CHAPTER 5

MEASURES OF CENTRAL TENDENCY

OBJECTIVES

After completing this chapter, you should

- be able to define, discuss, and compute the most commonly encountered measures of central tendency–the mean, median, and mode.
- understand the properties and advantages and disadvantages of each measure.

CHAPTER REVIEW

Central tendency is the trend for scores in a distribution to be concentrated near the middle of the distribution. The measures of central tendency discussed in the chapter are the mean, the median, and the mode.

The *mode* is the most frequently occurring score in a frequency distribution. The mode is abbreviated *Mo;* a distribution with two modes is called *bimodal,* and both values are reported.

The *median* (*Md*) is a point along the score scale separating the top 50% of the scores from the bottom 50%. The median is the score value at the 50th percentile, with *percentile* defined as the score at or below which a given percentage of the scores lie. The procedure detailed for finding the median is the counting method.

The counting method is used for locating the median in a frequency distribution or in a simple array of scores ranked from highest to lowest. First, the total N is considered. If N is odd, the median is the score in the middle. For example, if there are 9 scores, the median will be the 5th score because there are 4 scores below it and 4 scores above it. If the total N is even, the median will be halfway between the two scores in the middle. Thus, if there are 10 scores, the median will be halfway between the 5th and 6th scores because there are 4 scores below the 5th and 4 scores above the 6th.

The *mean* ($\bar{X}$ for a sample, μ for a population) or arithmetic average is the sum of the scores divided by the number of scores. The mean is often called the balancing point in the distribution because if we sum

algebraically all of the positive and negative deviations about the mean, the result is zero; that is, the positive and negative differences about the mean are in balance.

Four rounding rules are given in the chapter: (1) Round everything to hundredths; (2) if possible, round only the final answer; (3) in all preliminary calculations leading up to the final answer, maintain at least three decimal places; (4) if the digit in the thousandths place is less than 5, drop it and everything that follows it; (5) if the number in the thousandths place is 5 or more, round the preceding digit up.

The *mode* is the best measure in three instances: (1) when you need the quickest estimate of central tendency, (2) when you want to report the most frequently occurring score, or (3) when you have nominal scale data. The *median* is preferred when (1) you have a small, badly skewed distribution, or (2) there are missing or arbitrarily determined scores. The *mean* is the most useful of the measures of central tendency because many important statistical procedures depend on it. Also, the mean is the most stable of the measures from sample to sample. The sample mean is an unbiased estimate of the population mean.

In a symmetrical, unimodal distribution, the mean, median, and mode all have the same value. The mean is pulled in the direction of the tail in a skewed distribution, and the median will be between the mean and the mode.

SYMBOLS

Symbol	Stands For
Mo	mode
Md	median
$\overline{X}$	sample mean
Σ	capital sigma, indicates to sum the following values
$X - \overline{X}$	deviation of a score from the mean
μ	population mean, read "mu"

FORMULAS

Formulas 5-1 and **5-2.** *Equations for computing the sample mean*

$$\overline{X} \text{ (read "ex-bar")} = \frac{\Sigma X}{N} \text{ or } \frac{\Sigma fX}{N}$$

Formula 5-1 tells you to add all the Xs or scores and then to divide the result by the total number of scores (N). In a frequency distribution, N is the sum of frequencies. Formula 5-2 tells you to multiply each score by its frequency before summing and dividing by N.

TERMS TO DEFINE AND/OR IDENTIFY

central tendency

mode

bimodal

multimodal

median

percentile

mean

deviations

unbiased estimate

FILL-IN-THE-BLANK ITEMS

Introduction

Measures of central tendency are values near the (1) _____ of the distribution. The measures

discussed in the chapter are the (2) _____, the (3) _____, and the

(4) _____.

The Mode

The most frequently occurring score is called the (5) _____, symbolized by

(6) _____. The mode is the (7) _____ stable of the measures of central

tendency. A distribution with two modes is called (8) _____, and both values are reported.

The Median

The median is the score value at the (9) _____ percentile. A (10) _____ is the

score at or below which a given percentage of the scores lie.

Locating the median by the counting method: Even number of scores

In a frequency distribution with an even number of scores, the median will be halfway between the

(11) _____ score and the (12) _____ score.

Locating the median by the counting method: Odd number of scores

If there is an odd number of scores, the median will be the (13) _____ score.

The Mean, or Arithmetic Average

The mean or arithmetic average is the sum of the (14) _____ divided by the

(15) _____ of scores in a distribution. In a frequency distribution, the

(16) _____ must be taken into account when the mean is determined. The symbol for the

mean of a sample is (17) _____, whereas the symbol for the corresponding population

parameter is (18) _____.

Rounding conventions

Rounding rules are as follows:

1. Round your final answer to (19) _____.
2. If possible, round only the (20) _____ answer.
3. In all preliminary calculations leading up to the final answer, maintain at least

    (21) _____ decimal places.
4. If the digit in the thousandths place is less than 5, (22) _____ it and everything that

    follows it.
5. If the digit in the thousandths place is 5 or more, round the preceding digit (23) _____.

The mean as a balancing point

The mean is called the (24) _____ point in the distribution because the sum of the deviations

about it is equal to (25) _____.

Comparing Measures of Central Tendency

The (26) _____ is useful for summarizing nominal scale data and for obtaining a rough

estimate of the mean and the median. The (27) _____ is the best measure of central tendency

when a distribution is badly skewed or when there are (28) _____ scores. The mean is the

most useful of the measures because most other (29) _____ procedures are based on it. Also,

it is the most (30) _____ of the measures from sample to sample. Because the mean shows

no systematic tendencies in relation to the population mean, it is called an (31) _____

estimate of μ.

Positions of Measures of Central Tendency on a Frequency Polygon

In a unimodal, symmetrical distribution, the mean, median, and mode will all be the

(32) _____. In skewed distributions, the (33) _____ is most affected and is

pulled in the direction of the (34) _____. The median will be between the

(35) _____ and the mean in a skewed distribution.

Troubleshooting Your Computations

The most important thing to remember in locating the measures of central tendency is that your answer

should be a value near the (36) _____ of the distribution. Also, if you are trying to find the

mean in a frequency distribution, you must remember to take the (37) _____ into account.

To help prevent computational errors, you should perform all computations

(38) _____, being sure you get the same answer each time.

PROBLEMS

1. For the following distributions, try to give the three measures of central tendency without computing
   them. After you have done this, compute $\overline{X}$ for each distribution.

   a. 6, 10, 10, 14
   b. 7, 8, 9
   c. 1, 2, 3, 3, 4, 5

d.

X	f
20	1
18	2
16	4
14	2
12	1

2. Using the following distribution, find each of the three measures of central tendency. Show that the mean is the balancing point in the distribution—that is, $\Sigma f(X - \bar{X}) = 0$.

X	f
10	1
9	2
8	1
7	4
6	6
5	5
4	2
3	1
2	1

3. A neuroticism scale consisting of 10 items has been given to 50 individuals; higher scores mean greater neuroticism. The frequency distribution of their scores is as follows:

X	f
10	4
9	4
8	5
7	7
6	8
5	7
4	5
3	4
2	4
1	2

Find the mean, median, and mode of the scores.

4. A recall test of retention has been given after 25 students have studied briefly a list of 15 nouns. Determine the mean, median, and mode of the results.

X	f
6	2
5	0
4	6
3	5
2	8
1	3
0	1

5. The following data were obtained for 25 students given a recognition test of retention of 15 previously studied nouns. Find the median, mode, and mean.

X	f
15	7
14	6
13	4
12	2
10	3
8	1
6	2

6. ACT scores have been determined for 26 students in an honors introductory psychology class and are shown in the following frequency distribution. Compute the mean, the median, and the mode.

X	f
33	1
31	1
30	3
29	4
28	4
27	8
26	2
25	2
24	1
	$N = 26$

7. In a study of 15 rats, the latency to enter a darkened chamber has been recorded. If the animal didn't enter within 60 seconds, the trial was terminated and the animal received a score of 60. Determine the most appropriate measure of central tendency.

X	f
60	2
45	2
43	1
42	1
35	2
33	3
32	1
25	1
20	1
10	1

Omitting the nonresponders, determine all three measures of central tendency.

8. Using the rules for rounding, round each of the following. For each of your answers, cite the applicable rule.
   a. 1.4549 to hundredths

   b. 1.5551 to hundredths

   c. 3.66666 to hundredths

   d. 23.33333 to hundredths

   e. 7.825137 to hundredths

USING SPSS—EXAMPLE AND EXERCISE

From now on, we will assume that you have read Appendix 4 on using SPSS.

SPSS has several procedures that can provide measures of central tendency and dispersion. For example, Frequencies, Descriptives, or Explore could be used.

Example: We will use SPSS to obtain the three measures of central tendency (mean, median, and mode) using the data in Problem 6. The steps are as follows:

1. Start SPSS, enter the data, and name the variable **act.**
2. We will use Frequencies to obtain a frequency distribution and descriptive statistics.
3. Click *Analyze>Descriptive Statistics>Frequencies*.
4. Move **act** into the Variables box.
5. Click the *Statistics* box to open the Frequencies: Statistics dialog box. There click *Mean, Median, Mode* under Central Tendency, and then click *Continue>OK*.
6. The statistics and a frequency distribution should appear in the output Viewer window. The output showing the descriptive statistics and frequency distribution for Problem 6 should appear as follows:

```
FREQUENCIES
  VARIABLES=act
  /STATISTICS=MEAN MEDIAN MODE
  /ORDER  ANALYSIS .
```

Frequencies

Statistics

ACT

N	Valid	26
	Missing	0
Mean		27.8462
Median		27.5000
Mode		27.00

ACT

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	24.00	1	3.8	3.8	3.8
	25.00	2	7.7	7.7	11.5
	26.00	2	7.7	7.7	19.2
	27.00	8	30.8	30.8	50.0
	28.00	4	15.4	15.4	65.4
	29.00	4	15.4	15.4	80.8
	30.00	3	11.5	11.5	92.3
	31.00	1	3.8	3.8	96.2
	33.00	1	3.8	3.8	100.0
	Total	26	100.0	100.0	

Exercise Using SPSS

1. Using the data in Problem 3, use SPSS to obtain the mean, median, and mode of the scores.

CHECKING YOUR PROGRESS: A SELF-TEST

1. Match the following:
    ____ the measure used with nominal data
    ____ the most frequently occurring score in a distribution
    ____ the arithmetic average
    ____ the least stable measure of central tendency
    ____ the preferred measure of central tendency in a badly skewed distribution
    ____ the score at the 50th percentile
    ____ the measure of central tendency most affected by extreme scores
    ____ the sum of the deviations about it is zero
    a.   mode   **b.**   median   **c.**   mean

2. Give all three measures of central tendency for the following distribution of golf scores after the final round of a tournament. Each score tells how a golfer finished relative to par; for example, a score of –4 means the golfer was 4 strokes under par.

X	f		X	f
–14	1		0	4
–12	1		+1	3
–11	2		+2	1
–10	3		+3	4
–8	3		+4	3
–7	2		+5	1
–6	2		+6	3
–5	1		+7	2
–4	3		+8	1
–1	5		+17	1

3. Find the mean, median, and mode for the following distribution of IQ scores.

X	f
147	2
142	3
137	4
132	6
127	5
122	8
117	4
112	3
107	2
102	2