

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 3.6—Integers and Algorithms



— Page references correspond to locations of Extra Examples icons in the textbook.

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#1. Find the decimal expansion of $(D5A3)_{16}$.

Solution:

We expand $(D5A3)_{16}$ in terms of powers of 16: $(D5A3)_{16} = 13 \cdot 16^3 + 5 \cdot 16^2 + 10 \cdot 16^1 + 3 \cdot 16^0 = 54,691$.

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#2. Find the hexadecimal expansion of $(35,491)_{10}$.

Solution:

$$35,491 = 16 \cdot 2,218 + 3$$

$$2,218 = 16 \cdot 138 + 10$$

$$138 = 16 \cdot 8 + 10$$

$$8 = 16 \cdot 0 + 8$$

We use the remainders as the “digits”, using A for 10. Reading the remainders from bottom to top, we obtain $35,491 = (8AA3)_{16}$.

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#3. Find the binary expansion of 547.

Solution:

$$547 = 2 \cdot 273 + 1$$

$$273 = 2 \cdot 136 + 1$$

$$136 = 2 \cdot 68 + 0$$

$$68 = 2 \cdot 34 + 0$$

$$34 = 2 \cdot 17 + 0$$

$$17 = 2 \cdot 8 + 1$$

$$8 = 2 \cdot 4 + 0$$

$$4 = 2 \cdot 2 + 0$$

$$2 = 2 \cdot 1 + 0$$

$$1 = 2 \cdot 0 + 1$$

Using the remainders as the digits, and reading from bottom to top, we have $547 = (10\ 0010\ 0011)_2$.

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#4. Find values a , b , and c (not all 0) such that $(abc)_5 = (cba)_8$, or prove that there are none.

Solution:

Note that each of a , b , and c must be between 0 and 4 because the base of the number on the left is 5. Expanding $(abc)_5$ and $(cba)_8$ in terms of base 5 and 8 respectively yields

$$(abc)_5 = 25a + 5b + c \text{ and } (cba)_8 = 64c + 8b + a.$$

If $(abc)_5 = (cba)_8$, then

$$25a + 5b + c = 64c + 8b + a,$$

or

$$24a - 3b - 63c = 0.$$

This simplifies to

$$8a - b - 21c = 0.$$

The only solution with each variable between 0 and 4 (and not all 0) is $a = b = 3$ and $c = 1$. (This is easily seen by trial and error.) Hence $(331)_5 = (133)_8 = 91$.
