(Concluded)

The input to the suspension system is the road surface profile, which generates both displacement and velocity inputs x_{road} and $\dot{x}_{road}$. One objective of the suspension is to isolate the body of the car (i.e., the passengers) from any vibration caused by unevenness in the road surface. Automotive suspension systems are also very important in guaranteeing vehicle stability and in providing acceptable handling. In this illustration we simply consider the response of the vehicle to a sharp step of amplitude 10 cm (see Figure 5.49) for two cases, corresponding to new and worn-out shock absorbers, respectively. Which ride would you prefer?

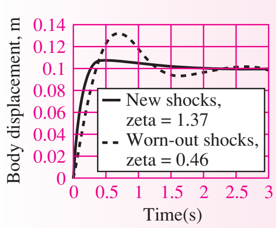

Figure 5.49 "Step" response of automotive suspension

Solution

Known Quantities: Circuit elements.

Find: The complete response of the differential equation in $i_L(t)$ describing the circuit of Figure 5.50.

Schematics, Diagrams, Circuits, and Given Data: $V_S = 25$ V; $R = 5$ kΩ; $C = 1\ \mu$F; $L = 1$ H.

Assumptions: The capacitor has been charged (through a separate circuit, not shown) prior to the switch closing, such that $v_C(0) = 5$ V.

Analysis:

Step 1: Steady-state response. Before the switch closes, the current in the circuit must be zero. We are therefore sure that the inductor current is initially zero: $i_L(0^-) = 0$. We cannot know, in general, what the state of charge of the capacitor is. The problem statement tells us that $v_C(0^-) = 5$ V. This fact will be useful later, when we determine the initial conditions.

After the switch has been closed for a long time and all the transients have died, the capacitor becomes an open circuit, and the inductor behaves as a short circuit. Since the open circuit prevents any current flow, the voltage across the resistor will be zero. Similarly, the inductor voltage is zero, and therefore the source voltage will appear across the capacitor. Hence, $i_L(\infty) = 0$ and $v_C(\infty) = 25$ V.

Step 2: Initial conditions. Recall that for a second-order circuit, we need to determine two initial conditions; and recall that, from continuity of inductor voltage and capacitor currents, we know that $i_L(0^-) = i_L(0^+) = 0$ and $v_C(0^-) = v_C(0^+) = 5$ V. Since the differential equation is in the variable i_L, the two initial conditions we need to determine are $i_L(0^+)$ and $di_L(0^+)/dt$. These can actually be found rather easily by applying KVL at $t = 0^+$:

$$V_S - v_C(0^+) - Ri_L(0^+) - v_L(0^+) = 0$$

$$V_S - v_C(0^+) - Ri_L(0^+) - L\frac{di_L(0^+)}{dt} = 0$$

$$\frac{di_L(0^+)}{dt} = \frac{V_S}{L} - \frac{v_C(0^+)}{L} = 25 - 5 = 20 \text{ V}$$

Step 3: Differential equation. The differential equation for the series circuit can be obtained by KVL:

$$V_S - v_C - Ri_L(t) - L\frac{di_L(t)}{dt} = 0$$

After substituting

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_L(t')\,dt'$$

we have the equation

$$LC\frac{di_L(t)}{dt} + RCi_L(t) + \int_{-\infty}^{t} i_L(t')\,dt' = CV_S$$