CHAPTER 7

PROBABILITY

OBJECTIVES

After completing this chapter, you should

- be able to define and discuss intuitively probability in our everyday lives.
- understand and be able to use some basic rules from probability theory.
- have an appreciation of Bayesian statistics and the binomial probability distribution.

CHAPTER REVIEW

Probability theory is the key to testing statistical hypotheses. *Statistical hypotheses* are assumptions about populations based on sample results. Probability theory enables us to determine the amount of certainty we have in the conclusions we draw from our sample results.

The *probability* of an event is defined as the proportion of times an event would occur if the chances for occurrence were infinite. Another way to say this is that the probability of an event is the ratio of the number of ways the event can occur to the number of ways any event can occur. Although we constantly assess probabilities in our daily lives, there are times when our intuitive ideas about probability are incorrect. For example, the *gambler's fallacy* is the mistaken belief that the probability of a particular event changes with a long string of the same event. We often say that a team that has lost many times in a row is due to win, as though the string of losses had changed the probability of the team's winning.

To you as an individual, probability means probability, not certainty. Probabilities should be considered patterns or tendencies, not guarantees about what will happen to you personally. Theoretical probability, on the other hand, is the way events are supposed to work in terms of formal probability theory. Real-world, or empirical, probability is based on experience, whereas subjective probability is our personal probability, formed on the basis of our own perspective on the world. Bayesian statistics uses subjective probability as a starting point for assessing a subsequent probability. The Bayesian approach to making statistical inferences involves the use of these subjective, prior probabilities, which makes this approach controversial.

The *addition rule* states that for independent events the probability of either one event or another is equal to the sum of the probabilities of the individual events. The multiplication rule determines the probability of a series of events. The *multiplication rule* states that the probability of two or more independent events occurring on separate occasions is the product of their individual probabilities. *Conditional probability* is the probability of an event given that another event has already occurred. The multiplication rule can be modified to determine the probability of nonindependent events.

The binomial distribution—a simple theoretical probability distribution—is based on events for which there are only two possible outcomes on each occurrence of the event. Coin-flipping examples are often used to illustrate the construction of a binomial distribution. Two important features of the binomial are that when $p = .5$, the distribution is symmetrical, and as the number of trials increases, the binomial more and more closely approximates the normal probability distribution.

SYMBOLS

Symbol	*Stands For*
$p(\text{A})$	probability of event A
$p(\text{A or B})$	probability of event A or event B
$p(\text{A, B})$	probability of both A and B
$p(\text{B}\vert\text{A})$	probability of event B given that event A has occurred

FORMULAS

Formula 7-1. *Equation for the addition rule of probability*

$$p(\text{A or B}) = p(\text{A}) + p(\text{B})$$

$p(\text{A or B})$ means the probability of either event A *or* event B, and it is equal to the probability of event A [$p(\text{A})$] plus the probability of event B [$p(\text{B})$].

Formula 7-2. *Equation for the multiplication rule of probability*

$$p(\text{A, B}) = p(\text{A}) \times p(\text{B})$$

$p(\text{A, B})$ is the probability of occurrence of both event A *and* event B, which is equal to the product of their individual probabilities. This equation is used when events A and B are independent.

Formula 7-3. *Equation for determining the probability of a sequence of nonindependent events*

$$p(\text{A, B}) = p(\text{A}) \times p(\text{B}|\text{A})$$

When events A and B are not independent—that is, when the probability of B depends on whether A has occurred—then the multiplication rule must be modified as shown. $p(\text{B}|\text{A})$ reads "probability of B given A."

TERMS TO DEFINE AND/OR IDENTIFY

statistical hypotheses

gambler's fallacy

probability

theoretical probability

real-world probability

personal (subjective) probability

Bayesian statistics

addition rule of probability

multiplication rule of probability

independent events

nonindependent events

conditional probability

binomial distribution

FILL-IN-THE-BLANK ITEMS

Introduction

(1) _______________, _______________ are guesses about populations based on sample results. Although we can never be certain that our guesses are correct, (2) _________________ theory will help us determine the degree of certainty we have in our conclusions. The essence of inferential statistics is in using sample (3) _________________ to attach a probability to the estimates of (4) _________________ parameters.

Thinking About Probability

One intuitive idea about probability is called (5) _________________ _________________, the mistaken belief that the probability of an event changes with a long string of the event. Formally, (6) _________________ is defined as the proportion of times an event would occur if the chances for occurrence were infinite. In other words, the probability of an event is equal to the number of times the event can occur divided by the number of ways (7) _________________ event can occur.

Probability and the individual

In terms of what will happen to you personally, probabilities should be considered long-run (8) _________________, and not (9) _________________.

Theoretical probability; Real-world probability

(10) _________________ probability is the way events are supposed to work in terms of formal probability theory. Probabilities based on past behavior and counting are called real-world, or (11) _________________ probabilities, and these are the basis for many assessments of chance that affect our lives. This type of probability is sometimes called (12) _________________ probability because the occurrence of events has been tallied relative to the number of opportunities for the event to occur.

Subjective probability

Probabilities based on our own perspectives are called (13) _________________ or (14) _________________ probabilities. Such probabilities are used in an area called

(15) _________________ statistics. The classical approach to (16) _________________ tells us to make our decision about our experiment's outcome on the basis of the data, without making any prior assumptions. The (17) _________________ approach, on the other hand, would have us use the data from our experiment to adjust our prior beliefs. The weak point of this approach is that prior probabilities may be (18) _________________, and experimenters could reach different (19) _________________ from the same data if they started with different prior beliefs.

Rules of Probability

The addition rule

For mutually exclusive, random events, the probability of either one event or another event is the (20) _________________ of the probabilities of the individual events. This is called the (21) _________________ rule of probability. The formula for the rule is as follows: p(A or B) = (22) _________________.

The multiplication rule

The (23) _________________ rule states that the probability of two or more independent events occurring on separate occasions is the product of their individual probabilities. The rule is shown symbolically as follows: (24) _________________ = _________________.

Events are (25) _________________ if the occurrence of one event does not alter the probability of any other event. (26) _________________ _________________ is the probability of an event given that another event has already occurred, expressed symbolically by (27) _________________. The multiplication rule for independent events can be modified to include nonindependent events. Thus, the probability for events A and B, where the probability of B depends on A, is found with this formula: p(A, B) = (28) _________________.

More on conditional probabilities

(29) _________________ probability can help us assess probabilities of events in our world by providing a way to add information to probabilities we already know. On an intuitive level, if $p(B|A) = p(B)$, then the occurrence or nonoccurrence of (30) _________________ has nothing to do with the occurrence of (31) _________________.

Bayesian statistics

Thomas Bayes initiated using (32) _________________ to help establish a mathematical basis for statistical inference. The Bayesian approach to probability and statistics is (33) _________________, and it has not been widely adopted.

The Binomial Probability Distribution

The binomial distribution is based on events for which there are only (34) _________________ possible outcomes on each occurrence. Two important features of the binomial distribution are that (a) when $p = .5$, the distribution is (35) _________________, and, as N (the number of trials) increases in value, the distribution more closely approximates the (36) _________________ _________________ _________________.

PROBLEMS

1. You've flipped a coin 9 times, and it has come up heads each time. There's no reason to believe the coin is biased. Do you think it's more or less likely that the next flip will produce another head? What is the probability of another head on the 10th flip?

2. Suppose you draw a single card from a standard 52-card deck. What is the probability of drawing the following cards?

a. the ace of hearts

b. an ace

c. a face card (jack through ace)

d. a heart

e. a card that is not a heart

3. In a sock drawer, there are 6 brown socks, 12 blue socks, 4 red socks, and 8 green socks randomly mixed together. With your eyes shut, you take a single sock from the drawer. What is the probability of getting the following socks?

a. a brown sock

b. a blue sock

c. a yellow sock

d. a sock that is not red

e. a red or a green sock

f. a sock that is neither blue nor green

4. Suppose you answer a single question on a true–false test purely by chance.

a. What is the probability that you will miss it?

b. What is the probability that you'll get it right?

5. Suppose you have a multiple-choice question with four choices.

a. What is the probability of guessing correctly?

b. What is the probability of an incorrect guess?

c. In terms of probability, which is easier, a multiple-choice question or a true–false question?

6. A slot machine has three identical reels with 10 different symbols on each reel. You pull the lever, and the reels spin independently until they stop. Assuming the symbols include stars, whistles, and clowns, what is the probability of getting the following?

a. three stars

b. two stars and a whistle

c. a star, a whistle, and a clown

d. the same three of any of the 10 symbols.

7. A box contains six pieces of currency: three \$1 bills, two \$5 bills, and one \$20 bill. You can remove only one bill at a time. On two successive draws, *with* replacement, what is the probability of getting the following?

a. the \$20 bill twice

b. two \$1 bills

c. two \$5 bills

d. a \$1 bill and the \$20 bill, in that order

e. \$21 for the sum of the two bills

f. \$6 for the sum of the two bills

8. Consider again the box from Problem 7. On two successive draws, *without* replacement, what is the probability of getting the following?

a. \$40

b. \$25

c. \$2

d. \$6

9. The psychology club conducted a lottery and sold 150 tickets. You bought 1 ticket, and your friend bought 3 tickets. There will be a first-place winner and a second-place winner.

a. What is the probability that you will win first place?

b. What is the probability that you will win first or second place?

c. What is the probability that you or your friend will win first place?

d. What is the probability that you or your friend will win something (either first or second place)?

e. What is the probability that both you and your friend will win something?

10. Suppose you obtain a biased coin with $p(\text{head}) = .6$ and $p(\text{tail}) = .4$. What is the probability of three heads in five flips? Of four heads in five flips?

11. A study was made of the relationship between personality type (extraversion/introversion) and whether a student held an office in a club or student organization. The results are summarized in the following table of probabilities:

Personality Type

		Extravert	Introvert	Total
Held Office	Yes	.28	.10	.38
	No	.24	.38	.62
	Total	.52	.48	1.00

a. What is the probability that a student will hold some office?

b. What is the probability that, given the student is an extravert, he or she will hold some office?

c. Does the added information about personality aid in predicting whether the student holds office?

d. Is personality type extravert or introvert (E–I) independent of holding office? Explain.

CHECKING YOUR PROGRESS: A SELF-TEST

1. An unbiased die is rolled 9 times, coming up 6 each time. What is the probability of its coming up 6 on the 10th roll?
  a. 1/6
  b. less than 1/6
  c. more than 1/6
  d. none of the above

2. The probability of an event given that another event has already occurred is called which of the following?
  a. gambler's fallacy
  b. the addition rule of probability
  c. the multiplication rule of probability
  d. conditional probability

3. Your psychology instructor says that it is impossible for you to locate accurately sounds in your median plane (the plane that splits you down the middle) if you are blindfolded. To prove her point, you are blindfolded, and a clacker is used to make a brief sound either directly in front of you, directly overhead, or directly behind.

  a. What is the probability of correctly picking the location on the first presentation?

  b. What is the probability of picking the correct location by chance on three consecutive trials?

4. A fruit basket contains 1 apple, 2 oranges, and 3 large plums. *With* replacement, what is the probability of selecting, on three consecutive draws blindfolded, the following?

  a. 3 apples

  b. 2 oranges and 1 plum, in that order

  c. 2 oranges and 1 plum, in any order

5. Using the same basket from Problem 4, what is the probability of selecting, *without* replacement, the following on three consecutive blindfolded draws?

a. an apple, an orange, and a plum

b. 2 oranges and 1 plum, in that order

c. 3 plums

6. A study of personality type (intuition vs. sensing) and the probability of holding student office was made. The results were as follows:

Personality Type

		Sensing	Intuition	Total
Held Office	Yes	.23	.15	.38
	No	.37	.25	.62
	Total	.60	.40	1.00

a. What is the probability that a student has held some office?

b. Given that a student is an intuitive type, what is the probability that he or she has held office?

c. Does this added information about personality type aid in predicting whether the student has held office?

d. In this problem, is personality type independent of holding office? Explain.