
CHAPTER 5

Introduction to Modern Symmetric-Key Ciphers

(Solution to Odd-Numbered Problems)

Review Questions

1. The traditional symmetric-key ciphers are character-oriented ciphers. The modern symmetric-key ciphers are bit-oriented ciphers.
3. A transposition is definitely a permutation of bits. A substitution of bits can be thought of a permutation if we add decoding and encoding to the operation.
5. A P-box (permutation box) transposes bits. We have three types of P-boxes in modern block ciphers: straight P-boxes, expansion P-boxes, and compression P-boxes. A straight P-box is invertible; the other two are not.
7. A product cipher is a complex cipher combining substitution, permutation, and other components discussed in this chapter. We discussed two classes of product ciphers: Feistel and non-Feistel ciphers.
9. A Feistel block cipher uses both invertible and noninvertible components. A non-Feistel block cipher uses only invertible components.
11. In a synchronous stream cipher the key stream is independent of the plaintext or ciphertext. In a nonsynchronous stream cipher the key stream is somehow dependent on the plaintext or ciphertext.

Exercises

13. The order of the group is $10! = 3,626,800$. The key size is $\lceil \log_2(10!) \rceil = 22$ bits. Note that a key of 22 bits can select $2^{22} = 4,194,304$ different permutations, but only 3,626,800 of them are used here.
15.
 - a. $\text{CircularLeftShift}_3(\mathbf{10011011}) \rightarrow 11011\mathbf{100}$
 - b. $\text{CircularRightShift}_3(11011\mathbf{100}) \rightarrow \mathbf{100}11011$

c. The original word in Part *a* and the result of Part *b* are the same, which shows that circular left shift and circular right shift operations are inverses of each other.

17. We show the complement of A with \bar{A} .

a. $(01001101) \oplus (01001101) = (00000000) \rightarrow (A \oplus A = \text{All } 0\text{s})$

b. $(01001101) \oplus (10110010) = (11111111) \rightarrow (A \oplus \bar{A} = \text{All } 1\text{s})$

c. $(01001101) \oplus (00000000) = (01001101) \rightarrow (A \oplus \text{All } 0\text{s} = A)$

d. $(01001101) \oplus (11111111) = (10110010) \rightarrow (A \oplus \text{All } 1\text{s} = \bar{A})$

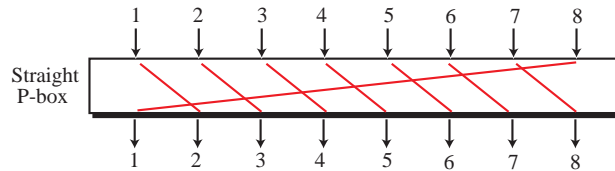
19. Using eight bits for each character, $|M| = 8 \times 2000 = 16,000$ bits. Therefore, we have

$$|M| + |\text{Pad}| = 0 \pmod{64} \rightarrow |\text{Pad}| = -|M| \pmod{64} \rightarrow |\text{Pad}| = -16,000 \pmod{64} = 0$$

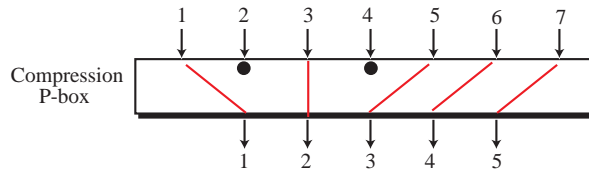
This means no padding is needed. The message is divided into 250 blocks.

21. The permutation table is [1 3 5].

23. See the figure below:



25. It is a compression P-box with 7 inputs and 5 outputs as shown below:



27. We use the following procedure:

a. We first find the input/output relation based on the given S-box:

Input:	00	01	10	11
Output:	01	11	10	00

b. We then find the inverse input/output relation (sorted on input):

Input:	00	01	10	11
Output:	11	00	10	01

c. Now we create the table for the inverse S-box (the left row defines the first input bit, the first column defines the second input bit, and the entries define the output):

	0	1
0	11	00
1	10	01

29. The characteristic polynomial is $x^4 + x^3 + x^2 + 1$ or $(11101)_2$ or $(1D)_{16}$. The polynomial is not primitive (see Appendix G). The maximum period is then less than $2^4 - 1$ or less than 15.

31. To have a maximum period of 32, the characteristic polynomial should be of degree 6 because $2^5 - 1 = 31 < 32$. However, if the characteristic polynomial is primitive, the maximum period is $2^6 - 1 = 63$. But the problem says that the maximum period is 32, therefore, the characteristic polynomial is not primitive. In other words, we have an LFSR of degree 6, with 6 cells (6-bit register) whose characteristic polynomial is not primitive.

33.

Input: 1 011	→	Left rotate	→	Output: 110
Input: 0 110	→	Right rotate	→	Output: 011

35.

```

combine (rightWord [1 ... m], leftWord [1 ... m], m)
{
    i ← 1
    while (i ≤ m)
    {
        word[i] ← rightWord[i]
        word[i + m] ← leftWord[i]
        i ← i + 1
    }
    return (word[1 ... n])           // n = 2m
}

```

37.

```

circularShift (shift, word [1 ... n], n, k)
{
    i ← 1
    while (i ≤ k)
    {
        if (shift = left)
            word ← circularShiftLeft(word, n)
        else
            word ← circularShiftRight(word, n)
        i ← i + 1
    }
    return (word[1 ... n])
}

circularShiftLeft (word [1 ... n], n)
{
    temp ← word[n]
    j ← n - 1
    while (j ≥ 0)
    {
        word[j + 1] ← word[j]
        j ← j - 1
    }
    word[1] ← temp
    return (word[0 ... n])
}

circularShiftRight (word [0 ... n], n)
{
    temp ← word[1]
    j ← 1
    while (j ≤ n)
    {
        word[j - 1] ← word[j]
        j ← j + 1
    }
    word[n] ← temp
    return (word[0 ... n])
}

```

39. The table can be designed in many different ways. We assume, we have a linear table of n cells, in which each cell contains a value of m bits. The input defines the cell number, the contents of the cell defines the output. With this configuration, the routine looks like the one shown below:

```

S-box (inputBits [1 ...  $n$ ], Table [1 ...  $n$ ],  $n$ ,  $m$ )
{
    index  $\leftarrow$  binaryToDecimal (inputBits)
    value  $\leftarrow$  Table [index]
    outputBits  $\leftarrow$  decimalToBinary (value)
    return (outputBits [0 ...  $m$ ])
}

```

41.

```

FeistelRound (inputBits [1 ...  $n$ ], roundKey[1 ...  $n$ ],  $n$ )
{
    (tempLeft, tempRight)  $\leftarrow$  split (word,  $n$ )
    tempLeft  $\leftarrow$  tempLeft  $\oplus$  function (tempRight, roundKey)
    (tempLeft, tempRight)  $\leftarrow$  swap (tempLeft, tempRight)
    outputBits  $\leftarrow$  combine(tempLeft, tempRight)
    return (outputBits [1 ...  $n$ ])
}

```

