decreased by the same factor, relative to the maximum value (at the resonant frequency). Since power in an electric signal is proportional to the square of the voltage or current, a drop by a factor $1/\sqrt{2}$ in the output voltage or current corresponds to the power being reduced by a factor of $\frac{1}{2}$. Thus, we term the frequencies at which the intersection of the 0.707 line with the frequency response occurs the **half-power frequencies.** Another useful definition of bandwidth B is as follows. We shall make use of this definition in the following examples. Note that a high-Q filter has a narrow bandwidth, and a low-Q filter has a wide bandwidth.

$$\boxed{B = \frac{\omega_n}{Q} \qquad \text{bandwidth}} \tag{6.46}$$

EXAMPLE 6.11 Frequency Response of Bandpass Filter

Problem

Compute the frequency response of the bandpass filter of Figure 6.25 for two sets of component values.

Solution

Known Quantities:

(a) $R = 1\ \text{k}\Omega$; $C = 10\ \mu\text{F}$; $L = 5$ mH.

(b) $R = 10\ \Omega$; $C = 10\ \mu\text{F}$; $L = 5$ mH.

Find: The frequency response $H_V(j\omega)$.

Assumptions: None.

Analysis: We write the frequency response of the bandpass filter as in equation 6.40:

$$H_V(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i}(j\omega) = \frac{j\omega CR}{1 + j\omega CR + (j\omega)^2 LC}$$

$$= \frac{\omega CR}{\sqrt{\left(1 - \omega^2 LC\right)^2 + (\omega CR)^2}} \angle \left[\frac{\pi}{2} - \arctan\left(\frac{\omega CR}{1 - \omega^2 LC}\right)\right]$$

We can now evaluate the response for two different values of the series resistance. The frequency response plots for case a (large series resistance) are shown in Figure 6.28. Those for case b (small series resistance) are shown in Figure 6.29. Let us calculate some quantities for each case. Since L and C are the same in both cases, the *resonant frequency* of the two circuits will be the same:

$$\omega_n = \frac{1}{\sqrt{LC}} = 4.47 \times 10^3 \text{ rad/s}$$

On the other hand, the *quality factor* Q will be substantially different:

$$Q_a = \frac{1}{\omega_n CR} \approx 2.22 \qquad \text{case a}$$

$$Q_b = \frac{1}{\omega_n CR} \approx 0.022 \qquad \text{case b}$$