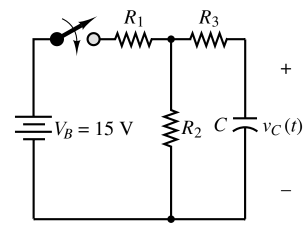

Figure 5.29 Circuit for Example 5.10

Solution

Known Quantities: Battery voltage; resistor and capacitor values.

Find: Capacitor voltage as a function of time $v_C(t)$ for all t.

Schematics, Diagrams, Circuits, and Given Data: $R_1 = R_2 = 1{,}000\ \Omega$, $R_3 = 500\ \Omega$, and $C = 25\ \mu$F. Figure 5.29.

Assumptions: None.

Analysis:

Part 1— $0 \le t < 50$ ms

Step 1: Steady-state response. We first observe that any charge stored in the capacitor has had a discharge path through resistors R_3 and R_2. Thus, the capacitor must be completely discharged. Hence,

$$v_C(t) = 0\text{ V} \qquad t < 0 \qquad \text{and} \qquad v_C(0^-) = 0\text{ V}$$

To determine the steady-state response, we look at the circuit a long time after the switch has been closed. At steady state, the capacitor behaves as an open circuit, and we can calculate the equivalent open circuit (Thévenin) voltage and equivalent resistance to be

$$v_C(\infty) = V_B \frac{R_2}{R_1 + R_2}$$
$$= 7.5\text{ V}$$
$$R_T = R_3 + R_1 \| R_2 = 1\text{ k}\Omega$$

Step 2: Initial condition. We can determine the initial condition for the variable $v_C(t)$ by virtue of the continuity of capacitor voltage (equation 5.22):

$$v_C(0^+) = v_C(0^-) = 0\text{ V}$$

Step 3: Writing the differential equation. To write the differential equation, we use the Thévenin equivalent circuit for $t \ge 0$, with $V_T = v_c(\infty)$ and we write the resulting differential equation

$$V_T - R_T i_C(t) - v_C(t) = V_T - R_T C \frac{dv_C(t)}{dt} - v_C(t) = 0 \qquad 0 \le t < 50\text{ ms}$$

$$R_T C \frac{dv_C(t)}{dt} + v_C(t) = V_T \qquad 0 \le t < 50\text{ ms}$$

Step 4: Time constant. In the above equation we recognize, with reference to equation 5.22, the following variables and parameters:

$$x = v_C; \qquad \tau = R_T C = 0.025\text{ s}; \qquad K_S = 1;$$
$$f(t) = V_T = 7.5\text{ V} \qquad 0 \le t < 50\text{ ms}$$

Step 5: Complete solution. The complete solution is

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-t/\tau} \qquad 0 \le t < 50\text{ ms}$$
$$v_C(t) = V_T + (0 - V_T)e^{-t/R_T C} = 7.5(1 - e^{-t/0.025})\text{ V} \qquad 0 \le t < 50\text{ ms}$$