Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 4.1—Mathematical Induction

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.267, icon at Example 1

#1. Use the Principle of Mathematical Induction to prove that

$$1 + 5 + 5^2 + 5^3 + \cdots + 5^n = \frac{5^{n+1} - 1}{4} \quad \text{for all } n \geq 0.$$

See Solution

p.267, icon at Example 1

#2. Use the Principle of Mathematical Induction to prove the "generalized" distributive law

$$a(b_1 + b_2 + \cdots + b_n) = ab_1 + ab_2 + \cdots + ab_n$$

for all integers $n \geq 2$.

See Solution

p.267, icon at Example 1

#3. Use the Principle of Mathematical Induction to prove that

$$\sum_{i=1}^{n}(2i+3) = n(n+4) \quad \text{for all } n \geq 1.$$

See Solution

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p.267, icon at Example 1

#4. Find a formula for

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\dots\left(1-\frac{1}{n^2}\right)$$

for $n \geq 2$, and use the Principle of Mathematical Induction to prove that the formula is correct.

See Solution

p.271, icon at Example 5

#1. Use the Principle of Mathematical Induction to show that the following inequality is true for all integers $n \geq 2$:

$$\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > \sqrt{n}.$$

See Solution

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p.271, icon at Example 5

#2. Use the Principle of Mathematical Induction to prove that $n^2 - 5n + 3 > 0$ for all $n \geq 5$.

See Solution

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p.273, icon at Example 8

#1. Prove that 6 is a divisor of $4^n + 7^n + 1$ for all positive integers n.

See Solution

p.273, icon at Example 8

#2. Use the Principle of Mathematical Induction to prove that $2 \mid (n^2 - n)$ for all $n \geq 0$.

See Solution

p.273, icon at Example 8

#3. Use mathematical induction to prove for all positive integers n, then $3^{2^n} - 1$ is divisible by 2^{n+2}.

See Solution

p.273, icon at Example 8

#4. Use mathematical induction to show that every positive integer not exceeding $n!$ can be expressed as the sum of at most n distinct divisors of $n!$

See Solution

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p.276, icon at Example 12

#1. Prove that for all positive integers n, $\frac{(2n)!}{2^n n!}$ is odd.

See Solution

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p.276, icon at Example 12

#2. *Muddy Children Puzzle*: The teacher of a group of children tells these children to play in their schoolyard without getting dirty. However, while playing, exactly n children get mud on their foreheads. When the children come back to the classroom after playing, the teacher states: "At least one of you has a muddy forehead" and then asks the children to answer "Yes" or "No" to the question: "Do you know whether you have a muddy forehead?"

The teacher asks this question over and over. What will the children with the muddy foreheads answer each time this question is asked, assuming that a child can see whether other children have muddy foreheads, but cannot see his or her own forehead? Furthermore, we assume that each child is honest, intelligent, and perceptive, and that all children answer each question simultaneously. (Note that the reason that the answer can differ when the question is asked repeatedly is that the children might learn from the previous answers of the other children.)

See Solution