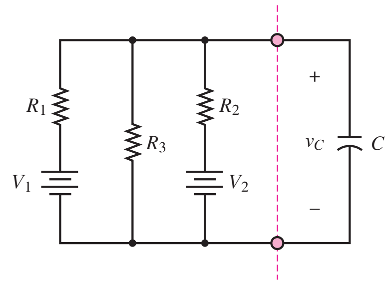

Figure 5.26 The circuit of Figure 5.22 for $t \geq 0$

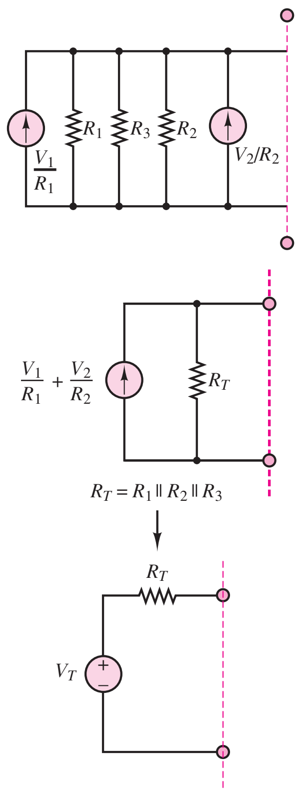

Figure 5.27 Reduction of the circuit of Figure 5.26 to Thévenin equivalent form

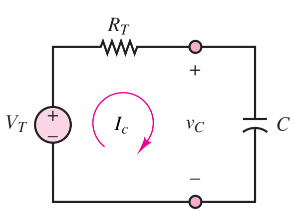

Figure 5.28 The circuit of Figure 5.25 in equivalent form for $t \geq 0$

to Thévenin form. Figure 5.28 depicts the final appearance of the equivalent circuit for $t \geq 0$.

When the switch has been closed for a long time, the capacitor sees the Thévenin equivalent circuit computed in Figures 5.27 and 5.28. Thus, when the capacitor is replaced with an open circuit, $v_C(\infty) = V_T$. Further, we can determine the initial condition for the variable $v_C(t)$ by virtue of the continuity of capacitor voltage (equation 5.22): $v_C(0^+) = v_C(0^-) = V_2$. At $t = 0$ the switch closes, and the circuit is described by the following differential equation, obtained by application of KVL for the circuit of Figure 5.28:

$$V_T - R_T i_C(t) - v_C(t) = V_T - R_T C \frac{dv_C(t)}{dt} - v_C(t) = 0 \qquad t > 0$$

$$R_T C \frac{dv_C(t)}{dt} + v_C(t) = V_T \qquad t > 0$$

with

$$R_T = R_1 || R_2 || R_3$$

$$V_T = R_T \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

In the above equation we recognize, with reference to equation 5.22, the following variables and parameters: $x = v_C$; $\tau = R_T C$; $K_S = 1$; $f(t) = V_T$ for $t > 0$. And we can write the complete solution

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-t/\tau} \qquad t \geq 0$$

$$v_C(t) = V_T + (V_2 - V_T)e^{-t/R_T C}$$

The use of Thévenin equivalent circuits to obtain transient responses is emphasized in the next few examples.

LO4

EXAMPLE 5.10 Use of Thévenin Equivalent Circuits in Solving First-Order Transients

Problem

The circuit of Figure 5.29 includes a switch that can be used to connect and disconnect a battery. The switch has been open for a very long time. At $t = 0$ the switch closes, and then at $t = 50$ ms the switch opens again. Determine the capacitor voltage as a function of time.