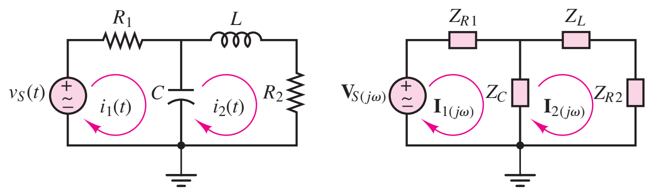

(a) (b)

Figure 4.53 (a) Circuit for Example 4.21; (b) same circuit ready for phasor analysis

Step 1: $v_S(t) = 15\cos(1{,}500t) \qquad \omega = 1{,}500$ rad/s
Step 2: $V_S(j\omega) = 15\angle 0$
Step 3: $Z_{R_1} = R_1 \qquad Z_{R_2} = R_2 \qquad Z_C = \dfrac{1}{j\omega C} \qquad Z_L = j\omega L$

The resulting phasor circuit is shown in Figure 4.53(b).

Step 4: We solve for the source current using mesh analysis. First, we write the mesh equations:

$$\mathbf{V}_S(j\omega) - Z_{R_1}\mathbf{I}_1(j\omega) - Z_C[\mathbf{I}_1(j\omega) - \mathbf{I}_2(j\omega)] = 0 \qquad \text{mesh 1}$$
$$Z_C[\mathbf{I}_2(j\omega) - \mathbf{I}_1(j\omega)] - Z_L\mathbf{I}_2(j\omega) + Z_{R_2}\mathbf{I}_2(j\omega) = 0 \qquad \text{mesh 2}$$

$Z_C[\mathbf{I}_2(j\omega)-\mathbf{I}_1(j\omega)]-Z_L\mathbf{I}_2(j\omega)+Z_{R_2}\mathbf{I}_2(j\omega)=0$
should be changed to
$Z_C[\mathbf{I}_2(j\omega)-\mathbf{I}_1(j\omega)]+Z_L\mathbf{I}_2(j\omega)+Z_{R_2}\mathbf{I}_2(j\omega)=0$

Next, we write the matrix form of the equations:

$$\begin{bmatrix} Z_{R_1} + Z_C & -Z_C \\ -Z_C & Z_L - Z_{R_2} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1(j\omega) \\ \mathbf{I}_2(j\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_S(j\omega) \\ 0 \end{bmatrix}$$

$\begin{bmatrix} Z_{R1}+Z_C & -Z_C \\ -Z_C & Z_C - Z_{R2} \end{bmatrix}$
should be changed to
$\begin{bmatrix} Z_{R_1}+Z_C & -Z_C \\ -Z_C & Z_C+Z_L+Z_{R_2} \end{bmatrix}$

and we use Cramer's rule to solve for the two currents:

$$\mathbf{I}_1(j\omega) = \frac{\begin{vmatrix} \mathbf{V}_S(j\omega) & -Z_C \\ 0 & Z_L - Z_{R_2} \end{vmatrix}}{\begin{vmatrix} Z_{R_1} + Z_C & -Z_C \\ -Z_C & Z_L - Z_{R_2} \end{vmatrix}} = \frac{Z_L - Z_{R_2}}{(Z_{R_1}+Z_C)(Z_L - Z_{R_2}) - Z_C^2}\mathbf{V}_S(j\omega)$$

$$\mathbf{I}_2(j\omega) = \frac{\begin{vmatrix} Z_{R_1} + Z_C & \mathbf{V}_S(j\omega) \\ -Z_C & 0 \end{vmatrix}}{\begin{vmatrix} Z_{R_1} + Z_C & -Z_C \\ -Z_C & Z_L - Z_{R_2} \end{vmatrix}} = \frac{Z_C}{(Z_{R_1}+Z_C)(Z_L - Z_{R_2}) - Z_C^2}\mathbf{V}_S(j\omega)$$

and we use Cramer's rule to solve for the two currents:

$$\mathbf{I}_1(j\omega) = \frac{\begin{vmatrix} \mathbf{V}_S(j\omega) & -Z_C \\ 0 & Z_C + Z_L + Z_{R_2} \end{vmatrix}}{\begin{vmatrix} Z_{R_1} + Z_C & -Z_C \\ -Z_C & Z_C + Z_L + Z_{R_2} \end{vmatrix}} = \frac{Z_C + Z_L + Z_{R_2}}{(Z_{R_1}+Z_C)(Z_C + Z_L + Z_{R_2}) - Z_C^2}\mathbf{V}_S(j\omega)$$

$$\mathbf{I}_2(j\omega) = \frac{\begin{vmatrix} Z_{R_1} + Z_C & \mathbf{V}_S(j\omega) \\ -Z_C & 0 \end{vmatrix}}{\begin{vmatrix} Z_{R_1} + Z_C & -Z_C \\ -Z_C & Z_C + Z_L + Z_{R_2} \end{vmatrix}} = \frac{Z_C}{(Z_{R_1}+Z_C)(Z_C + Z_L + Z_{R_2}) - Z_C^2}\mathbf{V}_S(j\omega)$$

Now we substitute the impedance values in the above expressions:

$$\mathbf{I}_1(j\omega) = \frac{Z_L + Z_{R_2}}{(Z_{R_1}+Z_C)(Z_L - Z_{R_2}) - Z_C^2} = \frac{j\omega L - R_2}{(R_1 + 1/j\omega C)(j\omega L - R_2) - (1/j\omega C)^2}$$
$$= \frac{j\omega C(j\omega L - R_2)}{(j\omega C R_1 + 1)(j\omega L - R_2) - 1/j\omega C}$$

$$\mathbf{I}_2(j\omega) = \frac{Z_C}{(Z_{R_1}+Z_C)(Z_L - Z_{R_2}) - Z_C^2} = \frac{1}{(j\omega C R_1 + 1)(j\omega L - R_2) - 1/j\omega C}$$

Now we substitute the impedance values in the above expressions

$$\mathbf{I}_1(j\omega) = \frac{1/j\omega C + j\omega L + R_2}{(R_1 + 1/j\omega C)(1/j\omega C + j\omega L + R_2) - (1/j\omega C)^2}\mathbf{V}_S(j\omega)$$
$$= \frac{j\omega C + (j\omega C)^2(j\omega L) + (j\omega C)^2 R_2}{(j\omega C R_1 + 1)(1 + (j\omega C)(j\omega L) + j\omega C R_2) - 1}\mathbf{V}_S(j\omega)$$
$$\mathbf{I}_2(j\omega) = \frac{1/j\omega C}{(R_1 + 1/j\omega C)(1/j\omega C + j\omega L + R_2) - (1/j\omega C)^2}\mathbf{V}_S(j\omega)$$
$$= \frac{j\omega C}{(j\omega C R_1 + 1)(1 + (j\omega C)(j\omega L) + j\omega C R_2) - 1}\mathbf{V}_S(j\omega)$$

and use numerical values to obtain

$$\mathbf{I}_1(j\omega) = 7.974 \times 10^{-4}\angle(1.5378)\mathbf{V}_S(j\omega) = 0.012\angle(1.5378) \text{ A}$$
$$\mathbf{I}_2(j\omega) = 7.0528 \times 10^{-4}\angle(-1.7034)\mathbf{V}_S(j\omega) = 0.0106\angle(-1.7034) \text{ A}$$

and use numerical values to obtain

$\mathbf{I}_1(j\omega) = 0.0033\angle 0.92$

$\mathbf{I}_2(j\omega) = 0.0196\angle -1.49$