resulting in the integrodifferential equation

$$V_S - L\frac{di_L(t)}{dt} - \frac{1}{C}\int_{-\infty}^{t} i_L(t')dt' - Ri_L(t) = 0 \qquad t \geq 0$$

which can be differentiated on both sides to obtain

$$LC\frac{d^2 i_L(t)}{dt^2} + RC\frac{di_L(t)}{dt} + i_L(t) = C\frac{dV_S}{dt} \qquad t \geq 0$$

Note that the right-hand side (forcing function) of this differential equation is exactly zero, since V_S is a constant.

Step 4: Solve for ω_n and ζ. If we now compare the second-order differential equations to the standard form of equation 5.50, we can make the following observations:

$$\omega_n = \sqrt{\frac{1}{LC}} = 447 \text{ rad/s}$$

$$\zeta = RC\frac{\omega_n}{2} = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.447$$

Thus, the second-order circuit is underdamped.

Step 5: Write the complete solution. Knowing that the circuit is underdamped ($\zeta < 1$), we write the complete solution for this case as

$$x(t) = x_N(t) + x_F(t) = \alpha_1 e^{\left(-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)t} + \alpha_2 e^{\left(-\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}\right)t} + x(\infty) \quad t \geq 0$$

and since $x_F = i_{LF} = i_L(\infty) = 0$, the complete solution is identical to the homogeneous solution:

$$i_L(t) = i_{LN}(t) = \alpha_1 e^{\left(-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)t} + \alpha_2 e^{\left(-\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}\right)t} \qquad t \geq 0$$

Step 6: Solve for the constants α_1 and α_2. Finally, we solve for the initial conditions to evaluate the constants α_1 and α_2. The first initial condition yields

$$i_L(0^+) = \alpha_1 e^0 + \alpha_2 e^0 = 0$$

$$\alpha_1 = -\alpha_2$$

The second initial condition is evaluated as follows:

$$\frac{di_L(t)}{dt} = \left(-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)\alpha_1 e^{\left(-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)t} + \left(-\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}\right)\alpha_2 e^{\left(-\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}\right)t}$$

$$\frac{di_L(0^+)}{dt} = \left(-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)\alpha_1 e^0 + \left(-\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}\right)\alpha_2 e^0$$

Substituting $\alpha_1 = -\alpha_2$, we get

$$\frac{di_L(0^+)}{dt} = \left(-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)\alpha_1 - \left(-\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}\right)\alpha_1 = 20 \text{ V}$$

$$2\left(j\omega_n\sqrt{1-\zeta^2}\right)\alpha_1 = 20$$

$$\alpha_1 = \frac{10}{j\omega_n\sqrt{1-\zeta^2}} = -j\frac{10}{\omega_n\sqrt{1-\zeta^2}} = -j0.025$$

$$\alpha_2 = -\alpha_1 = j0.025$$