

Rosen, Discrete Mathematics and Its Applications, 6th edition
 Extra Examples

Section 10.2—Applications of Trees



— Page references correspond to locations of Extra Examples icons in the textbook.

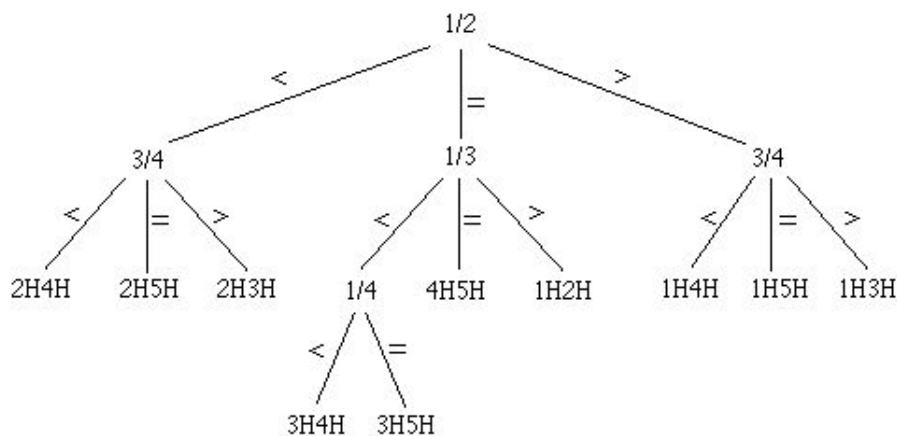
p.698, icon at Example 3

#1. Suppose you have five coins — three are good, but two are counterfeit. Assume that a counterfeit coin is heavier than a good coin and that the two heavy coins have the same weight. A balance scale will be used to find the bad coins and determine which are the heavy coins. Draw a decision tree that describes the process.

Solution:

The following decision tree finds the two heavy coins in at most three weighings. A vertex labeled “X/Y” corresponds to a weighing with coin X on the left pan and coin Y on the right pan. The symbols “<”, “>”, and “=” on the edges mean that the left pan goes up and the right pan goes down, the left pan goes down and the right pan goes up, and the pans balance, respectively. The leaves in the tree are the ten possibilities for the two heavy coins. (For example, the leaf labeled “2H4H” means that coins 2 and 4 are heavy, and therefore counterfeit.) Note that if coins 1 and 2 balance, this does not mean that both coins are good; they could both be counterfeit.

It is not possible to guarantee that the two heavy coins can be found in two weighings. A decision tree for coin weighing is a 3-ary tree, and such a tree can have at most nine leaves if its height is two; but there are ten possibilities, which forces the tree to have more than nine leaves.



p.702, icon at Example 5

#1. Use Huffman coding to encode the following five symbols with given frequencies:

A: 0.23 B: 0.14 C: 0.16 D: 0.18 E: 0.29

Solution:

The following shows the steps in constructing the Huffman code. The resulting code is:

A: 10 B: 001 C:000 D: 11 E: 01.

