ADD-ON 21B

INCENTIVES AND RISK AVERSION

Incentive schemes based on observed performance usually expose the individual facing them to earnings risk because some factors that affect measured performance are out of the individual's control. In Section 21.4, we mentioned that this consideration limits the ability of incentive schemes to achieve efficiency in settings with moral hazard. In such cases, optimal incentive schemes balance the benefits of inducing greater effort against risk-bearing costs. In this Add-On we explore this trade-off in greater detail. This material draws heavily on Chapter 11, "Risk and Uncertainty." Here we assume a working knowledge of that chapter, particularly Section 11.2, "Risk Preference."

We'll consider the case of a salesperson who may exert effort to sell more cars. To keep things simple, we'll assume the salesperson has only two possible actions, exerting "high effort" or "low effort." (In Section 21.4, we allowed the salesperson to exert high effort for any fraction of his 40-hour workweek.) We'll also assume that sales can be either "high" or "low." The probability of high sales if the salesperson exerts high effort is Π_H. It is some smaller probability Π_L if he exerts low effort (so $\Pi_H > \Pi_L$). The car dealership's expected profit, excluding the salesperson's pay, is R_H if the sales are high, and the smaller amount R_L if sales are low.

Figure 21B.1(a) shows possible incentives schemes for the salesperson. We measure his income if sales are high, W_H, on the horizontal axis and his income if sales are low, W_L, on the vertical axis. For example, if the dealership's owner pays the salesperson the dealership's full profit at all sales levels, then the salesperson's pay corresponds to the point R; he receives R_H if sales are high and R_L if sales are low. We assume that, for each possible sales realization, the salesperson's consumption equals his compensation.

The figure also shows the incentive schemes that induce the salesperson to exert high effort. These are the points in the green-shaded region, whose boundary is the curve labeled *MH* (for "moral hazard"). For example, if the salesperson is an expected utility maximizer (see Section 11.2, pages 381–385) and incurs a personal disutility $C > 0$ from exerting high effort, then he will exert high effort if

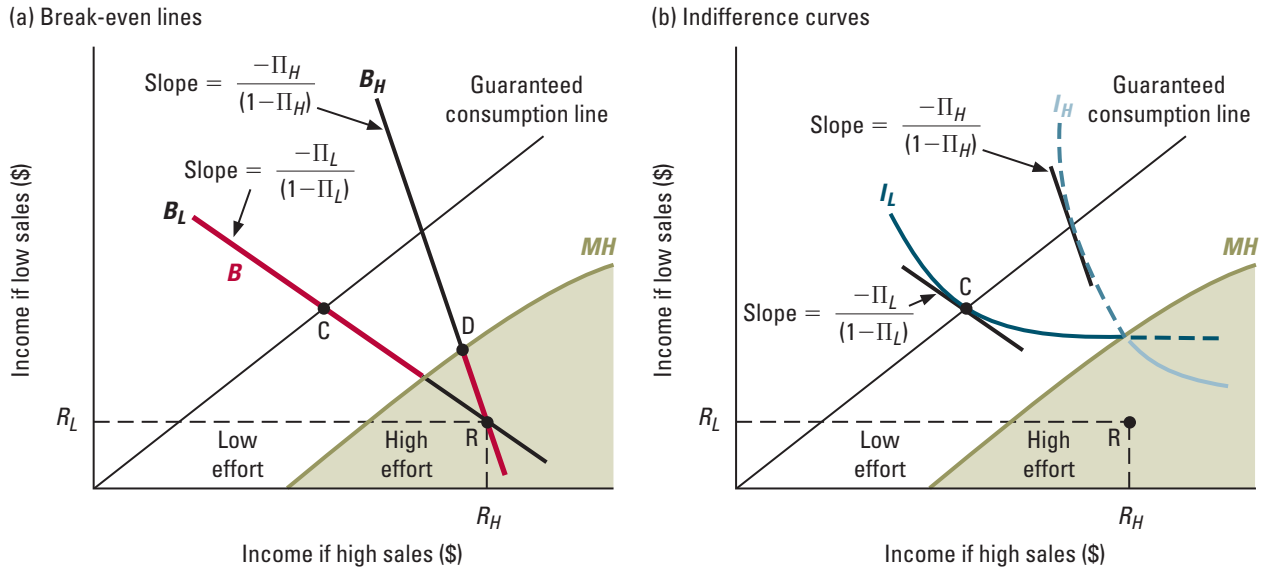
$$\Pi_H U(W_H)+(1-\Pi_H)U(W_L) - C \geq \Pi_L U(W_H)+(1-\Pi_L)U(W_L) \qquad \textbf{(1)}$$

where $U(W)$ is the benefit from consumption W. [The left-hand side of inequality (1) is the salesperson's expected utility if he chooses to exert high effort, taking account of both the resulting probability of high sales and the disutility of effort, while the right-hand side is his expected utility if he chooses to exert low effort.] We can rewrite expression (1) as:

$$(\Pi_H - \Pi_L)[U(W_H) - U(W_L)] \geq C \qquad \textbf{(2)}$$

Figure 21B.1

Break-Even Lines and Indifference Curves with Moral Hazard The figures show possible incentive schemes for a salesperson who may either exert high or low effort, which affects the likelihood of high versus low sales. The point R is the dealership's profit for the two possible sales realizations. For schemes that lie in the green-shaded region, the salesperson exerts high effort. Outside of that region, he exerts low effort. In figure (a), the dealership breaks even at points on the red line segments labeled *B,* which jump from B_L to B_H at the green *MH* curve because of the change in the salesperson's effort. Figure (b) shows an indifference curve for the salesperson, which consists of the solid parts of the dark- and light-blue curves. The dark- and light-blue indifference curves reflect the salesperson's risk aversion because along the lines with constant expected income, the salesperson prefers to be on the guaranteed consumption line, with the same compensation regardless of whether sales are high or low.



Since the cost C of exerting high effort is positive, and the probability of high sales is greater if there is high effort ($\Pi_H > \Pi_L$), expression (2) tells us that to induce the salesperson to work hard his pay must be greater when sales are high than when sales are low. In other words, we must have $W_H > W_L$, which implies that the entire green-shaded "high-effort" area lies below the guaranteed consumption line. (We introduced the concept of a guaranteed consumption line on page 374.) In Figure 21B.1(a), the salesperson exerts high effort if the dealership pays him its full profit. We'll maintain this assumption for now and consider later the case in which paying the salesperson the full profit does not induce him to work hard.

We'll suppose that the labor market is competitive and that competition among dealerships for employees causes dealerships to just break even on average. Figure 21B.1(a) also shows the break-even incentive schemes. We have drawn two lines, B_H and B_L. Line B_H contains those incentive schemes that cause the dealership to break even on average if

the salesperson exerts high effort. Since the dealership earns its profit less the amount it pays the salesperson, such schemes satisfy the equation

$$\Pi_H(R_H - W_H) + (1 - \Pi_H)(R_L - W_L) = 0 \quad \textbf{(3)}$$

or, after rearranging terms,

$$W_L = \left[R_L + \left(\frac{\Pi_H}{1 - \Pi_H}\right)R_h\right] - \left(\frac{\Pi_H}{1 - \Pi_H}\right)W_H \quad \textbf{(4)}$$

So the slope of that line is $-\Pi_H / (1 - \Pi_H)$. Likewise, the line B_L contains those incentive schemes that cause the dealership to break even on average if the salesperson exerts low effort. Following similar reasoning, its slope is $-\Pi_L / (1 - \Pi_L)$. Since $\Pi_H > \Pi_L$, B_L is flatter than B_H. Because the salesperson exerts high effort only for incentive schemes in the green-shaded region, the true break-even schemes, which lie on the red line segments labeled *B,* switch from B_H below *MH* to B_L above *MH.*

Figure 21B.1(b) shows how to construct the salesperson's indifference curves. The dark blue curve labeled I_L is an indifference curve if the salesperson exerts low effort. The curve reflects risk aversion on the part of the salesperson because, along the constant expected consumption line with slope $-\Pi_L / (1 - \Pi_L)$, the salesperson prefers point C, which lies on the guaranteed consumption line (see Chapter 11, page 376). The light blue curve, labeled I_H, is an indifference curve if the salesperson exerts high effort. It corresponds to the same level of well-being as I_L (taking the cost of effort into account). That curve also reflects the salesperson's risk aversion because, along the constant expected consumption line associated with high effort, the salesperson prefers the point on the guaranteed consumption line. Because he exerts high effort only in the green-shaded region, his overall indifference curve is the solid scallop-shaped curve consisting of the light-blue curve in the green-shaded region and the dark-blue curve in the unshaded region.

If competition for salespeople among dealerships drives dealerships' profits to zero on average, they will end up offering the incentive scheme that maximizes a salesperson's well-being subject to breaking even—in other words, the incentive scheme on the red break-even line segments labeled *B* that the salesperson most prefers.[1] Sometimes they will offer a higher wage when sales are high to induce high effort; other times the dealerships will forgo high effort in order to avoid the costs of making the salesperson face risk. Figures 21B.2(a) and (b) illustrate these cases.

Among the incentive schemes that induce high effort and allow the dealership to break even, the best one for the salesperson is point D in Figures 21B.2(a) and (b). Any scheme that lies on the red break-even line segments labeled *B* and inside the green-shaded region, such as point E in Figure 21B.2(a), is worse for the salesperson than D. The reason is that the salesperson prefers points along this line that are closer to the guaranteed consumption line.[2] In other words, because the salesperson is risk averse, the dealership should impose the smallest amount of consumption variation on the salesperson that still induces high effort.

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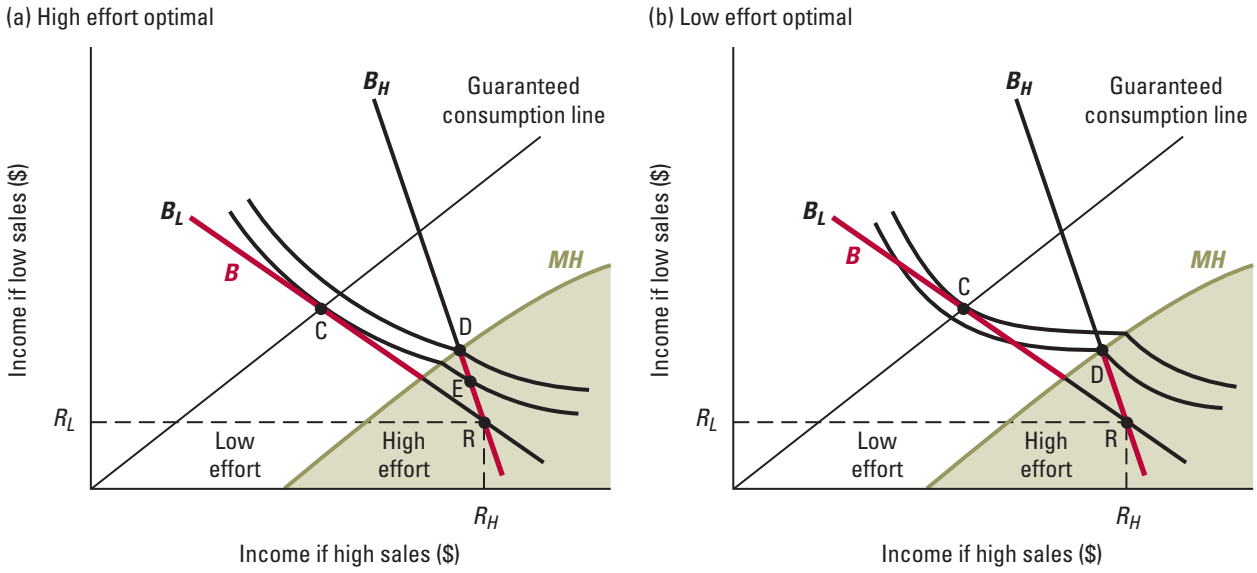
[1] Even if competition does not drive the dealership's profit to zero, similar conclusions will apply since any optimal contract must maximize the salesperson's well-being, holding fixed the dealership's expected profit.

[2] Risk aversion, as defined in Chapter 11, implies that a point on the guaranteed consumption line is better than any point with varying consumption and the same expected consumption level. Here we make the slightly stronger assumption that moving along the line B_H toward the guaranteed consumption line, which reduces risk while holding expected consumption constant, makes the salesperson better off.

Figure 21B.2

The Optimal Incentive Scheme In both figures, the point D represents the best incentive scheme that induces high effort, while point C—which gives a guaranteed consumption level—is best when inducing low effort. In each case, these points involve the lowest level of risk among schemes that induce the given level of effort. In figure (a), the salesperson is better off at point D, so competition among dealerships will lead them to offer point D in a competitive equilibrium. In figure (b), the salesperson is more risk averse than in figure (a) and so prefers point C to point D. Competition among dealerships for salespeople will in that case involve compensation that is independent of a salesperson's success in making sales.



Among the incentive schemes that induce low effort and allow the dealership to break even, the best one for the salesperson is point C in Figures 21B.2(a) and (b). That scheme ensures the salesperson as much income as possible given that he'll choose low effort, and given that the dealership must break even. Because point C lies on the guaranteed consumption line, the salesperson faces no consumption risk at all—he is paid the same amount regardless of how many cars are sold.

In Figure 21B.2(a), the salesperson prefers point D to point C. Therefore, the best incentive scheme induces high effort. In contrast, in Figure 21B.2(b) the salesperson is more risk averse and prefers point C to point D. In that case, the best incentive scheme does not link compensation to profits, so the salesperson exerts low effort. Risk-bearing costs lead the dealership to forgo completely the use of incentive pay in Figure 21B.2(b).

What happens if the salesperson exerts low effort even when he receives all of the dealership's profits? That case is shown in Figure 21B.3. Notice that point R lies in the unshaded region. Because B_H is below B_L to the right of point R (rather than above B_L as

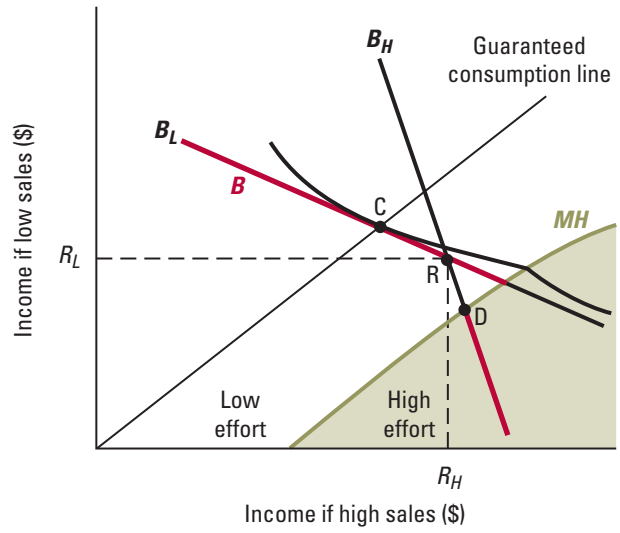


Figure 21B.3

The Optimal Compensation Policy when Receiving All Profits Does Not Induce High Effort The figure shows a case in which point R does not lie in the green-shaded region, so offering all the profits to the salesperson does not induce high effort. The optimal compensation policy in this case is always point C, a fixed wage.

it is to the left of point R), the red break-even line, *B,* jumps downward when it hits the green *MH* curve, rather than upward. As a result, the salesperson always prefers point C to point D when he is risk averse, and the optimal contract pays the salesperson a fixed wage. This makes sense. To induce high effort, the dealership must pay the salesperson more than its profits, and therefore lose money, when sales are high. Clearly, the dealership's owner has no interest in providing incentives for the salesperson to take actions that result in negative profits. Given that the salesperson will exert low effort, it is best to given him a fixed wage to avoid imposing risk-bearing costs.