the wall. Does the velocity of the ball change in this process? Explain.

Q10. A ball attached to a string is whirled in a horizontal circle such that it moves with constant speed.
a. Does the velocity of the ball change in this process? Explain.
b. Is the acceleration of the ball equal to zero? Explain.

*Q11. A ball tied to a string fastened at the other end to a rigid support forms a pendulum. If we pull the ball to one side and release it, the ball moves back and forth along an arc determined by the string length.
a. Is the velocity constant in this process? Explain.
b. Is the speed likely to be constant in this process? What happens to the speed when the ball reverses direction?

Q12. A dropped ball gains speed as it falls. Can the velocity of the ball be constant in this process? Explain.

Q13. A driver of a car steps on the brakes, causing the velocity of the car to decrease. According to the definition of acceleration provided in this chapter, does the car accelerate in this process? Explain.

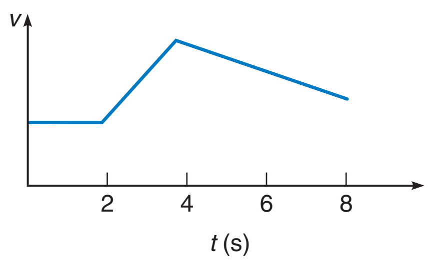
Q14. At a given instant in time, two cars are traveling at different velocities, one twice as large as the other. Based upon this information is it possible to say which of these two cars has the larger acceleration at this instant in time? Explain.

Q15. A car just starting up from a stop sign has zero velocity at the instant that it starts. Must the acceleration of the car also be zero at this instant? Explain.

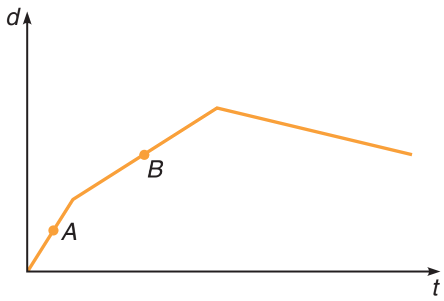
Q16. A car traveling with constant speed rounds a curve in the highway. Is the acceleration of the car equal to zero in this situation? Explain.

Q17. A racing sports car traveling with a constant velocity of 100 MPH due west startles a turtle by the side of the road who begins to move out of the way. Which of these two objects is likely to have the larger acceleration at that instant? Explain.

Q18. In the graph shown here, velocity is plotted as a function of time for an object traveling in a straight line.
a. Is the velocity constant for any time interval shown? Explain.
b. During which time interval shown does the object have the greatest acceleration? Explain.

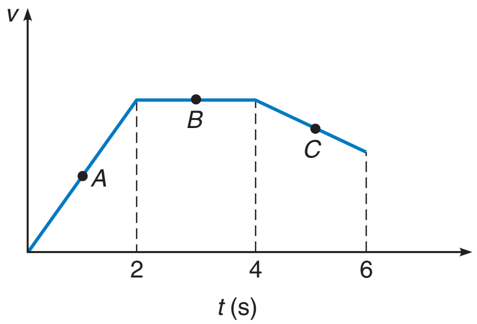


Q19. A car moves along a straight line so that its position (distance from some starting point) varies with time as described by the graph shown here.
a. Does the car ever go backward? Explain.
b. Is the instantaneous velocity at point A greater or less than that at point B? Explain.



Q20. For the car whose distance is plotted against time in question 19, is the velocity constant during any time interval shown in the graph? Explain.

Q21. A car moves along a straight section of road so that its velocity varies with time as shown in the graph.
a. Does the car ever go backward? Explain.
b. At which of the labeled points on the graph, A, B, or C, is the magnitude of the acceleration the greatest? Explain.



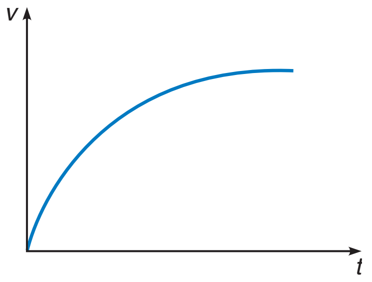
Q22. For the car whose velocity is plotted in question 21, in which of the equal time segments 0–2 seconds, 2–4 seconds, or 4–6 seconds, is the distance traveled by the car the greatest? Explain.

Q23. Look again at the velocity-versus-time graph for the toy car shown in figure 2.15.
a. Is the instantaneous speed greater at any time during this motion than the average speed for the entire trip? Explain.
b. Is the car accelerated when the direction of the car is reversed at $t = 50$ s? Explain.

Q24. Suppose that the acceleration of a car increases with time. Could we use the relationship $v = v_0 + at$ in this situation? Explain.

Q25. When a car accelerates uniformly from rest, which of these quantities increases with time: acceleration, velocity, and/or distance traveled? Explain.

Q26. The velocity-versus-time graph of an object curves as shown in the diagram. Is the acceleration of the object constant? Explain.



Q27. For a uniformly accelerated car, is the average acceleration equal to the instantaneous acceleration? Explain.

Q28. A car traveling in the forward direction experiences a *negative* uniform acceleration for 10 seconds. Is the distance covered during the first 5 seconds equal to, greater than, or less than the distance covered during the second 5 seconds? Explain.

Q29. A car starts from rest, accelerates uniformly for 5 seconds, travels at constant velocity for 5 seconds, and finally decelerates uniformly for 5 seconds. Sketch graphs of velocity versus time and acceleration versus time for this situation.

Q30. Suppose that two runners run a 100-meter dash, but the first runner reaches maximum speed more quickly than the second runner. Both runners maintain constant speed once they have reached their maximum speed and cross the finish line at the same time. Which runner has the larger maximum speed? Explain.

Q31. Sketch a graph showing velocity-versus-time curves for the two runners described in question 30. (Sketch both curves on the same graph, so that the differences are apparent.)

*Q32. A physics instructor walks with increasing speed across the front of the room then suddenly reverses direction and walks backward with constant speed. Sketch graphs of velocity and acceleration consistent with this description.

exercises

E1. A traveler covers a distance of 460 miles in a time of 8 hours. What is the average speed for this trip?

E2. A walker covers a distance of 1.8 km in a time of 30 minutes. What is the average speed of the walker for this distance in km/h?

E3. Grass clippings are found to have an average length of 4.8 cm when a lawn is mowed 12 days after the previous mowing. What is the average speed of growth of this grass in cm/day?

E4. A driver drives for 2.5 hours at an average speed of 54 MPH. What distance does she travel in this time?

E5. A woman walks a distance of 240 m with an average speed of 1.2 m/s. What time was required to walk this distance?

E6. A person in a hurry averages 62 MPH on a trip covering a distance of 300 miles. What time was required to travel that distance?

E7. A hiker walks with an average speed of 1.2 m/s. What distance in kilometers does the hiker travel in a time of 1 hour?

E8. A car travels with an average speed of 22 m/s.
a. What is this speed in km/s?
b. What is this speed in km/h?

E9. A car travels with an average speed of 58 MPH. What is this speed in km/h? (See example box 2.1.)

E10. Starting from rest and moving in a straight line, a runner achieves a velocity of 7 m/s in a time of 2 s. What is the average acceleration of the runner?

E11. Starting from rest, a car accelerates at a rate of 4.2 m/s^2 for a time of 5 seconds. What is its velocity at the end of this time?

E12. The velocity of a car decreases from 30 m/s to 18 m/s in a time of 4 seconds. What is the average acceleration of the car in this process?

E13. A car traveling with an initial velocity of 12 m/s accelerates at a constant rate of 2.5 m/s^2 for a time of 2 seconds.
a. What is its velocity at the end of this time?
b. What distance does the car travel during this process?

E14. A runner traveling with an initial velocity of 2.0 m/s accelerates at a constant rate of 1.2 m/s^2 for a time of 2 seconds.
a. What is his velocity at the end of this time?
b. What distance does the runner cover during this process?

E15. A car moving with an initial velocity of 30 m/s slows down at a constant rate of -3 m/s^2.
a. What is its velocity after 3 seconds of deceleration?
b. What distance does the car cover in this time?

E16. A runner moving with an initial velocity of 4.0 m/s slows down at a constant rate of -1.5 m/s^2 over a period of 2 seconds.
a. What is her velocity at the end of this time?
b. What distance does she travel during this process?

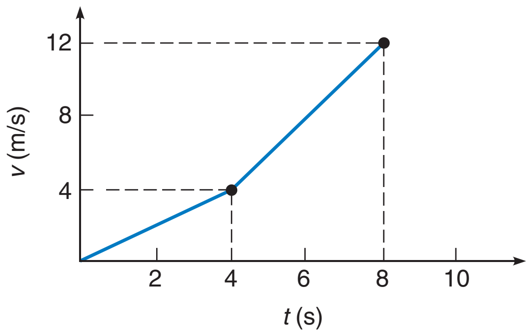
E17. If a world-class sprinter ran a distance of 100 meters starting at his top speed of 11 m/s and running with constant speed throughout, how long would it take him to cover the distance?

E18. Starting from rest, a car accelerates at a constant rate of 3.0 m/s^2 for a time of 5 seconds.
a. Compute the velocity of the car at 1 s, 2 s, 3 s, 4 s, and 5 s and plot these velocity values against time.
b. Compute the distance traveled by the car for these same times and plot the distance values against time.

synthesis problems

SP1. A railroad engine moves forward along a straight section of track for a distance of 80 m due west at a constant speed of 5 m/s. It then reverses its direction and travels 20 m due east at a constant speed of 4 m/s. The time required for this deceleration and reversal is very short due to the small speeds involved.
  a. What is the time required for the entire process?
  b. Sketch a graph of average speed versus time for this process. Show the deceleration and reacceleration upon reversal as occurring over a very short time interval.
  c. Using negative values of velocity to represent reversed motion, sketch a graph of velocity versus time for the engine.
  d. Sketch a graph of acceleration versus time for the engine.

SP2. The velocity of a car increases with time as shown in the graph.
  a. What is the average acceleration between 0 seconds and 4 seconds?
  b. What is the average acceleration between 4 seconds and 8 seconds?
  c. What is the average acceleration between 0 seconds and 8 seconds?
  d. Is the result in part c equal to the average of the two values in parts a and b? Compare and explain.



SP3. A car traveling due west on a straight road accelerates at a constant rate for 10 seconds increasing its velocity from 0 to 24 m/s. It then travels at constant speed for 10 seconds and then decelerates at a steady rate for the next 5 seconds to a velocity of 10 m/s. It travels at this velocity for 5 seconds and then decelerates rapidly to a stop in a time of 2 seconds.
  a. Sketch a graph of the car's velocity versus time for the entire motion just described. Label the axes of your graph with the appropriate velocities and times.
  b. Sketch a graph of acceleration versus time for the car.
  c. Does the distance traveled by the car continually increase in the motion described? Explain.

SP4. A car traveling in a straight line with an initial velocity of 14 m/s accelerates at a rate of 2.0 m/s^2 to a velocity of 24 m/s.
  a. How much time does it take for the car to reach the velocity of 24 m/s?
  b. What is the distance covered by the car in this process?
  c. Compute values of the distance traveled at 1-second intervals and carefully draw a graph of distance plotted against time for this motion.

SP5. Just as car A is starting up, it is passed by car B. Car B travels with a constant velocity of 10 m/s, while car A accelerates with a constant acceleration of 4.5 m/s^2, starting from rest.
  a. Compute the distance traveled by each car for times of 1 s, 2 s, 3 s, and 4 s.
  b. At what time, approximately, does car A overtake car B?
  c. How might you go about finding this time exactly? Explain.

home experiments and observations

HE1. How fast do you normally walk? Using a meter stick or a string of known length, lay out a straight course of 40 or 50 meters. Then use a watch with a second hand or a stopwatch to determine:
  a. Your normal walking speed in m/s.
  b. Your walking speed for a brisk walk.
  c. Your jogging speed for this same distance.
  d. Your sprinting speed for this distance.

Record and compare the results for these different cases. Is your sprinting speed more than twice your speed for a brisk walk?

HE2. The speed with which hair or fingernails grow provides some interesting measurement challenges. Using a millimeter rule, estimate the speed of growth for one or more of: fingernails, toenails, facial hair if you shave regularly, or hair near your face (such as sideburns) that will provide an easy reference point. Measure the average size of clippings or of growth at regular time intervals.
  a. What is the average speed of growth? What units are most appropriate for describing this speed?
  b. Does the speed appear to be constant with time? Does the speed appear to be the same for different nails (thumb versus fingers, fingernails versus toenails), or in the case of hair, for different positions on your face?