Activity Set 3.3 MULTIPLYING WITH BASE-TEN PIECES

PURPOSE

To use base-ten pieces to build visual models for multiplication.

MATERIALS

Base-ten number pieces from the Manipulative Kit or from the Virtual Manipulatives.

INTRODUCTION

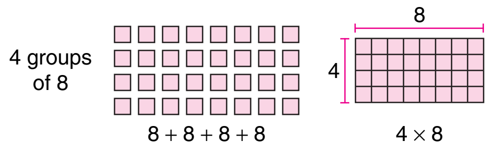
Multiplication is often thought of as repeated addition. For example,

$$4 \times 8$$

can be thought of as 4 copies of 8, or

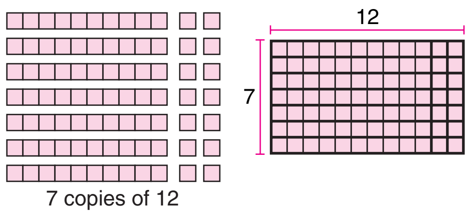
$$8 + 8 + 8 + 8$$

If repeated addition is illustrated with 4 sets of 8 tiles, the tiles can be pushed together to form a rectangular array of squares with dimensions 4 by 8.



So, one possibility for a conceptual model for multiplication is a rectangle. The two factors are the dimensions of the rectangle, and the number of squares in the rectangle is the product. This rectangular model (or area model) and base-ten pieces will be used to illustrate multiplication algorithms.

1. The product 7×12 can be represented with base-ten pieces as 7 copies of 12. When the pieces are pushed together, they form a rectangle with dimensions 7×12. By regrouping the base pieces to form a minimal collection of 8 longs and 4 units, we can determine that the product is 84.



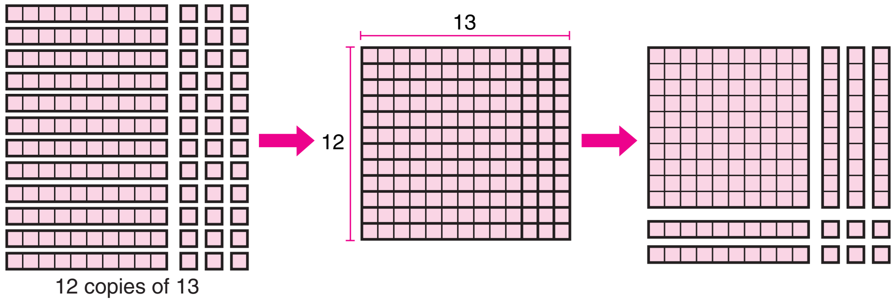
Determine the following products by forming rectangles with your base-ten pieces and then regrouping to obtain the minimal collection. Sketch each rectangle of base-ten pieces, and write the numeral representing the minimal collection beneath it.

a. 6×14

b. 7×23

***2.** Explain how it is possible, using base-ten number pieces, to multiply a one-digit number by a two-digit number without using any multiplication facts. For example, think of representing 7×64 with your base-ten number pieces. Draw a diagram to illustrate your explanation.

3. The product 12×13 is represented below with base-ten pieces as 12 copies of 13. When pushed together, the pieces form a rectangle with dimensions 12×13. Several exchanges can be made without affecting the dimensions of the rectangle. Ten longs can be exchanged for 1 flat, and groups of 10 adjacent units can be exchanged for longs. The resulting rectangle is composed of 1 flat, 5 longs, and 6 units, so the product of 12 and 13 is 156.



Use your number pieces to determine the following products. Make exchanges when possible, and use as few base pieces as you can when building your rectangle. Sketch each rectangle, regroup to show the minimal set of base-ten pieces, and record the numeral for this collection.

a. 13×13

***b.** 21×23

c. 17×12

***4.** Explain how it is possible, using base-ten pieces, to multiply a two-digit number by a two-digit number without using any multiplication facts. Draw a diagram to illustrate your explanation for 21×23.

5. The rectangular model for multiplication corresponds very closely to the traditional paper-and-pencil algorithm for multiplication. Use your base-ten number pieces to form the rectangle representing 23 × 24.

***a.** Here is a paper-and-pencil procedure that uses four partial products for multiplication. Make a diagram of the base-ten number piece rectangle representing 23 × 24. Clearly match each partial product with the corresponding region of your diagram.

$$\begin{array}{rll} & 24 & \\ \times & \underline{23} & \\ & 12 & (3 \times 4) \\ & 60 & (3 \times 20) \\ & 80 & (20 \times 4) \\ & \underline{400} & (20 \times 20) \\ & 552 & \end{array} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{Partial products}$$

b. The following procedure uses two partial products for multiplication. Make another sketch of your number piece rectangle, and match the parts of the rectangle with the corresponding partial products.

$$\begin{array}{rll} & 24 & \\ \times & \underline{23} & \\ & 72 & (3 \times 24) \\ & \underline{480} & (20 \times 24) \\ & 552 & \end{array} \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Partial products}$$

6. Once you become familiar with the base-ten number piece model, it is easy to sketch diagrams of rectangles in order to compute products. For example, to compute 33 × 41, you would outline the rectangle as in the first sketch below and fill it with flats, longs, and units as in the second sketch. The product is obtained by counting the number pieces, 12 hundreds, 15 tens, and 3 units, and regrouping to obtain 1 thousand, 3 hundreds, 5 tens, and 3 units (1353).

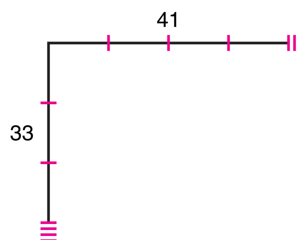


Figure 1

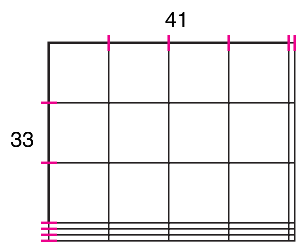


Figure 2

For each of the products on the following page,

(1) Draw a base-ten number piece sketch, similar to figure 2, in the first column.

(2) Determine the product from the sketch and record it below the sketch.

(3) In column 2, compute the product using the paper-and-pencil algorithm with *four* partial products and match the partial products to the corresponding parts of your sketch by marking the partial products on the sketch.

(4) In column 3 compute the product using the paper-and-pencil algorithm with *two* partial products and match the partial products to the corresponding parts of your sketch by marking the partial products on the sketch.

a. 12×23

Sketch and Product from Sketch	Four-Partial-Product Algorithm	Two-Partial-Product Algorithm

***b.** 22×43

Sketch and Product from Sketch	Four-Partial-Product Algorithm	Two-Partial-Product Algorithm

c. 45×45

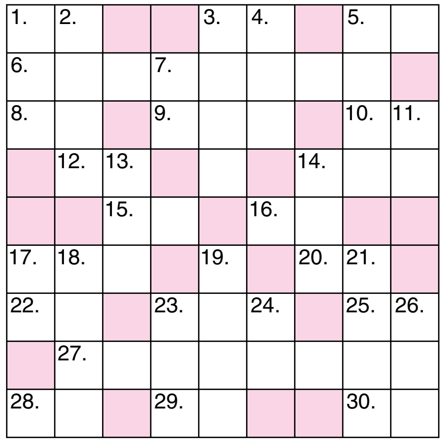
Sketch and Product from Sketch	Four-Partial-Product Algorithm	Two-Partial-Product Algorithm

7. Examine the relationship between the four-partial-product algorithm and the two-partial-product algorithm in activity 6 and explain how they are related.

JUST FOR FUN

CROSS-NUMBERS FOR CALCULATORS

Sit back, relax, and exercise your calculator's multiplication key with this cross-number puzzle.



Across

1. $[(18 + 2) \times 4] + 19$
5. $\dfrac{14 + 3}{9 + 8} \times 66$
6. 500,000,000 ÷ 9 (Omit the decimal part of the number.)
8. The middle two digits of the product of $2 \times 2 \times 2 \times 2 \times 3 \times 37$
12. $\dfrac{(41 - 29) \times (63 - 23)}{95 \div 19}$
15. The sum of the digits in the product of $13 \times 9004 \times 77$
16. The sum of the first seven odd numbers
17. March 30, 1974: A "freeze model" in a department store posed motionless for 5 hours, 32 minutes. How many minutes in all did he pose?
23. $\dfrac{13{,}334 - 6566}{8 \times 9} \times \dfrac{992 \div 4}{17 + 14}$
27. $(10{,}000{,}000 \div 81) \times 100$ (nearest whole number)
28. Take the current year, subtract 17, multiply by 2, add 6, divide by 4, add 21, subtract $\frac{1}{2}$ of the current year.
29. The first two digits of the product of 150×12

Down

3. $(34 \times 53) - (137 \times 2)$
4. Asphalt roof shingles are sold in bundles of 27. Three bundles cover 100 ft^2. How many bundles are needed for 11,700 ft^2 of roofing in a housing development?
5. $\dfrac{88{,}984 \times 493}{17 \times 392}$
7. $\dfrac{51 \times 153 \times 62 \times 57}{17 \times 969 \times 31}$
11. $\dfrac{(62 \times 21) + 23}{25} - 11$
14. The number of different combinations of U.S. coins into which a dollar can be changed. (This number is a palindrome, and the sum of its digits is 13.)
17. Remainder of $1816 \div 35$
18. $(343 \times 5 + 242) \times 2$
19. $4 \times [(37 \times 24) - 1]$
21. The number halfway between 7741 and 8611
26. $\dfrac{119 \times 207 \times 415}{23 \times 747}$

Connections 3.3 MULTIPLYING WITH BASE-TEN PIECES

1. *School Classroom:* How would you answer the following elementary school student's question: "I know that to multiply a number by 10 you just put a zero at the end, but why? Write your reply and include any necessary diagrams or sketches.

2. *School Classroom:* Design an activity that you believe is appropriate for helping an elementary school student understand what partial products are and how they work while multiplying one two-digit number by another two-digit number. Write a few questions to go with your activity. Explain how your activity will help a typical child learning about multiplication.

3. *Math Concept:* Use base-five number pieces to model $13_{five} \times 4_{five}$, regroup the base pieces to form the minimal collection, and give the solution to $13_{five} \times 4_{five}$ in base five. Illustrate your model and explain your thinking.

4. *Math Concept:* Using your base-five number pieces only (no pencil and paper), build a rectangle similar to the base-ten rectangle in activity 3 to determine the product $12_{five} \times 13_{five}$. Draw a sketch of your rectangle, label the dimensions, and record the product in base five. Explain how you used the base-five number pieces to determine the product.

5. *NCTM Standards:* Read the **Grades 3–5 Number and Operations Standards** in the back pages of this book. Pick two *Expectations* that the activities in this section address. State the *Expectations* and the *Standards* they are under. Explain which activities address these *Expectations* and how they do so.

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