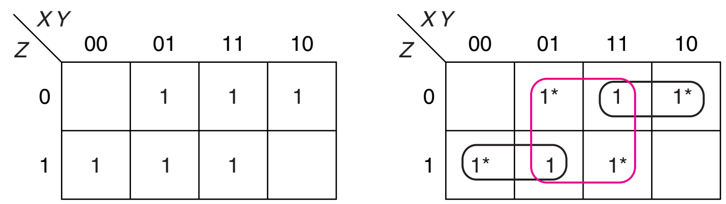
c. $g(a, b, c, d) = \Sigma m(0, 6, 8, 9, 10, 11, 13, 14, 15)$
(2 solutions)

d. $f(a, b, c, d) = \Sigma m(0, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$
(2 solutions)

e. $f(a, b, c, d) = \Sigma m(0, 1, 2, 4, 6, 7, 8, 9, 10, 11, 12, 15)$

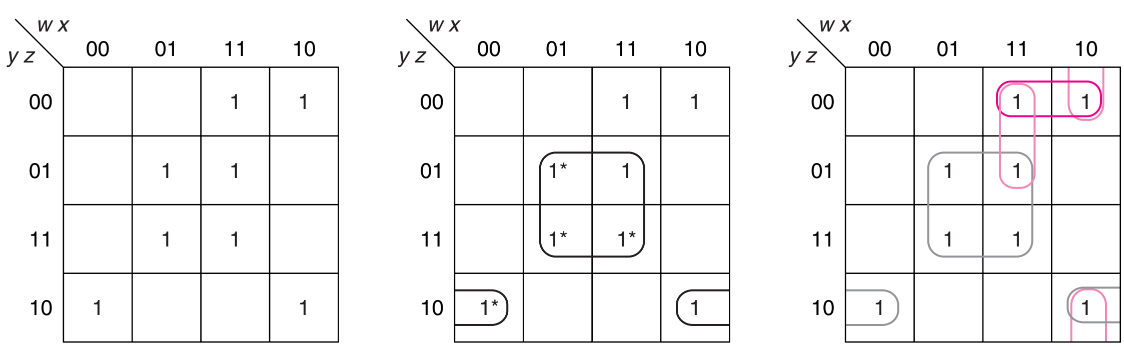
f. $g(a, b, c, d) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 11, 12, 13, 14, 15)$
(4 solutions)

a. All of the prime implicants are essential, as shown on the map to the right.



$G = Y + XZ' + X'Z$

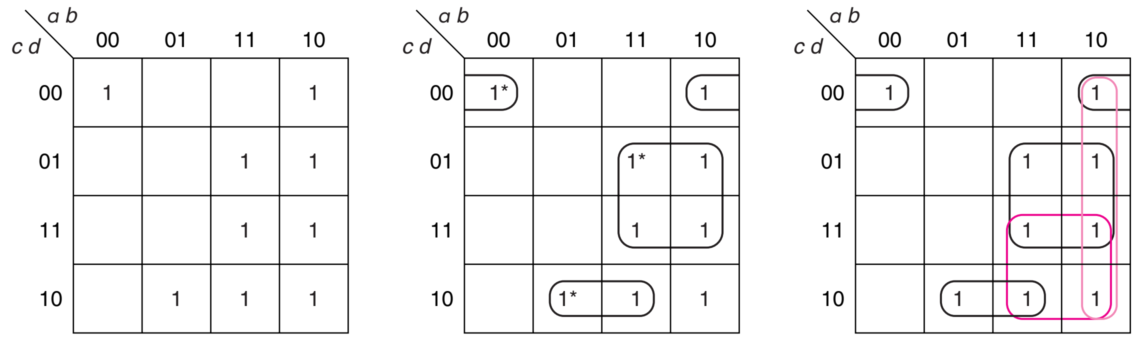
b.



The essential prime implicants are shown on the second map, leaving two 1's to be covered. The third map shows that each can be covered by two different prime implicants, but the green group shown is the only one that covers both with one term. We would require both light green terms. The minimum is

$$f = xz + x'yz' + wy'z'$$

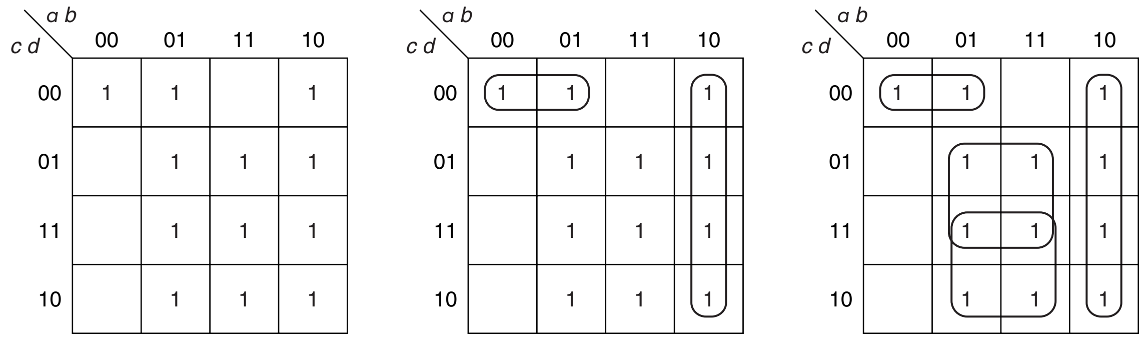
c.



The three essential prime implicants are shown on the center map. The only 1 left to be covered can be covered by either of two groups of four, as shown circled in green on the third map, producing

$$g = b'c'd' + bcd' + ad + ab'$$
$$g = b'c'd' + bcd' + ad + ac$$

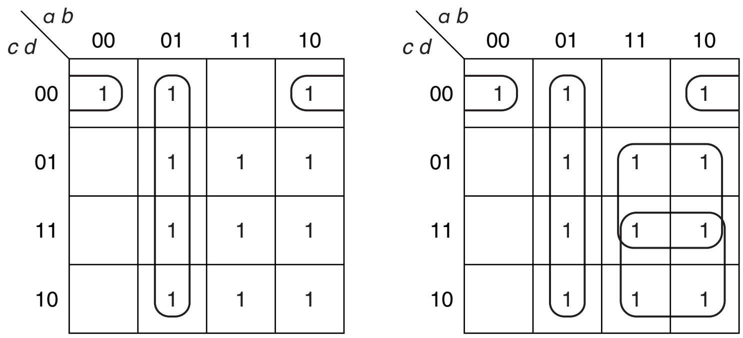
d.



There are no essential prime implicants. We need one group of two to cover m_0; all other 1's can be covered by groups of four. Once we have chosen $a'c'd'$ to cover m_0 (center map), we would choose ab' to cover m_8. (Otherwise, we must use $b'c'd'$, a group of two to cover that 1. Not only is that more literals, it covers nothing else new; whereas ab' covered three additional uncovered 1's.) Once that has been done, the other two prime implicants become obvious, giving

$$f = a'c'd' + ab' + bc + bd$$

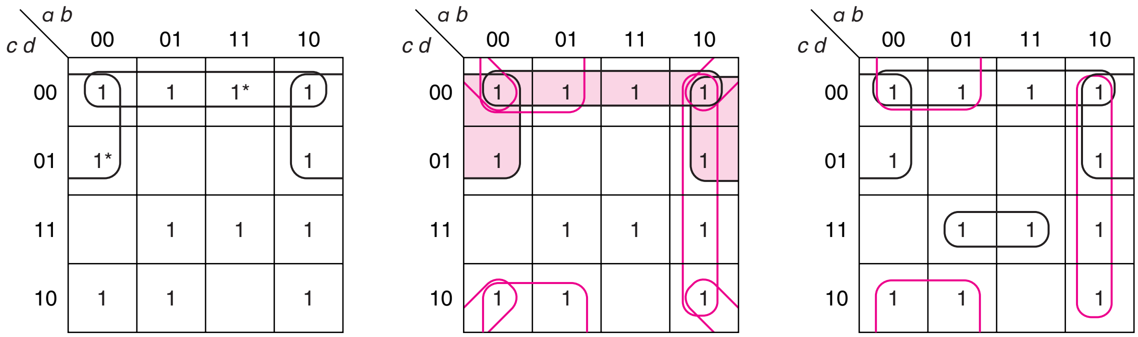
In a similar fashion (on the next map), once we choose $b'c'd'$ (the other prime implicant that covers m_0), $a'b$ is the appropriate choice to cover m_4:



The only way to cover the remaining 1's in two terms is with ac and ad, as shown on the second map, leaving

$$f = b'c'd' + a'b + ac + ad$$

e. There are two essential prime implicants, as indicated on the first map, leaving six 1's to be covered. The essential prime implicants are shaded on the second map.



No prime implicant covers more than two of the remaining 1's; thus three more terms are needed. The three groups of four (two literal terms) are circled in green on the second map. We can cover four new 1's only using $a'd'$ and ab'. Note that m_7 and m_{15} are uncovered; they require a group of two, bcd. The only minimum solution, requiring five terms and 11 literals,

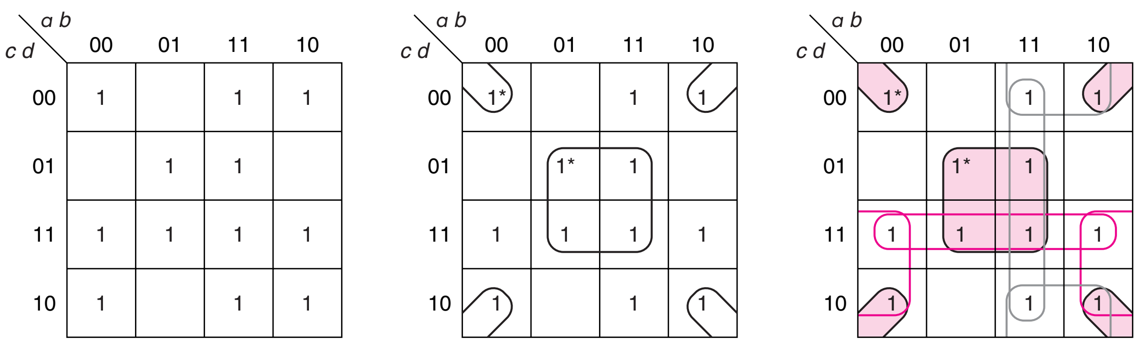
$$f = c'd' + b'c' + a'd' + ab' + bcd$$

is shown on the third map. There is another solution that uses five terms, but it requires 12 literals, namely,

$$f = c'd' + b'c' + b'd' + a'bc + acd$$

Obviously, it is not minimum (since it has an extra literal); it only used one of the groups of four instead of two.

f. On the second map, the two essential prime implicants have been highlighted ($b'd' + bd$), leaving four 1's uncovered. On the third map, we have shown the 1's covered by these prime implicants shaded.



We can cover m_3 and m_{11} by either cd or $b'c$ (shown with green lines), and we can cover m_{12} and m_{14} by either ab or ad' (shown in gray lines). Thus, there are four solutions:

$$f = b'd' + bd + cd + ab$$
$$f = b'd' + bd + cd + ad'$$
$$f = b'd' + bd + b'c + ab$$
$$f = b'd' + bd + b'c + ad'$$

The term ac is also a prime implicant. However, it is not useful in a minimum solution since it leaves two isolated 1's to be covered, resulting in a five-term solution.

2. For the following functions,

i. List all prime implicants, indicating which are essential.

ii. Show the minimum sum of products expression(s).

a. $G(A, B, C, D) = \Sigma m(0, 1, 4, 5, 7, 8, 10, 13, 14, 15)$ (3 solutions)

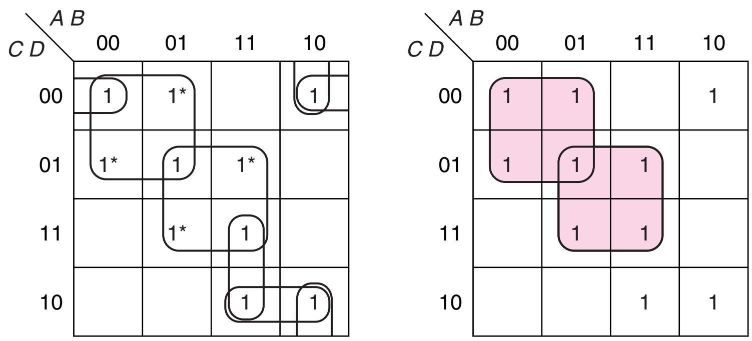
b. $f(w, x, y, z) = \Sigma m(2, 3, 4, 5, 6, 7, 9, 10, 11, 13)$

c. $h(a, b, c, d) = \Sigma m(1, 2, 3, 4, 8, 9, 10, 12, 13, 14, 15)$ (2 solutions)

a. The first map shows all of the prime implicants circled; the 1's that have been covered only once are indicated with an asterisk:

Essential prime implicants: $A'C'$, BD

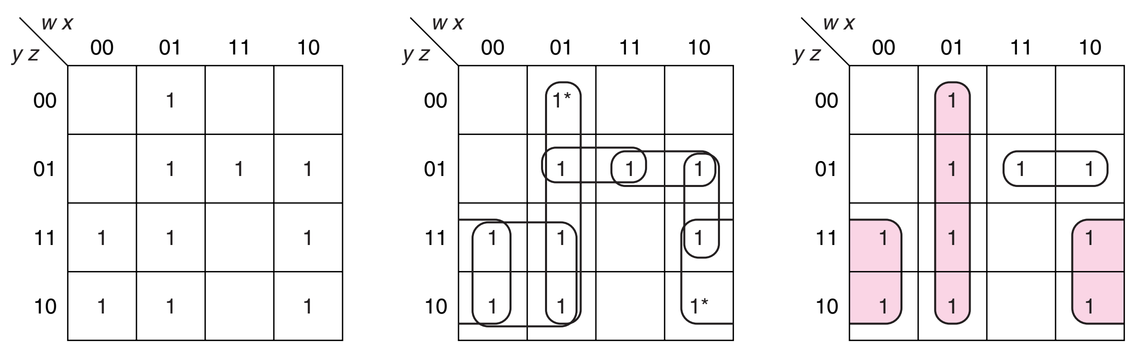
Other prime implicants: $B'C'D'$, $AB'D'$, ACD', ABC



On the second map, the essential prime implicants have been shaded, highlighting the three 1's remaining to be covered. We need two terms to cover them, at least one of which must cover two of these remaining 1's. The three solutions are thus

$$F = A'C' + BD + ACD' + B'C'D'$$
$$F = A'C' + BD + AB'D' + ACD'$$
$$F = A'C' + BD + AB'D' + ABC$$

b.



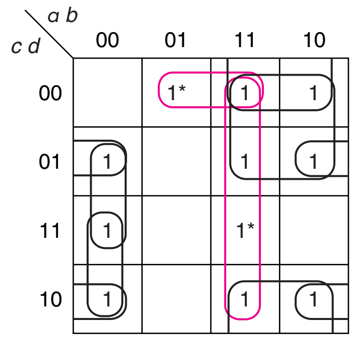
The second map shows all of the prime implicants circled and the 1's that have been covered only once are indicated with an asterisk:

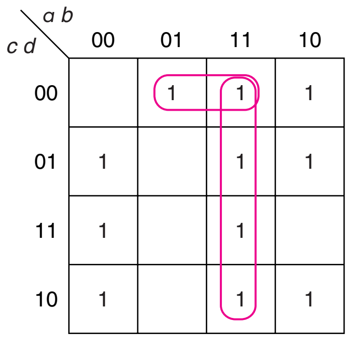
Essential prime implicants: $w'x$, $x'y$
Other prime implicants: $w'y$, $xy'z$, $wy'z$, $wx'z$

With the essential prime implicants shaded on the third map, it is clear that the only minimum solution is

$$f = w'x + x'y + wy'z$$

c. All of the prime implicants are circled on the first map, with the essential prime implicants shown in green.

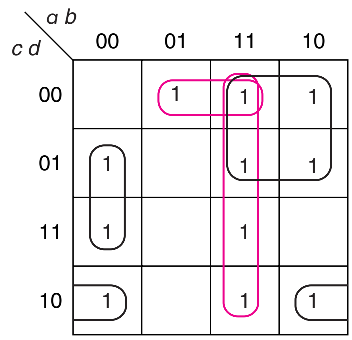


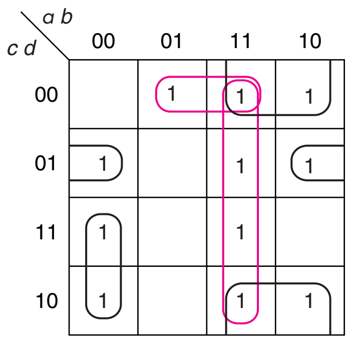


Essential prime implicants: ab, $bc'd'$

Other prime implicants: ac', ad', $b'c'd$, $b'cd'$, $a'b'c$, $a'b'd$

Once we chose the essential prime implicants, there are six 1's left to be covered. We can only cover two at a time. There are two groups of four 1's, either of which can be used. (We cannot use both, since that would only cover three 1's.) The two solutions are shown on the maps below.





$$h = ab + bc'd' + ac' + a'b'd + b'cd'$$
$$h = ab + bc'd' + ad' + b'c'd + a'b'c$$

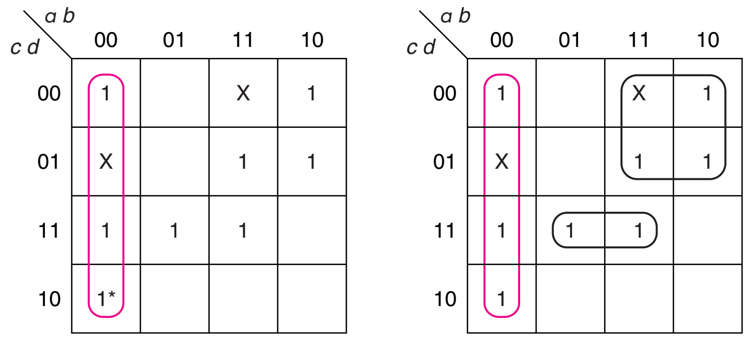
3. For each of the following, find all minimum sum of product expressions. (If there is more than one solution, the number of solutions is given in parentheses.)

a. $f(a, b, c, d) = \Sigma m(0, 2, 3, 7, 8, 9, 13, 15) + \Sigma d(1, 12)$

b. $F(W, X, Y, Z) = \Sigma m(1, 3, 5, 6, 7, 13, 14) + \Sigma d(8, 10, 12)$ (2 solutions)

c. $f(a, b, c, d) = \Sigma m(3, 8, 10, 13, 15) + \Sigma d(0, 2, 5, 7, 11, 12, 14)$ (8 solutions)

a.

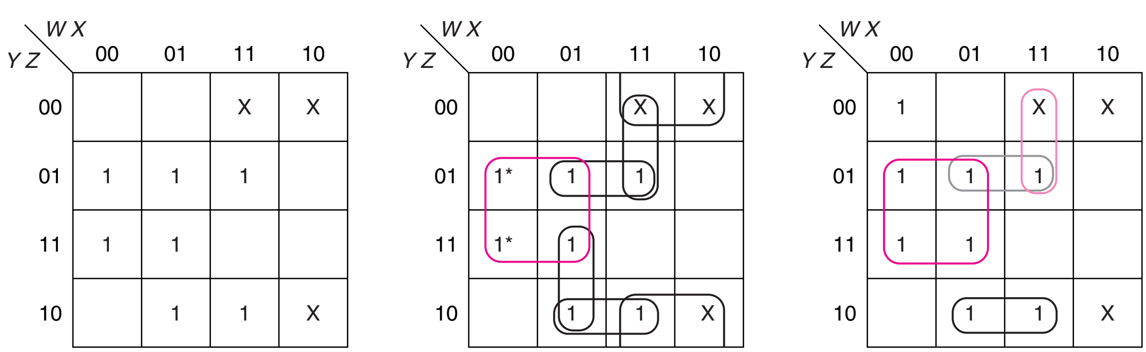


The first map shows the one essential prime implicant, $a'b'$. The remaining 1's can be covered by two additional terms, as shown on the second map. In this example, all don't cares are treated as 1's. The resulting solution is

$$f = a'b' + ac' + bcd$$

Although there are other prime implicants, such as $b'c'$, abd, and $a'cd$, three prime implicants would be needed in addition to $a'b'$ if any of them were chosen.

b.



The second map shows all of the prime implicants circled. It is clear that only $W'Z$ is essential, after which three 1's remain uncovered. The prime implicant XYZ' is the only one that can cover two of these and thus appears in both minimum solutions. That leaves a choice of two terms to cover the remaining one—either WXY' (light green) or $XY'Z$ (gray). Note that they treat the don't care at m_{12} differently and thus, although the two solutions shown below both satisfy the requirements of the problem, they are not equal:

$$F = W'Z + XYZ' + WXY'$$
$$F = W'Z + XYZ' + XY'Z$$

Also, the group of four (WZ') is not used; that would require a four term solution.

c. There are no essential prime implicants in this problem. The left map shows the only two prime implicants that cover m_8; they also cover m_{10}. We must choose one of these. The next map shows the only prime implicants that cover m_{13}; both also cover m_{15}. We must choose one of these also. Finally, the last map shows the only two prime implicants that cover m_3.

cd \ ab	00	01	11	10
00	X		X	1
01		X	1	
11	1	X	1	X
10	X		X	1

cd \ ab	00	01	11	10
00	X		X	1
01		X	1	
11	1	X	1	X
10	X		X	1

cd \ ab	00	01	11	10
00	X		X	1
01		X	1	
11	1	X	1	X
10	X		X	1

So, our final solution takes one from each group, giving us a total of eight solutions:

$$f = \begin{Bmatrix} ad' \\ b'd' \end{Bmatrix} + \begin{Bmatrix} ab \\ bd \end{Bmatrix} + \begin{Bmatrix} cd \\ b'c \end{Bmatrix}$$

or, written out

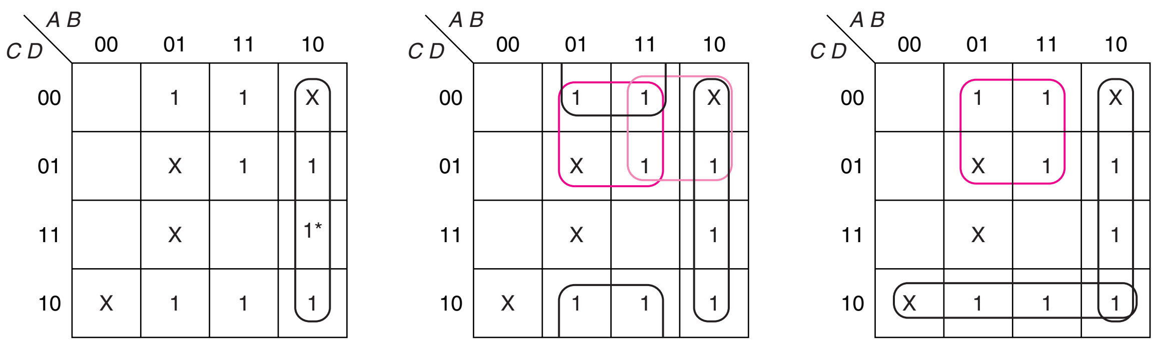
$$f = ad' + ab + cd$$
$$f = ad' + ab + b'c$$
$$f = ad' + bd + cd$$
$$f = ad' + bd + b'c$$
$$f = b'd' + ab + cd$$
$$f = b'd' + ab + b'c$$
$$f = b'd' + bd + cd$$
$$f = b'd' + bd + b'c$$

4. For each of the following, find all minimum sum of product expressions. Label the solutions $f_1, f_2, \ldots$ and indicate which solutions are equal.

a. $F(A, B, C, D) = \Sigma m(4, 6, 9, 10, 11, 12, 13, 14) + \Sigma d(2, 5, 7, 8)$ (3 solutions)

b. $f(a, b, c, d) = \Sigma m(0, 1, 4, 6, 10, 14) + \Sigma d(5, 7, 8, 9, 11, 12, 15)$ (13 solutions)

a.



On the first map, we have shown the one essential prime implicant, AB'. Neither $A'B$ nor CD' are essential, since the 1's covered by them can each be covered by some other prime implicant. (That there is a don't care that can only be covered by one of these terms does not make that term essential.) With five 1's left to be covered, we need two additional terms. The first that stands out is BD', circled on the middle map, since it covers four of the remaining 1's. If that is chosen, it leaves only m_{13}, which can be covered by BC' or AC'. However, the third map shows still another cover, utilizing BC' and CD'. Thus, the three solutions are

$$F_1 = AB' + BD' + BC'$$
$$F_2 = AB' + BD' + AC'$$
$$F_3 = AB' + BC' + CD'$$

Notice that none of the solutions utilize the remaining prime implicant, $A'B$.

Next is the question of whether or not these three solutions are equal. The answer can be determined by examining how the don't cares are treated by each of the functions. The following table shows that:

	2	**5**	**7**	**8**
F_1	0	1	0	1
F_2	0	0	0	1
F_3	1	1	0	1

In all functions, m_7 is treated as 0 (that is, it is not included in any prime implicant used) and m_8 as 1 (since it is included in the essential prime implicant, AB'); but the first two columns show that no two functions treat m_2 and m_5 the same. Thus, none of these is equal to any other.

b. There are no essential prime implicants. The best place to start is with a 1 that can only be covered in two ways; in this problem there is only one, m_1. Any solution must contain either the term $a'c'$ (as shown on the first four maps) or the term $b'c'$ (as shown on the remaining two maps). There is no reason to use both, since $b'c'$ does not cover any 1's that are not already covered by $a'c'$. The first map shows $a'c'$. Note that there are three 1's left, requiring two more terms. At least one of these terms must cover two of the remaining 1's.

cd \ ab	00	01	11	10
00	1	1	X	X
01	1	X		X
11		X	X	X
10		1	1	1

cd \ ab	00	01	11	10
00	1	1	X	X
01	1	X		X
11		X	X	X
10		1	1	1

cd \ ab	00	01	11	10
00	X	X	X	X
01	X	X		X
11		X	X	X
10		X	X	1

The second map shows two ways of covering m_6 and m_{14}, bc and bd'. In either case, only one 1 is left to be covered. The third map shows the previously covered 1's as don't cares and three ways of covering the last 1, m_{10}. Thus, we have as the first six solutions

$$f_1 = a'c' + bc + ab'$$
$$f_2 = a'c' + bc + ac$$
$$f_3 = a'c' + bc + ad'$$
$$f_4 = a'c' + bd' + ab'$$
$$f_5 = a'c' + bd' + ac$$
$$f_6 = a'c' + bd' + ad'$$

Next, we consider how we may cover both m_{10} and m_{14} with one term (in addition to those already found). That provides two more solutions shown on the left map below. (Other solutions that use these terms have already been listed.)

cd \ ab	00	01	11	10
00	1	1	X	X
01	1	X		X
11		X	X	X
10		1	1	1

cd \ ab	00	01	11	10
00	1	1	X	X
01	1	X		X
11		X	X	X
10		1	1	1

cd \ ab	00	01	11	10
00	1	1	X	X
01	1	X		X
11		X	X	X
10		1	1	1

$$f_7 = a'c' + a'b + ad'$$
$$f_8 = a'c' + a'b + ac$$

We next consider the solutions that use $b'c'$. The middle map shows two of these, utilizing $a'b$. The last map shows the final three, utilizing bd', instead; it has the same three last terms as in the first series. Thus, we have

$$f_9 = b'c' + a'b + ad'$$
$$f_{10} = b'c' + a'b + ac$$
$$f_{11} = b'c' + bd' + ab'$$
$$f_{12} = b'c' + bd' + ac$$
$$f_{13} = b'c' + bd' + ad'$$

Finally, the table below shows how each of the functions treats the don't cares:

	5	7	8	9	11	12	15
f_1	1	1	1	1	1	0	1
f_2	1	1	0	0	1	0	1
f_3	1	1	1	0	0	1	1
f_4	1	0	1	1	1	1	0
f_5	1	0	0	0	1	1	1
f_6	1	0	1	0	0	1	0
f_7	1	1	1	0	0	1	0
f_8	1	1	0	0	1	0	1
f_9	1	1	1	1	0	1	0
f_{10}	1	1	1	1	1	0	1
f_{11}	0	0	1	1	1	1	0
f_{12}	0	0	1	1	1	1	1
f_{13}	0	0	1	1	0	1	0

Comparing the rows, the only two pairs that are equal are

$$f_1 = f_{10} \quad \text{and} \quad f_2 = f_8.$$

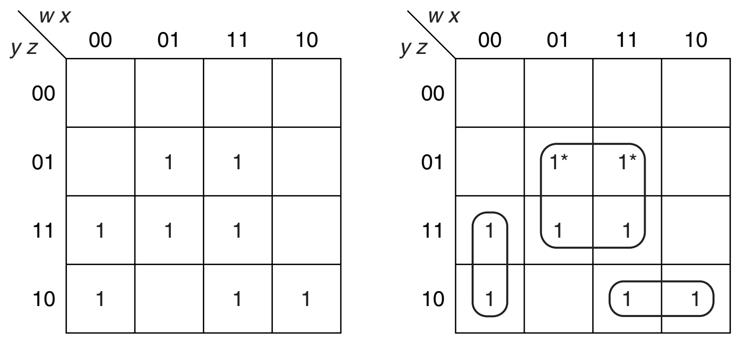
5. For each of the following functions, find all of the minimum sum of product expressions and all of the minimum product of sums expressions:

a. $f(w, x, y, z) = \Sigma m(2, 3, 5, 7, 10, 13, 14, 15)$
(1 SOP, 1 POS solution)

b. $f(a, b, c, d) = \Sigma m(3, 4, 9, 13, 14, 15) + \Sigma d(2, 5, 10, 12)$
(1 SOP, 2 POS solutions)

c. $f(a, b, c, d) = \Sigma m(4, 6, 11, 12, 13) + \Sigma d(3, 5, 7, 9, 10, 15)$
(2 SOP and 8 POS solutions)

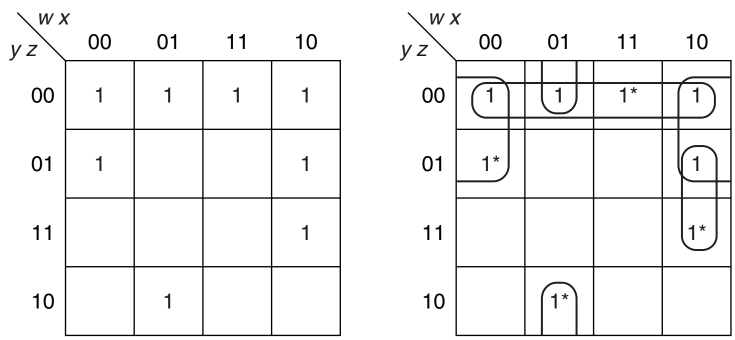
a. The map of f is shown below.



Although there is only one essential prime implicant, there is only one way to complete the cover with two more terms, namely,

$$f = xz + w'x'y + wyz'$$

By replacing all the 1's with 0's and 0's with 1's, or by plotting all the minterms not in f, we get the map for f'



There are four essential prime implicants, covering all of f', giving

$$f' = x'y' + y'z' + w'xz' + wx'z$$

Using DeMorgan's theorem, we get

$$f = (x + y)(y + z)(w + x' + z)(w' + x + z')$$

In this case, the sum of products solution requires fewer terms.

b. As indicated on the map below, all of the 1's are covered by essential prime implicants, producing the minimum sum of product expression

cd \ ab	00	01	11	10
00		1	X	
01		X	1	1
11	1		1	
10	X		1	X

cd \ ab	00	01	11	10
00		1*	X	
01		X	1	1*
11	1*		1*	
10	X		1	X

$$f_1 = bc' + ab + a'b'c + ac'd$$

Now, replacing all of the 1's by 0's and 0's by 1's and leaving the X's unchanged, we get the map for f'

cd \ ab	00	01	11	10
00	1		X	1
01	1	X		
11		1		1
10	X	1		X

cd \ ab	00	01	11	10
00	1		X	1
01	1	X		
11		1		1*
10	X	1		X

cd \ ab	00	01	11	10
00	1		X	1
01	1	X		
11		1		1*
10	X	1		X

There is one essential prime implicant, $ab'c$. Although m_6 and m_7 can each be covered in two ways, only $a'bc$ covers them both (and neither of the other terms cover additional 1's). The middle map shows each of these terms circled, leaving three 1's to be covered. There is a group of four, covering two of the 1's (as shown on the third map), $b'd'$. That leaves just m_1, which can be covered in two ways, as shown on the third map in green and light green lines. Thus, the two minimum sum of product expressions for f' are

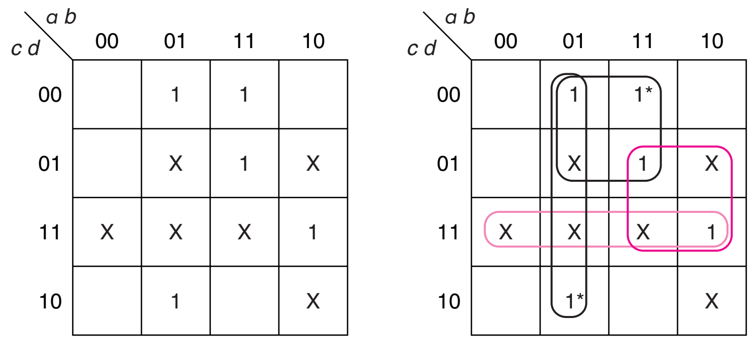
$$f_2' = ab'c + a'bc + b'd' + a'c'd$$
$$f_3' = ab'c + a'bc + b'd' + a'b'c'$$

producing the two minimum product of sums solutions

$$f_2 = (a' + b + c')(a + b' + c')(b + d)(a + c + d')$$
$$f_3 = (a' + b + c')(a + b' + c')(b + d)(a + b + c)$$

c. The map for f is shown next (on the left). There are two essential prime implicants, leaving only m_{11} to be covered.

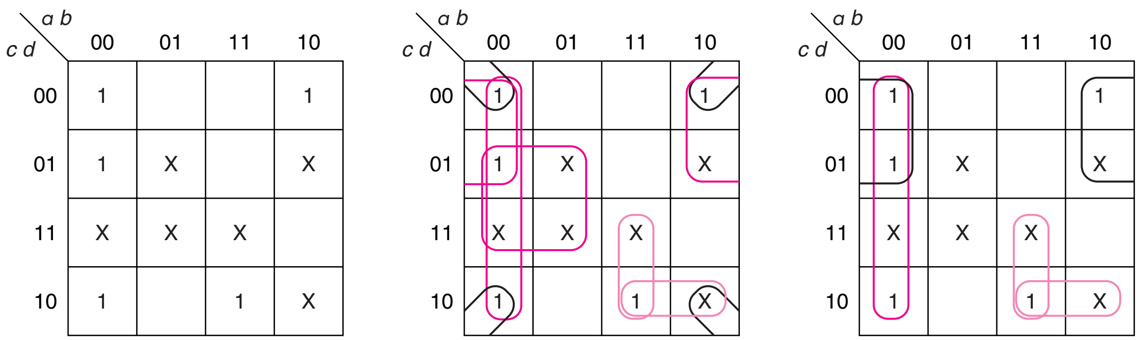
There are two groups of four that can be used, as indicated on the right map.



Thus the two sum of products solutions are

$$f_1 = a'b + bc' + ad$$
$$f_2 = a'b + bc' + cd$$

We then mapped f' and found no essential prime implicants.



We chose as a starting point m_8. It can be covered either by the four corners, $b'd'$ (as shown on the second map) or by $b'c'$, as shown on the third map. Whichever solution we choose, we need a group of two to cover m_{14} (as shown in light green); neither covers any other 1. After choosing one of these (and $b'd'$), all that remains to be covered is m_1. The three green lines show the covers. (Notice that one of those is $b'c'$.) If we don't choose $b'd'$, then we must choose $b'c'$ to cover m_0 and $a'b'$ to cover m_2 (since the only other prime implicant that covers m_2 is $b'd'$ and we have already found all of the solutions using that term). Thus, the eight solutions for f' are

$$f'_3 = b'd' + abc + a'b'$$
$$f'_4 = b'd' + abc + a'd$$
$$f'_5 = b'd' + abc + b'c'$$

$f_6' = b'd' + acd' + a'b'$
$f_7' = b'd' + acd' + a'd$
$f_8' = b'd' + acd' + b'c'$
$f_9' = b'c' + abc + a'b'$
$f_{10}' = b'c' + acd' + a'b'$

The product of sums solutions for f are thus

$f_3 = (b + d)(a' + b' + c')(a + b)$
$f_4 = (b + d)(a' + b' + c')(a + d')$
$f_5 = (b + d)(a' + b' + c')(b + c)$
$f_6 = (b + d)(a' + c' + d)(a + b)$
$f_7 = (b + d)(a' + c' + d)(a + d')$
$f_8 = (b + d)(a' + c' + d)(b + c)$
$f_9 = (b + c)(a' + b' + c')(a + b)$
$f_{10} = (b + c)(a' + c' + d)(a + b)$

6. Label the solutions of each part of problem 5 as $f_1, f_2, \ldots,$ and indicate which solutions are equal.

a. Since this problem does not involve don't cares, all solutions are equal.

b.

	2	5	10	12
f_1	1	1	0	1
f_2'	1	1	1	0
f_2	0	0	0	1
f_3'	1	0	1	0
f_3	0	1	0	1

All of the solutions are unique. The sum of products solution treats m_2 as a 1; the product of sums treats it as a 0. The two product of sums solutions treat m_5 differently.

c.

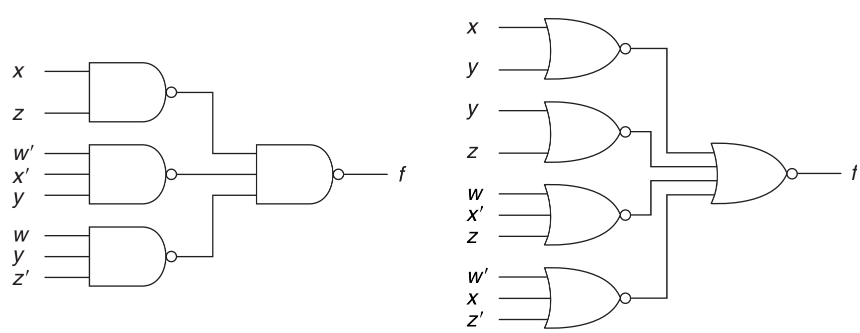
	3	5	7	9	10	15
f_1	0	1	1	1	0	1
f_2	1	1	1	0	0	1
f_3'	1	0	0	0	1	1
f_4'	1	1	1	0	1	1
f_5'	0	0	0	1	1	1
f_6'	1	0	0	0	1	0
f_7'	1	1	1	0	1	0
f_8'	0	0	0	1	1	0
f_9'	1	0	0	1	0	1
f_{10}'	1	0	0	1	1	0

For one of the sum of product expressions to be equal to one of the product of sum expressions, the pattern must be opposite (since we are showing the values of the don't cares for f' for the POS forms). Thus, $f_1 = f_6$, and $f_2 = f_8$, that is

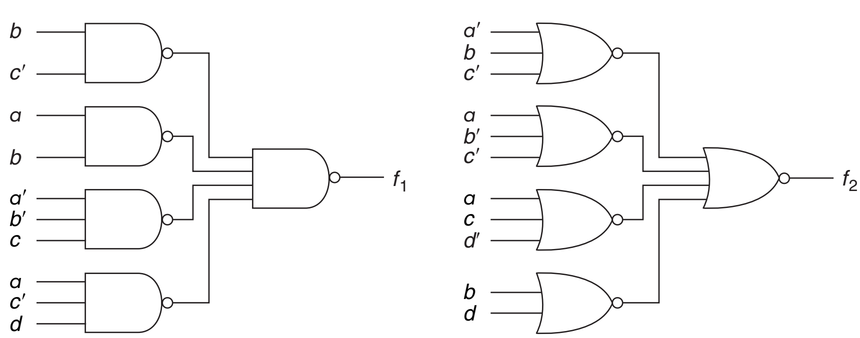
$$a'b + bc' + ad = (b + d)(a' + c' + d)(a + b)$$
$$a'b + bc' + cd = (b + d)(a' + c' + d)(b + c)$$

7. For each part of problem 5, draw the block diagram of a two-level NAND gate circuit and a two-level NOR gate circuit. (For those parts with multiple solutions, you need only draw one NAND and one NOR solution.)

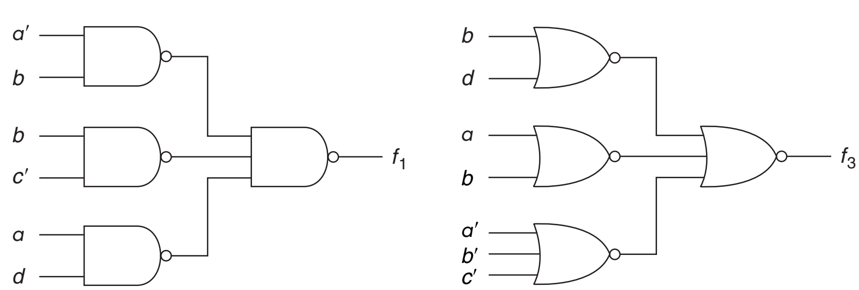
a.



b.



c.



8. Find the minimum sum of products solution(s) for each of the following:

a. $F(A, B, C, D, E) = \Sigma m(0, 5, 7, 9, 11, 13, 15, 18, 19, 22, 23, 25, 27, 28, 29, 31)$

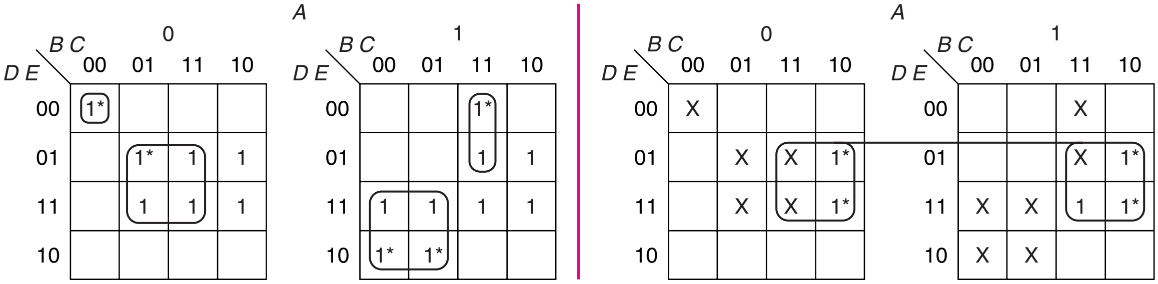
b. $F(A, B, C, D, E) = \Sigma m(0, 2, 4, 7, 8, 10, 15, 17, 20, 21, 23, 25, 26, 27, 29, 31)$

c. $G(V, W, X, Y, Z) = \Sigma m(0, 1, 4, 5, 6, 7, 10, 11, 14, 15, 21, 24, 25, 26, 27)$ (3 solutions)

d. $G(V, W, X, Y, Z) = \Sigma m(0, 1, 5, 6, 7, 8, 9, 14, 17, 20, 21, 22, 23, 25, 28, 29, 30)$ (3 solutions)

e. $H(A, B, C, D, E) = \Sigma m(1, 3, 10, 14, 21, 26, 28, 30) + \Sigma d(5, 12, 17, 29)$

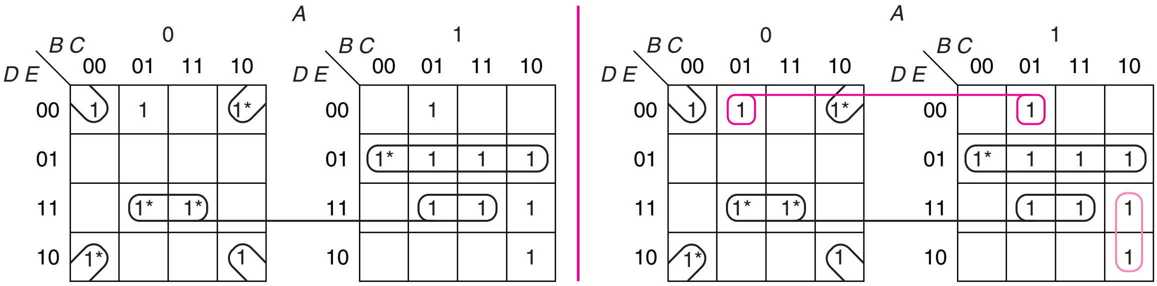
a. We begin by looking at 1's for which the corresponding position on the other layer is 0. On the first map, all of the essential prime implicants that are totally contained on one layer of the map, $A'B'C'D'E'$, $A'CE$, $AB'D$, and $ABCD'$, are circled.



The 1's covered by these essential prime implicants are shown as don't cares on the second map. The remaining 1's are all part of the group of eight, BE, shown on the second map. Thus, the minimum solution is

$$F = A'B'C'D'E' + A'CE + AB'D + ABCD' + BE$$

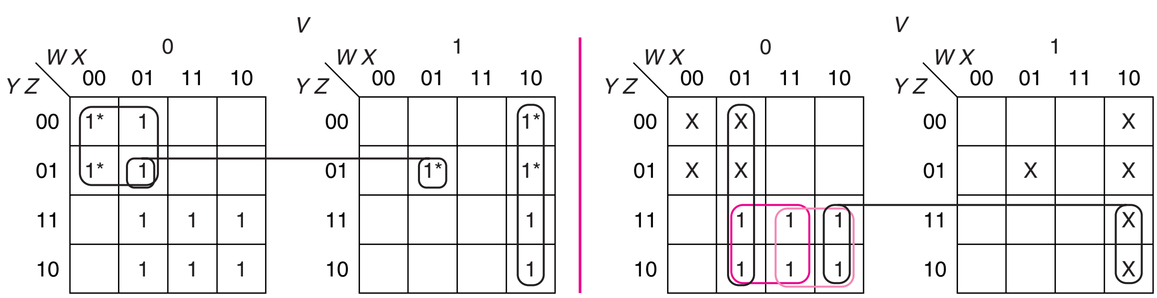
b. On the left map below, the essential prime implicants are circled. Note that $A'C'E'$ is on the top layer, $AD'E$ is on the lower layer and CDE is split between the layers.



That leaves four 1's to be covered, using two groups of two as shown on the right map. The minimum is thus

$$F = A'C'E' + AD'E + CDE + B'CD'E' + ABC'D$$

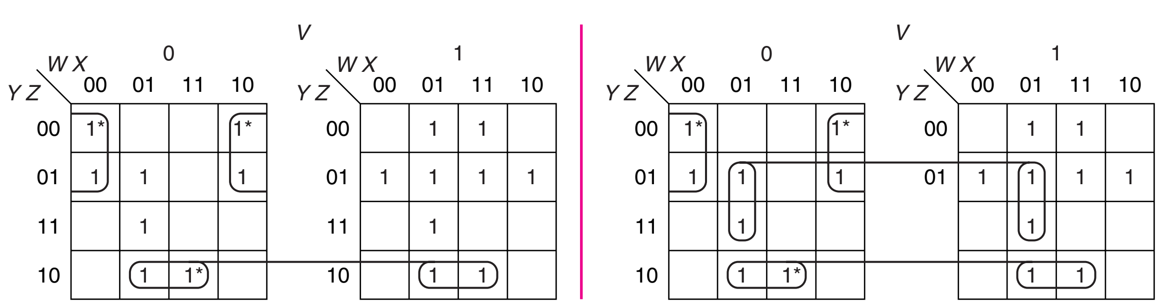
c. The map, with essential prime implicants circled, is shown on the left. After choosing $V'W'Y' + VWX' + W'XY'Z$, there are still six 1's uncovered. On the right map, the minterms covered by essential prime implicants are shown as don't cares. Each of the 1's can be covered by two different groups of four, which are shown on the map on the right.



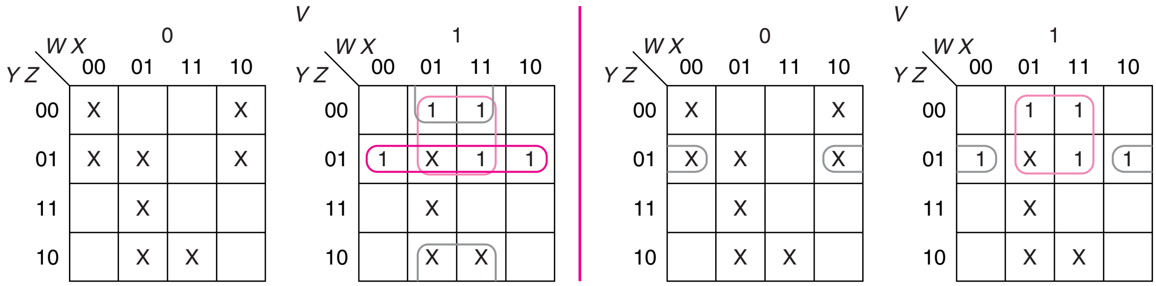
One group that covers four new 1's must be used (or both of them may be used), giving the following solutions:

$$G = V'W'Y' + VWX' + W'XY'Z + V'XY + V'WY$$
$$G = V'W'Y' + VWX' + W'XY'Z + V'XY + WX'Y$$
$$G = V'W'Y' + VWX' + W'XY'Z + V'WY + V'W'X$$

d. On the first map, the two essential prime implicants, $V'X'Y'$ and XYZ', are circled. The term $W'XZ$ is circled on the second map; if it is not used, $W'XY$ would be needed to cover m_7 and m_{23}. But then, three more terms would be needed to cover the function.



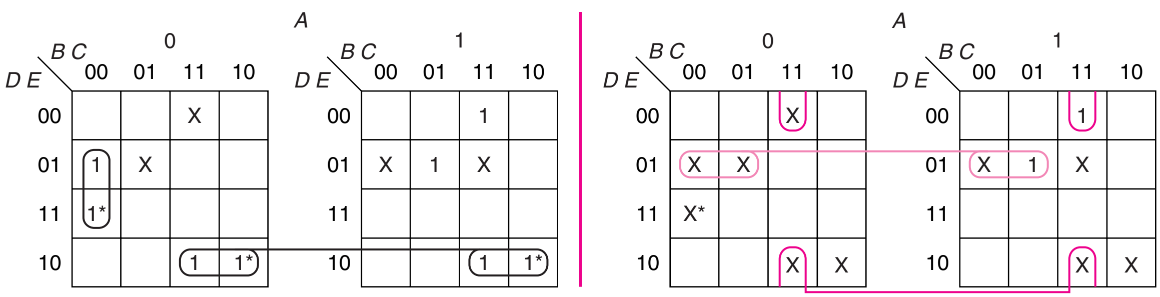
The following maps show the covered terms as don't cares and three ways of covering the remaining 1's. On the left map, the green term, $VY'Z$, is used with either of the other terms, VXY' or VXZ'. On the right map, VXY' and $X'Y'Z$ are used.



The three minimum solutions are thus

$$G = V'X'Y' + XYZ' + W'XZ + VY'Z + VXY'$$
$$G = V'X'Y' + XYZ' + W'XZ + VY'Z + VXZ'$$
$$G = V'X'Y' + XYZ' + W'XZ + VXY' + X'Y'Z$$

e. The two essential prime implicants, $A'B'C'E$ and BDE', are circled on the first map. Each of the remaining 1's can be covered in two ways, by a group of two contained completely on one layer or by the group of four shown.



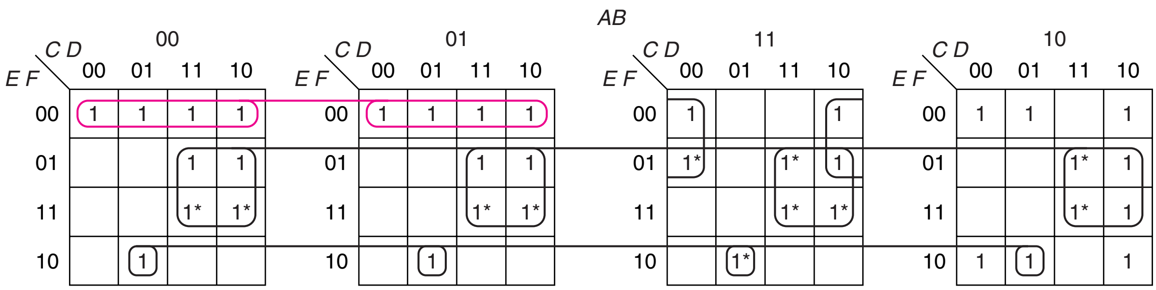
Thus, the minimum solution is

$$H = A'B'C'E + BDE' + BCE' + B'D'E$$

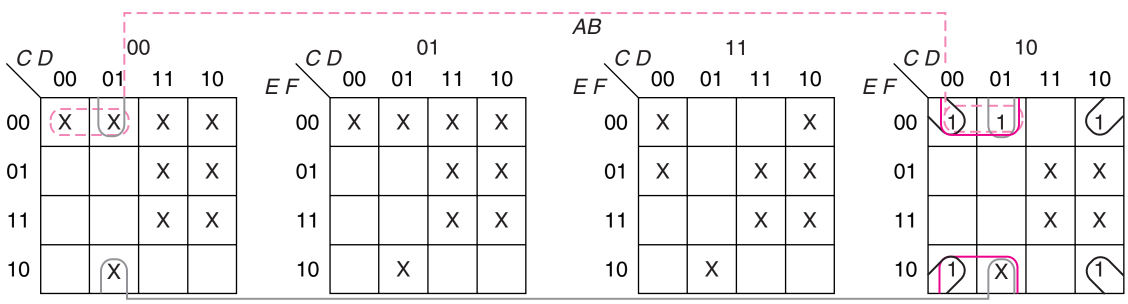
9. Find the four minimum sum of product expressions for the following six-variable function

$$G(A, B, C, D, E, F) = \Sigma m(0, 4, 6, 8, 9, 11, 12, 13, 15, 16, 20, 22, 24, 25, 27, 28, 29, 31, 32, 34, 36, 38, 40, 41, 42, 43, 45, 47, 48, 49, 54, 56, 57, 59, 61, 63)$$

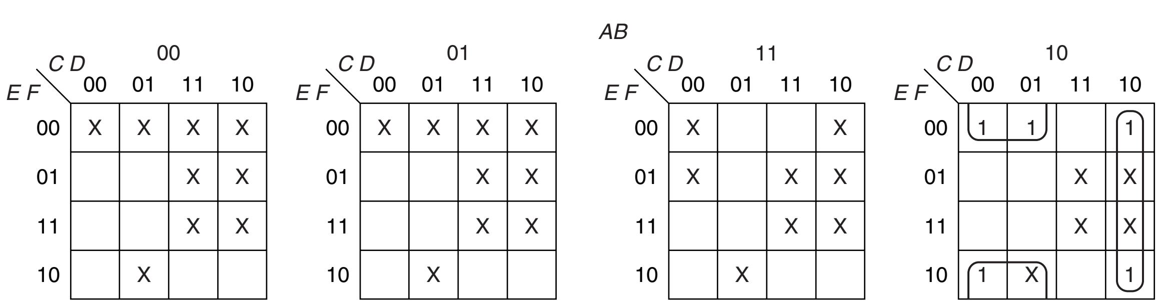
On the first map, the three essential prime implicants, $ABD'E'$, CF, and $C'DEF'$, are circled in black. The first is on just the third layer. The other two include 1's on all four layers (and thus do not involve the variable A and B). Also circled (in green) is a group of eight, $A'E'F'$, that is not essential (since each of the 1's is part of some other prime implicant). If that is not used, however, at least two terms would be needed to cover those 1's.



On the next map, the 1's that have been covered are shown as don't cares. The remaining 1's are all on the bottom (10) layer. The four corners, $AB'D'F'$, covers four of the five remaining 1's. Then, either $AB'C'F'$ (on the bottom layer) or $B'C'E'F'$ or $B'C'DF'$ (both half on the top layer and half on the bottom) can be used to cover the remaining 1's. These terms are circled below.



Also, as shown on the map below, $AB'C'F'$ could be used with $AB'CD'$.



Thus, we have the following four solutions

$$H = ABD'E' + CF + C'DEF' + A'E'F' + AB'D'F' + AB'C'F'$$

$$H = ABD'E' + CF + C'DEF' + A'E'F' + AB'D'F' + B'C'E'F'$$

$$H = ABD'E' + CF + C'DEF' + A'E'F' + AB'D'F' + B'C'DF'$$

$$H = ABD'E' + CF + C'DEF' + A'E'F' + AB'C'F' + AB'CD'$$

10. Find a minimum two-level circuit (corresponding to sum of products expressions) using AND gates and one OR gate per function for each of the following sets of functions:

a. $f(a, b, c, d) = \Sigma m(0, 1, 2, 3, 5, 7, 8, 10, 11, 13)$
$g(a, b, c, d) = \Sigma m(0, 2, 5, 8, 10, 11, 13, 15)$
(7 gates, 19 inputs)

b. $f(a, b, c, d) = \Sigma m(1, 2, 4, 5, 6, 9, 11, 13, 15)$
$g(a, b, c, d) = \Sigma m(0, 2, 4, 8, 9, 11, 12, 13, 14, 15)$
(8 gates, 23 inputs)

c. $F(W, X, Y, Z) = \Sigma m(2, 3, 6, 7, 8, 9, 13)$
$G(W, X, Y, Z) = \Sigma m(2, 3, 6, 7, 9, 10, 13, 14)$
$H(W, X, Y, Z) = \Sigma m(0, 1, 4, 5, 9, 10, 13, 14)$
(8 gates, 22 inputs)

d. $f(a, b, c, d) = \Sigma m(0, 2, 3, 8, 9, 10, 11, 12, 13, 15)$
$g(a, b, c, d) = \Sigma m(3, 5, 7, 12, 13, 15)$
$h(a, b, c, d) = \Sigma m(0, 2, 3, 4, 6, 8, 10, 14)$
(10 gates, 28 inputs)

e. $f(a, b, c, d) = \Sigma m(0, 3, 5, 7) + \Sigma d(10, 11, 12, 13, 14, 15)$
$g(a, b, c, d) = \Sigma m(0, 5, 6, 7, 8) + \Sigma d(10, 11, 12, 13, 14, 15)$
(7 gates, 19 inputs)

a. The maps below show the only prime implicant, $a'd$ in f, that covers a 1 not part of the other function.

cd \ ab	00	01	11	10
00	1			1
01	1	1	1	
11	1	1*		1
10	1			1

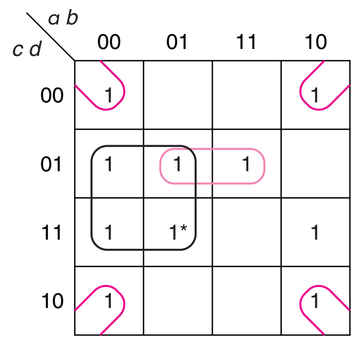
f

cd \ ab	00	01	11	10
00	1			1
01		1	1	
11			1	1
10	1			1

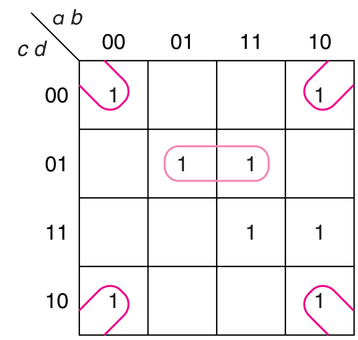
g

No other 1 (of either f or g) that is not shared makes a prime implicant essential (m_1 or m_3 in f or m_{15} in g). Two other terms,

$b'd'$ and $bc'd$, are essential prime implicants of both f and g and have been thus chosen in the maps below.

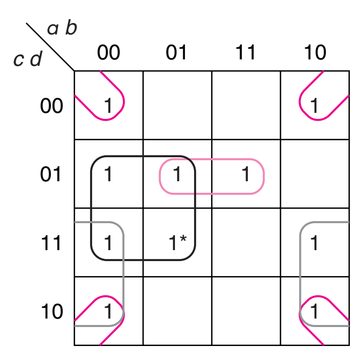


f

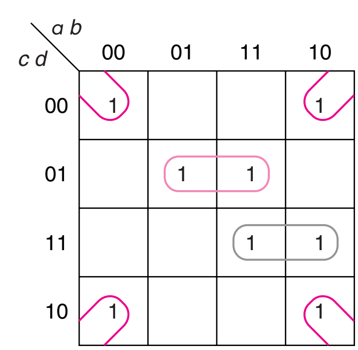


g

Although the term $ab'c$ could be shared, another term would be needed for g (either abd or acd). This would require seven gates and 20 gate inputs (one input too many). But, if acd is used for g, we could then complete covering both functions using $b'c$ for f as shown on the maps below.



f



g

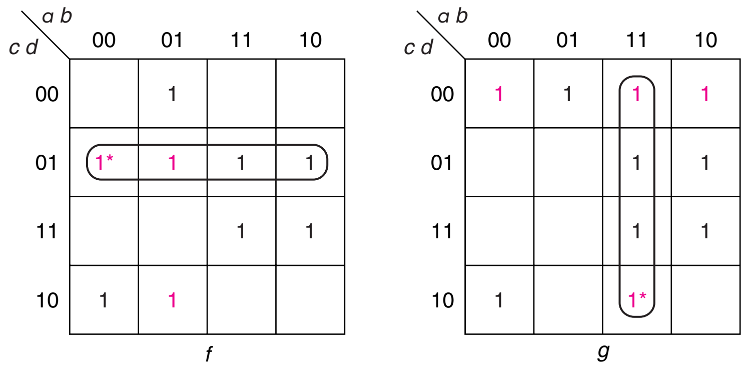
Thus,

$$f = a'd + b'd' + bc'd + b'c$$
$$g = b'd' + bc'd + acd$$

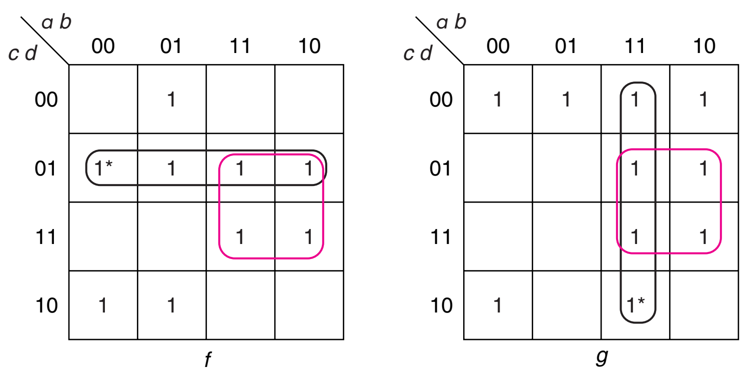
requiring seven gates and 19 inputs.

b. Scanning each function for 1's that are not part of the other function, we find m_1, m_5, and m_6 in f and m_0, m_8, m_{12}, and m_{14}

in g. The only ones that make a prime implicant essential are indicated on the map below.



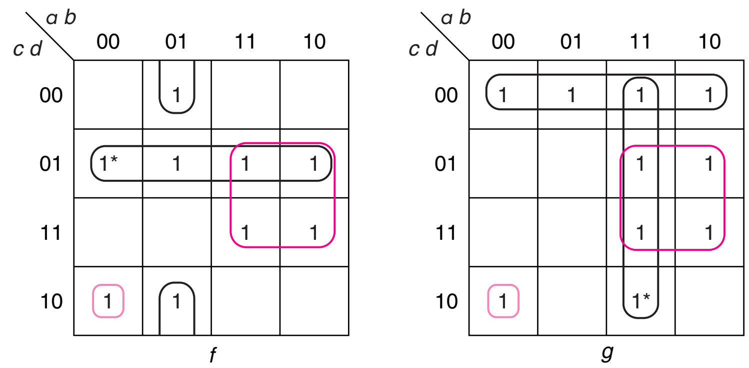
Next, we note that ad is an essential prime implicant of both functions, producing the following maps:



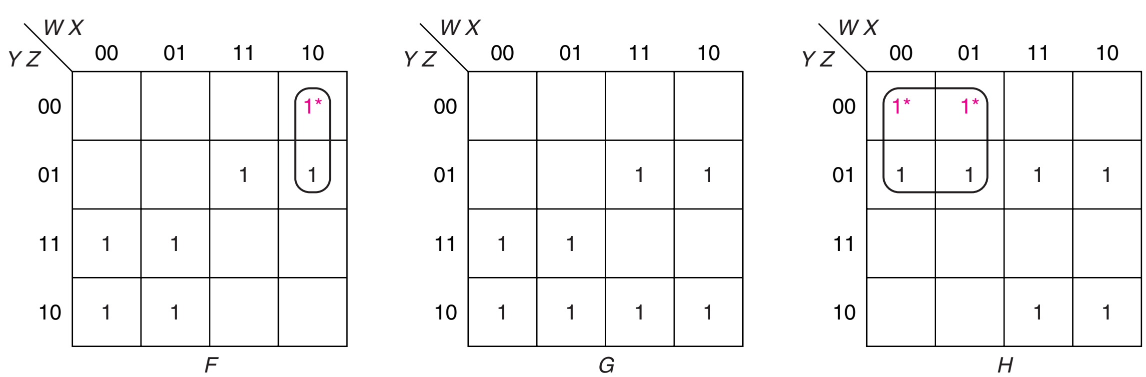
Unless we choose $c'd'$ to cover the remaining three 1's in the first row of g, we will need an extra term. Once we have done that, we see that the last 1 (m_2) of g can be covered by the minterm and shared with f. That leaves just two 1's of f that can be covered with the term $a'bd'$. The functions and the maps are shown next:

$$f = c'd + ad + a'b'cd' + a'bd'$$
$$g = ab + ad + c'd' + a'b'cd'$$

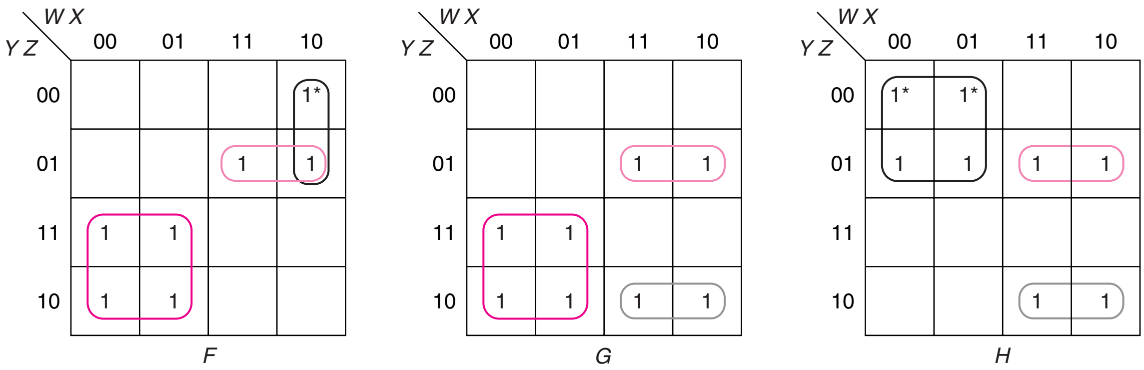
for a total of eight gates and 23 inputs.



c. When minimizing three functions, we still look for 1's that are only included in one of the functions and that make a prime implicant essential. In this problem, the only ones that satisfy these conditions are m_8 in F and m_0 and m_4 in H, as shown on the map below.



Next, notice that $W'Y$ is an essential prime implicant of both F and G. Once that is chosen, the term $WY'Z$ covers the remaining 1 of F and two 1's in G and H. (That term would be used for both F and G in any case since it is an essential prime implicant of both and is shareable. It is used for H since the remaining 1's in the prime implicant $Y'Z$ are already covered.) Finally, WYZ', an essential prime implicant of H, finishes the cover of G and H. The maps and functions below show the final solution, utilizing eight gates and 22 inputs.

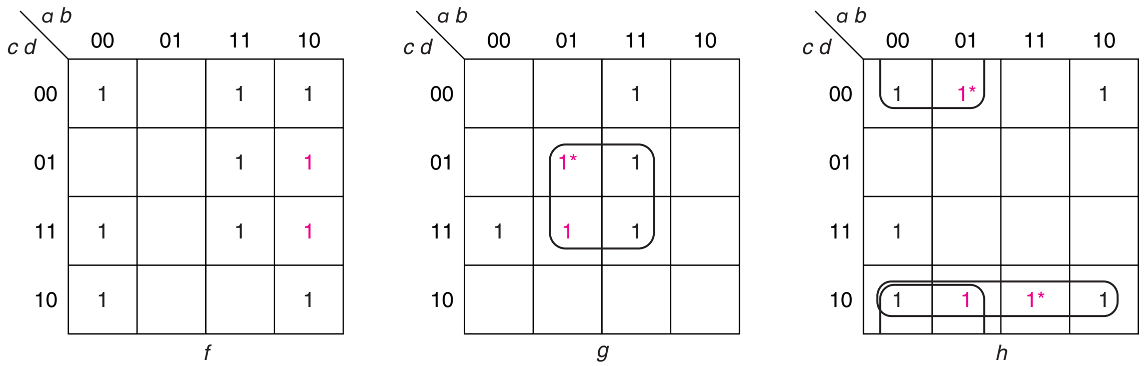


$F = WX'Y' + W'Y + WY'Z$

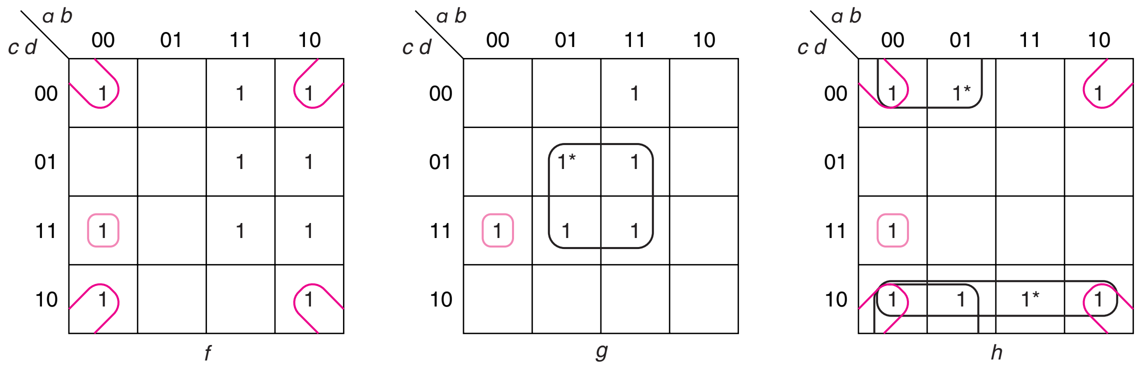
$G = W'Y + WY'Z + WYZ'$

$H = W'Y' + WY'Z + WYZ'$

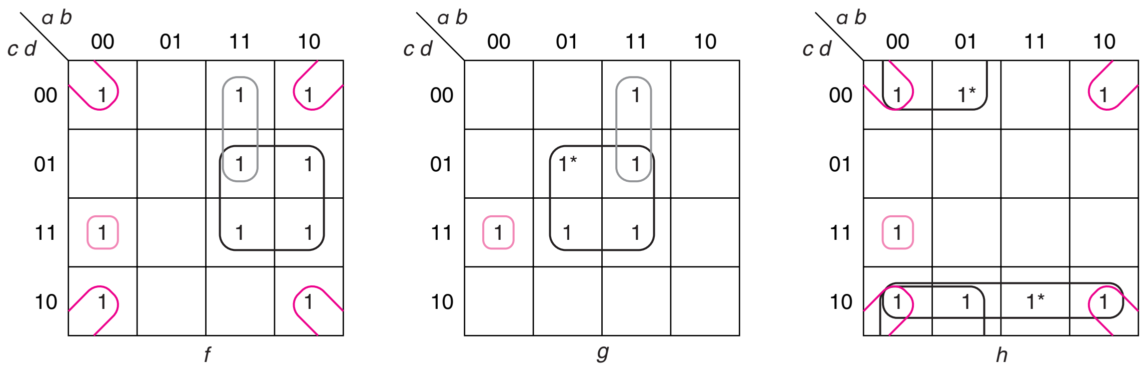
d. On the maps below, the essential prime implicants that cover 1's not part of any other function are circled. In f, m_9 and m_{11} can be covered with any of three prime implicants.



Next, we note that m_8 can only be covered by $b'd'$ in h and that $b'd'$ is also an essential prime implicant of f. That leaves only m_3 uncovered in h; by using the minterm for that, it can be shared with both f and g. (Otherwise, a new term would be required in each of those functions.) The resulting maps are shown below.

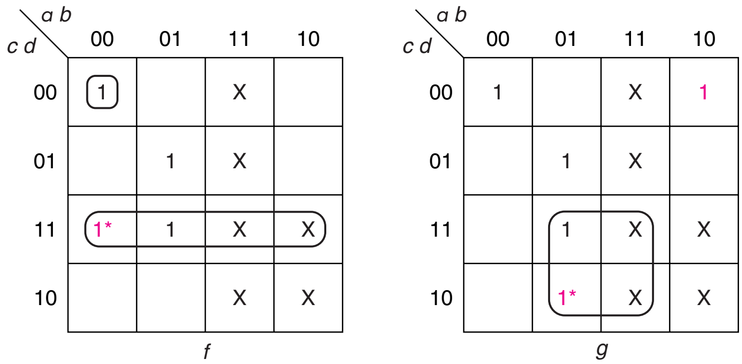


The only uncovered 1 in g is m_{12}. By using abc' for both that and for f, we can cover the three remaining 1's in f with ad, yielding the maps and equations below.

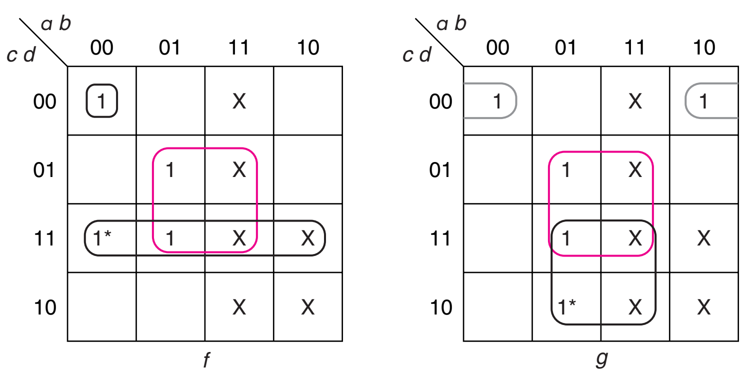


$$f = b'd' + a'b'cd + abc' + ad$$
$$g = bd + a'b'cd + abc'$$
$$h = a'd' + cd' + b'd' + a'b'cd$$

e. This example includes a number of don't cares, but that does not change the process significantly. There are two essential prime implicants, cd in f and bc in g, that cover 1's that cannot be shared. In addition, $a'b'c'd'$ must be used in f since it is the only prime implicant that covers m_0. (If a minterm is a prime implicant, we have no choice but to use it.) The maps below show these terms circled.



Next, we use bd to cover m_5 in both functions, and complete the cover of f. The obvious choice is to use $b'c'd'$ for the remaining 1's of g, producing the following maps and equations:

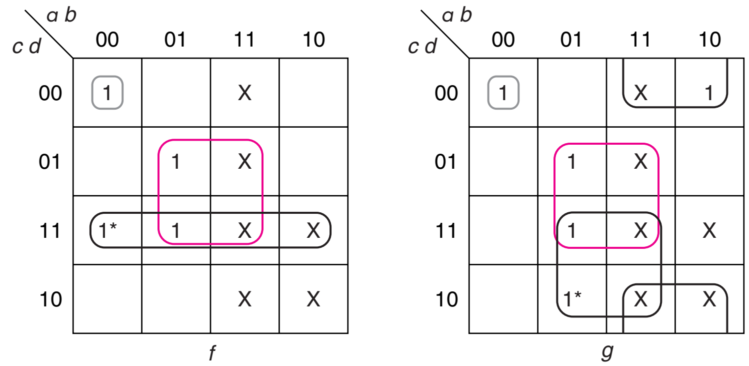


$$f = cd + a'b'c'd' + bd$$
$$g = bc + bd + b'c'd'$$

But, there is another solution, as illustrated below. By using $a'b'c'd'$ to cover m_0 in g (we already needed that term for f), we can cover the remaining 1 in g with a group of four, ad', producing the solution

$$f = cd + a'b'c'd' + bd$$
$$g = bc + bd + a'b'c'd' + ad'$$

as shown on the following maps. Both solutions require seven gates and 19 inputs.



3.8 EXERCISES

1. For each of the following, find all minimum sum of products expressions. (If there is more than one solution, the number of solutions is given in parentheses.)

a. $f(a, b, c) = \Sigma m(1, 2, 3, 6, 7)$

*b. $g(w, x, y) = \Sigma m(0, 1, 5, 6, 7)$ (2 solutions)

c. $h(a, b, c) = \Sigma m(0, 1, 2, 5, 6, 7)$ (2 solutions)

d. $f(a, b, c, d) = \Sigma m(1, 2, 3, 5, 6, 7, 8, 11, 13, 15)$

*e. $G(W, X, Y, Z) = \Sigma m(0, 2, 5, 7, 8, 10, 12, 13)$

f. $h(a, b, c, d) = \Sigma m(2, 4, 5, 6, 7, 8, 10, 12, 13, 15)$ (2 solutions)

g. $f(a, b, c, d) = \Sigma m(1, 3, 4, 5, 6, 11, 12, 13, 14, 15)$ (2 solutions)

h. $g(w, x, y, z) = \Sigma m(2, 3, 6, 7, 8, 10, 11, 12, 13, 15)$ (2 solutions)

*i. $h(p, q, r, s) = \Sigma m(0, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15)$ (3 solutions)

j. $F(W, X, Y, Z) = \Sigma m(0, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 15)$ (4 solutions)

k. $f(w, x, y, z) = \Sigma m(0, 1, 2, 4, 5, 6, 9, 10, 11, 13, 14, 15)$ (2 solutions)

l. $g(a, b, c, d) = \Sigma m(0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15)$

*m. $H(W, X, Y, Z) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 12, 13)$ (4 solutions)

*n. $f(a, b, c, d) = \Sigma m(0, 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$ (6 solutions)

o. $g(w, x, y, z) = \Sigma m(0, 1, 2, 3, 5, 6, 7, 8, 9, 10, 13, 14, 15)$
(6 solutions)

p. $f(a, b, c, d) = \Sigma m(0, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14)$
(32 solutions)

2. For the following functions,

i. List all prime implicants, indicating which are essential.

ii. Show the minimum sum of products expression(s).

a. $f(a, b, c, d) = \Sigma m(0, 3, 4, 5, 8, 11, 12, 13, 14, 15)$

*b. $g(w, x, y, z) = \Sigma m(0, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15)$

3. Map each of the following functions and find the minimum sum of products expression:

a. $F = AD + AB + A'CD' + B'CD + A'BC'D'$

*b. $g = w'yz + xy'z + wy + wxy'z' + wz + xyz'$

4. For each of the following, find all minimum sum of products expressions. (If there is more than one solution, the number of solutions is given in parentheses.) Label the solutions $f_1, f_2, \ldots$.

a. $f(w, x, y, z) = \Sigma m(1, 3, 6, 8, 11, 14) + \Sigma d(2, 4, 5, 13, 15)$
(3 solutions)

b. $f(a, b, c, d) = \Sigma m(0, 3, 6, 9, 11, 13, 14) + \Sigma d(5, 7, 10, 12)$

*c. $f(a, b, c, d) = \Sigma m(0, 2, 3, 5, 7, 8, 9, 10, 11) + \Sigma d(4, 15)$
(3 solutions)

d. $f(w, x, y, z) = \Sigma m(0, 2, 4, 5, 10, 12, 15) + \Sigma d(8, 14)$
(2 solutions)

e. $f(a, b, c, d) = \Sigma m(5, 7, 9, 11, 13, 14) + \Sigma d(2, 6, 10, 12, 15)$
(4 solutions)

*f. $f(a, b, c, d) = \Sigma m(0, 2, 4, 5, 6, 7, 8, 9, 10, 14) + \Sigma d(3, 13)$
(3 solutions)

g. $f(w, x, y, z) = \Sigma m(1, 2, 5, 10, 12) + \Sigma d(0, 3, 4, 8, 13, 14, 15)$
(7 solutions)

5. For each of the functions of problem 4, indicate which solutions are equal.

6. For each of the following functions, find all of the minimum sum of products expressions and all of the minimum product of sums expressions:

*a. $f(A, B, C, D) = \Sigma m(1, 4, 5, 6, 7, 9, 11, 13, 15)$

b. $f(W, X, Y, Z) = \Sigma m(2, 4, 5, 6, 7, 10, 11, 15)$

c. $f(A, B, C, D) = \Sigma m(1, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15)$
(1 SOP and 2 POS solutions)

*d. $f(a, b, c, d) = \Sigma m(0, 2, 4, 6, 7, 9, 11, 12, 13, 14, 15)$
(2 SOP and 1 POS solutions)

e. $f(w, x, y, z) = \Sigma m(0, 4, 6, 9, 10, 11, 14) + \Sigma d(1, 3, 5, 7)$

f. $f(a, b, c, d) = \Sigma m(0, 1, 2, 5, 7, 9) + \Sigma d(6, 8, 11, 13, 14, 15)$
(4 SOP and 2 POS solutions)

g. $f(w, x, y, z) = \Sigma m(4, 6, 9, 10, 11, 13) + \Sigma d(2, 12, 15)$
(2 SOP and 2 POS solutions)

h. $f(a, b, c, d) = \Sigma m(0, 1, 4, 6, 10, 14) + \Sigma d(5, 7, 8, 9, 11, 12, 15)$
(13 SOP and 3 POS solutions)

*i. $f(w, x, y, z) = \Sigma m(1, 3, 7, 11, 13, 14) + \Sigma d(0, 2, 5, 8, 10, 12, 15)$
(6 SOP and 1 POS solutions)

j. $f(a, b, c, d) = \Sigma m(0, 1, 6, 15) + \Sigma d(3, 5, 7, 11, 14)$
(1 SOP and 2 POS solutions)

7. Label the solutions of each part of problem 6 as $f_1, f_2, \ldots$ and indicate which solutions are equal.

8. For each part of problem 6, draw the block diagram of a two-level NAND gate circuit and a two-level NOR gate circuit. (For those parts with multiple solutions, you need only draw one NAND and one NOR solution.)

9. For each of the following five variable functions, find all minimum sum of products expressions. (If there is more than one solution, the number of solutions is given in parentheses.)

a. $F(A, B, C, D, E) = \Sigma m(0, 1, 5, 7, 8, 9, 10, 11, 13, 15, 18, 20, 21, 23, 26, 28, 29, 31)$

b. $G(A, B, C, D, E) = \Sigma m(0, 1, 2, 4, 5, 6, 10, 13, 14, 18, 21, 22, 24, 26, 29, 30)$

*c. $H(A, B, C, D, E) = \Sigma m(5, 8, 12, 13, 15, 17, 19, 21, 23, 24, 28, 31)$

d. $F(V, W, X, Y, Z) = \Sigma m(2, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 24, 25, 29, 30, 31)$

e. $G(V, W, X, Y, Z) = \Sigma m(0, 1, 4, 5, 8, 9, 10, 15, 16, 18, 19, 20, 24, 26, 28, 31)$

*f. $H(V, W, X, Y, Z) = \Sigma m(0, 1, 2, 3, 5, 7, 10, 11, 14, 15, 16, 18, 24, 25, 28, 29, 31)$ (2 solutions)

g. $F(A, B, C, D, E) = \Sigma m(0, 4, 6, 8, 12, 13, 14, 15, 16, 17, 18, 21, 24, 25, 26, 28, 29, 31)$ (6 solutions)

h. $G(A, B, C, D, E) = \Sigma m(0, 3, 5, 7\ 12, 13, 14, 15, 19, 20, 21, 22, 23, 25, 26, 29, 30)$ (3 solutions)

*i. $H(A, B, C, D, E) = \Sigma m(0, 1, 5, 6, 7, 8, 9, 14, 17, 20, 21, 22, 23, 25, 28, 29, 30)$ (3 solutions)

j. $F(V, W, X, Y, Z) = \Sigma m(0, 4, 5, 7, 10, 11, 14, 15, 16, 18, 20, 21, 23, 24, 25, 26, 29, 31)$ (4 solutions)

k. $G(V, W, X, Y, Z) = \Sigma m(0, 2, 5, 6, 8, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 29, 31)$
(3 solutions)

l. $H(V, W, X, Y, Z) = \Sigma m(0, 1, 2, 3, 5, 8, 9, 10, 13, 17, 18, 19, 20, 21, 26, 28, 29)$
(3 solutions)

m. $F(A, B, C, D, E) = \Sigma m(1, 2, 5, 8, 9, 10, 12, 13, 14, 15, 16, 18, 21, 22, 23, 24, 26, 29, 30, 31)$
(18 solutions)

*n. $G(V, W, X, Y, Z) = \Sigma m(0, 1, 5, 7, 8, 13, 24, 25, 29, 31) + \Sigma d(9, 15, 16, 17, 23, 26, 27, 30)$
(2 solutions)

o. $H(A, B, C, D, E) = \Sigma m(0, 4, 12, 15, 27, 29, 30) + \Sigma d(1, 5, 9, 10, 14, 16, 20, 28, 31)$
(4 solutions)

p. $F(A, B, C, D, E) = \Sigma m(8, 9, 11, 14, 28, 30) + d(0, 3, 4, 6, 7, 12, 13, 15, 20, 22, 27, 29, 31)$
(8 solutions)

10. For each of the following six-variable functions, find all minimum sum of products expressions. (The number of terms and literals and, if there is more than one solution, the number of solutions is given in parentheses.)

a. $G(A, B, C, D, E, F) = \Sigma m(4, 5, 6, 7, 8, 10, 13, 15, 18, 20, 21, 22, 23, 26, 29, 30, 31, 33, 36, 37, 38, 39, 40, 42, 49, 52, 53, 54, 55, 60, 61)$
(6 terms, 21 literals)

*b. $G(A, B, C, D, E, F) = \Sigma m(2, 3, 6, 7, 8, 12, 14, 17, 19, 21, 23, 25, 27, 28, 29, 30, 32, 33, 34, 35, 40, 44, 46, 49, 51, 53, 55, 57, 59, 61, 62, 63)$
(8 terms, 30 literals)

c. $G(A, B, C, D, E, F) = \Sigma m(0, 1, 2, 4, 5, 6, 7, 9, 13, 15, 17, 19, 21, 23, 26, 27, 29, 30, 31, 33, 37, 39, 40, 42, 44, 45, 46, 47, 49, 53, 55, 57, 59, 60, 61, 62, 63)$
(8 terms, 28 literals, 2 solutions)

11. Find a minimum two-level circuit (corresponding to sum of products expressions) using AND and one OR gate per function for each of the following sets of functions.

*a. $f(a, b, c, d) = \Sigma m(1, 3, 5, 8, 9, 10, 13, 14)$
$g(a, b, c, d) = \Sigma m(4, 5, 6, 7, 10, 13, 14)$ (7 gates, 21 inputs)

b. $f(a, b, c, d) = \Sigma m(0, 1, 2, 3, 4, 5, 8, 10, 13)$
$g(a, b, c, d) = \Sigma m(0, 1, 2, 3, 8, 9, 10, 11, 13)$
(6 gates, 16 inputs)

c. $f(a, b, c, d) = \Sigma m(5, 8, 9, 12, 13, 14)$
$g(a, b, c, d) = \Sigma m(1, 3, 5, 8, 9, 10)$
(3 solutions, 8 gates, 25 inputs)

d. $f(a, b, c, d) = \Sigma m(1, 3, 4, 5, 10, 11, 12, 14, 15)$
$g(a, b, c, d) = \Sigma m(0, 1, 2, 8, 10, 11, 12, 15)$
(9 gates, 28 inputs)

*e. $F(W, X, Y, Z) = \Sigma m(1, 5, 7, 8, 10, 11, 12, 14, 15)$
$G(W, X, Y, Z) = \Sigma m(0, 1, 4, 6, 7, 8, 12)$ (8 gates, 23 inputs)

f. $F(W, X, Y, Z) = \Sigma m(0, 2, 3, 7, 8, 9, 13, 15)$
$G(W, X, Y, Z) = \Sigma m(0, 2, 8, 9, 10, 12, 13, 14)$
(2 solutions, 8 gates, 23 inputs)

g. $f(a, b, c, d) = \Sigma m(1, 3, 5, 7, 8, 9, 10)$
$g(a, b, c, d) = \Sigma m(0, 2, 4, 5, 6, 8, 10, 11, 12)$
$h(a, b, c, d) = \Sigma m(1, 2, 3, 5, 7, 10, 12, 13, 14, 15)$
(2 solutions, 12 gates, 33 inputs)

*h. $f(a, b, c, d) = \Sigma m(0, 3, 4, 5, 7, 8, 12, 13, 15)$
$g(a, b, c, d) = \Sigma m(1, 5, 7, 8, 9, 10, 11, 13, 14, 15)$
$h(a, b, c, d) = \Sigma m(1, 2, 4, 5, 7, 10, 13, 14, 15)$
(2 solutions, 11 gates, 33 inputs)

i. $f(a, b, c, d) = \Sigma m(0, 2, 3, 4, 6, 7, 9, 11, 13)$
$g(a, b, c, d) = \Sigma m(2, 3, 5, 6, 7, 8, 9, 10, 13)$
$h(a, b, c, d) = \Sigma m(0, 4, 8, 9, 10, 13, 15)$
(2 solutions for f and g, 10 gates, 32 inputs)

*j. $f(a, c, b, d) = \Sigma m(0, 1, 2, 3, 4, 9) + \Sigma d(10, 11, 12, 13, 14, 15)$
$g(a, c, b, d) = \Sigma m(1, 2, 6, 9) + \Sigma d(10, 11, 12, 13, 14, 15)$
(3 solutions for f, 6 gates, 15 inputs)

k. $f(a, c, b, d) = \Sigma m(5, 6, 11) + \Sigma d(0, 1, 2, 4, 8)$
$g(a, c, b, d) = \Sigma m(6, 9, 11, 12, 14) + \Sigma d(0, 1, 2, 4, 8)$
(2 solutions for g, 7 gates, 18 inputs)

12. In each of the following sets, the functions have been minimized individually. Find a minimum two-level circuit (corresponding to sum of products expressions) using AND and one OR gate per function for each.

a. $F = B'D' + CD' + AB'C$
$G = BC + ACD$ (6 gates, 15 inputs)

*b. $F = A'B'C'D + BC + ACD + AC'D'$
$G = A'B'C'D' + A'BC + BCD'$
$H = B'C'D' + BCD + AC' + AD$
(2 solutions for H, 10 gates, 35 inputs)

c. $f = a'b' + a'd + b'c'd'$
$g = b'c'd' + bd + acd + abc$
$h = a'd' + a'b + bc'd + b'c'd'$ (10 gates, 31 inputs)

3.9 CHAPTER 3 TEST (100 MINUTES, OR TWO 50-MINUTE TESTS)

1. Find the minimum sum of products expression for each of the following functions (that is, circle the terms on the map and write the algebraic expressions).

a.

yz \ wx	00	01	11	10
00		1		1
01		1		
11		1	1	1
10		1		

b.

cd \ ab	00	01	11	10
00	1	1	1	
01	1	1	1	
11			1	1
10	1	1	1	

2. Find all four minimum sum of product expressions for the following function. (Two copies of the map are given for your convenience.)

cd \ ab	00	01	11	10
00	1	1	1	
01	1		1	1
11	1		1	1
10		1	1	1

cd \ ab	00	01	11	10
00	1	1	1	
01	1		1	1
11	1		1	1
10		1	1	1

3. For the following function (three copies of the map are shown),

a. List all prime implicants, indicating which, if any, are essential.

b. Find all four minimum solutions.

yz \ wx	00	01	11	10
00		1	1	X
01	X	X		X
11	X			1
10			1	1

yz \ wx	00	01	11	10
00		1	1	X
01	X	X		X
11	X			1
10			1	1

yz \ wx	00	01	11	10
00		1	1	X
01	X	X		X
11	X			1
10			1	1

4. For the following function (three copies of the map are shown),

a. List all prime implicants, indicating which, if any, are essential.

b. Find both minimum solutions.

yz \ wx	00	01	11	10
00				
01		1	1	1
11	1	1		1
10	1	1	1	1

yz \ wx	00	01	11	10
00				
01		1	1	1
11	1	1		1
10	1	1	1	1

yz \ wx	00	01	11	10
00				
01		1	1	1
11	1	1		1
10	1	1	1	1

5. For the following four-variable function, *f*, find both minimum sum of products expressions and both minimum product of sums expressions.

cd \ ab	00	01	11	10
00			X	
01	X	1	X	1
11	1	1		X
10		X		

6. For the following function, *f*, find all four minimum sum of products expressions and all four minimum product of sums expressions.

yz \ wx	00	01	11	10
00	X		1	
01	X	1	1	
11	X		X	1
10	X		X	

7. For the following five-variable problem, find both minimum sum of products expressions.

A = 0

DE \ BC	00	01	11	10
00	1		1	
01	1	1		
11	1			
10	1			

A = 1

DE \ BC	00	01	11	10
00			1	
01		1	1	
11		1	1	1
10				1

8. For the following five-variable problem, find both minimum sum of products expressions. (5 terms, 15 literals)

A = 0

DE \ BC	00	01	11	10
00	1			1
01				1
11		1	1	1
10	1			1

A = 1

DE \ BC	00	01	11	10
00	1	1	1	
01	1			1
11	1	1	1	1
10	1	1	1	

9. a. For the following two functions, find the minimum sum of products expression for each (treating them as two separate problems).

$yz \backslash wx$	00	01	11	10
00		1	1	1
01				1
11				1
10			1	1

f

$yz \backslash wx$	00	01	11	10
00				
01	1	1		1
11	1	1		1
10		1		

g

b. For the same two functions, find a minimum sum of products solution (corresponding to minimum number of gates, and among those with the same number of gates, minimum number of gate inputs). (7 gates, 19 inputs)

10. Consider the three functions, the maps of which are shown below.

$yz \backslash wx$	00	01	11	10
00				1
01	1	1		
11	1	1	1	1
10	1	1		1

f

$yz \backslash wx$	00	01	11	10
00		1	1	1
01				
11			1	
10	1	1	1	

g

$yz \backslash wx$	00	01	11	10
00		1		1
01	1	1		
11	1	1	1	
10		1		

h

a. Find the minimum sum of products expression (individually) for each of the three functions. Indicate which, if any, prime implicants can be shared.

b. Find a minimum two-level NAND gate solution. Full credit for a solution using 10 gates and 32 inputs. All variables are available both uncomplemented and complemented. Show the equations *and* a block diagram.