

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 4.4—Recursive Algorithms



— Page references correspond to locations of Extra Examples icons in the textbook.

p.311, icon at Example 1

#1.

- (a) Write a recursive algorithm for finding the sum of the first n even positive integers.
- (b) Use mathematical induction to prove that the algorithm in (a) is correct.

Solution:

- (a) Let $evensum(n)$ be the sum of the first n even positive integers. A recursive algorithm is:

```
procedure  $evensum(n: integer \geq 1)$ 
  if  $n = 1$  then  $evensum(n) := 2$ 
  else  $evensum(n) := evensum(n - 1) + 2n$ 
```

- (b) Let $P(n)$ be “ $evensum(n)$ is the sum of the first n even positive integers.”

BASIS STEP: When $n = 1$, the “**then**” clause of the procedure takes effect, and gives $evensum(1) = 2$, which is the sum of the first even integer.

INDUCTION STEP: We assume $P(k)$ is true for some $k \geq 1$ and must show that $P(k + 1)$ is true. The proposition $P(k)$ states that “ $evensum(k)$ is the sum of the first k even positive integers”. According to the algorithm, because $k + 1 > 1$, the “**else**” clause is used (with $k + 1$ in place of n) to obtain $evensum(k + 1)$ and gives

$$\begin{aligned}evensum(k + 1) &= evensum(k) + 2(k + 1) \\ &= \text{sum of the first } k \text{ even integers} + 2(k + 1),\end{aligned}$$

which is the sum of the first $k + 1$ even integers. Therefore, the induction step follows.

Thus, the Principle of Mathematical Induction proves that the algorithm is correct.
