Chapter **Two**

Linear Programming: Basic Concepts

Learning objectives

After completing this chapter, you should be able to

1. Explain what linear programming is.
2. Identify the three key questions to be addressed in formulating any spreadsheet model.
3. Name and identify the purpose of the four kinds of cells used in linear programming spreadsheet models.
4. Formulate a basic linear programming model in a spreadsheet from a description of the problem.
5. Present the algebraic form of a linear programming model from its formulation on a spreadsheet.
6. Apply the graphical method to solve a two-variable linear programming problem.
7. Use Excel to solve a linear programming spreadsheet model.

The management of any organization regularly must make decisions about how to allocate its resources to various activities to best meet organizational objectives. Linear programming is a powerful problem-solving tool that aids management in making such decisions. It is applicable to both profit-making and not-for-profit organizations, as well as governmental agencies. The resources being allocated to activities can be, for example, money, different kinds of personnel, and different kinds of machinery and equipment. In many cases, a wide variety of resources must be allocated simultaneously. The activities needing these resources might be various production activities (e.g., producing different products), marketing activities (e.g., advertising in different media), financial activities (e.g., making capital investments), or some other activities. Some problems might even involve activities of *all* these types (and perhaps others), because they are competing for the same resources.

You will see as we progress that even this description of the scope of linear programming is not sufficiently broad. Some of its applications go beyond the allocation of resources. However, activities always are involved. Thus, a recurring theme in linear programming is the need to find the *best mix* of activities—which ones to pursue and at what levels.

Like the other management science techniques, linear programming uses a *mathematical model* to represent the problem being studied. The word *linear* in the name refers to the form of the mathematical expressions in this model. *Programming* does not refer to computer programming; rather, it is essentially a synonym for planning. Thus, linear programming means the *planning of activities* represented by a *linear* mathematical model.

Because it comprises a major part of management science, linear programming takes up several chapters of this book. Furthermore, many of the lessons learned about how to apply linear programming also will carry over to the application of other management science techniques.

This chapter focuses on the basic concepts of linear programming.

2.1 A CASE STUDY: THE WYNDOR GLASS CO. PRODUCT-MIX PROBLEM

Jim Baker is excited. The group he heads has really hit the jackpot this time. They have had some notable successes in the past, but he feels that this one will be really special. He can hardly wait for the reaction after his memorandum reaches top management.

Jim has had an excellent track record during his seven years as manager of new product development for the Wyndor Glass Company. Although the company is a small one, it has been experiencing considerable growth largely because of the innovative new products developed by Jim's group. Wyndor's president, John Hill, has often acknowledged publicly the key role that Jim has played in the recent success of the company.

Therefore, John felt considerable confidence six months ago in asking Jim's group to develop the following new products:

- An 8-foot glass door with aluminum framing.
- A 4-foot × 6-foot double-hung, wood-framed window.

Although several other companies already had products meeting these specifications, John felt that Jim would be able to work his usual magic in introducing exciting new features that would establish new industry standards.

Now, Jim can't remove the smile from his face. They have done it.

Background

The Wyndor Glass Co. produces high-quality glass products, including windows and glass doors that feature handcrafting and the finest workmanship. Although the products are expensive, they fill a market niche by providing the highest quality available in the industry for the most discriminating buyers. The company has three plants.

Plant 1 produces aluminum frames and hardware.

Plant 2 produces wood frames.

Plant 3 produces the glass and assembles the windows and doors.

Because of declining sales for certain products, top management has decided to revamp the company's product line. Unprofitable products are being discontinued, releasing production capacity to launch the two new products developed by Jim Baker's group if management approves their release.

The 8-foot glass door requires some of the production capacity in Plants 1 and 3, but not Plant 2. The 4-foot × 6-foot double-hung window needs only Plants 2 and 3.

Management now needs to address two issues:

1. Should the company go ahead with launching these two new products?
2. If so, what should be the *product mix*—the number of units of each produced per week—for the two new products?

Management's Discussion of the Issues

Having received Jim Baker's memorandum describing the two new products, John Hill now has called a meeting to discuss the current issues. In addition to John and Jim, the meeting includes Bill Tasto, vice president for manufacturing, and Ann Lester, vice president for marketing.

Let's eavesdrop on the meeting.

John Hill (president): Bill, we will want to rev up to start production of these products as soon as we can. About how much production output do you think we can achieve?

Bill Tasto (vice president for manufacturing): We do have a little available production capacity, because of the products we are discontinuing, but not a lot. We should be able to achieve a production rate of a few units per week for each of these two products.

John: Is that all?

Bill: Yes. These are complicated products requiring careful crafting. And, as I said, we don't have much production capacity available.

An Application Vignette

Swift & Company is a diversified protein-producing business based in Greeley, Colorado. With annual sales of over $8 billion, beef and related products are by far the largest portion of the company's business.

To improve the company's sales and manufacturing performance, upper management concluded that it needed to achieve three major objectives. One was to enable the company's customer service representatives to talk to their more than 8,000 customers with accurate information about the availability of current and future inventory while considering requested delivery dates and maximum product age upon delivery. A second was to produce an efficient shift-level schedule for each plant over a 28-day horizon. A third was to accurately determine whether a plant can ship a requested order-line-item quantity on the requested date and time given the availability of cattle and constraints on the plant's capacity.

To meet these three challenges, a management science team developed an integrated system of 45 linear programming models based on three model formulations to dynamically schedule its beef-fabrication operations at five plants in real time as it receives orders. The total audited benefits realized in the first year of operation of this system were $12.74 million, including $12 million due to optimizing the product mix. Other benefits include a reduction in orders lost, a reduction in price discounting, and better on-time delivery.

Source: A. Bixby, B. Downs, and M. Self, "A Scheduling and Capable-to-Promise Application for Swift & Company, *Interfaces* 36, no. 1 (January–February 2006), pp. 69–86.

John: Ann, will we be able to sell several of each per week?

Ann Lester (vice president for marketing): Easily.

John: OK, good. I would like to set the launch date for these products in six weeks. Bill and Ann, is that feasible?

Bill: Yes.

Ann: We'll have to scramble to give these products a proper marketing launch that soon. But we can do it.

John: Good. Now there's one more issue to resolve. With this limited production capacity, we need to decide how to split it between the two products. Do we want to produce the same number of both products? Or mostly one of them? Or even just produce as much as we can of one and postpone launching the other one for a little while?

Jim Baker (manager of new product development): It would be dangerous to hold one of the products back and give our competition a chance to scoop us.

Ann: I agree. Furthermore, launching them together has some advantages from a marketing standpoint. Since they share a lot of the same special features, we can combine the advertising for the two products. This is going to make a big splash.

The issue is to find the most profitable mix of the two new products.

John: OK. But which mixture of the two products is going to be most profitable for the company?

Bill: I have a suggestion.

John: What's that?

Bill: A couple times in the past, our Management Science Group has helped us with these same kinds of product-mix decisions, and they've done a good job. They ferret out all the relevant data and then dig into some detailed analysis of the issue. I've found their input very helpful. And this is right down their alley.

John: Yes, you're right. That's a good idea. Let's get our Management Science Group working on this issue. Bill, will you coordinate with them?

The meeting ends.

The Management Science Group Begins Its Work

At the outset, the Management Science Group spends considerable time with Bill Tasto to clarify the general problem and specific issues that management wants addressed. A particular concern is to ascertain the appropriate objective for the problem from management's viewpoint. Bill points out that John Hill posed the issue as determining which mixture of the two products is going to be most profitable for the company.

Therefore, with Bill's concurrence, the group defines the key issue to be addressed as follows.

> **Question:** Which combination of *production rates* (the number of units produced per week) for the two new products would *maximize the total profit* from both of them?

The group also concludes that it should consider *all* possible combinations of production rates of both new products permitted by the available production capacities in the three plants. For example, one alternative (despite Jim Baker's and Ann Lester's objections) is to forgo producing one of the products for now (thereby setting its production rate equal to zero) in order to produce as much as possible of the other product. (We must not neglect the possibility that maximum profit from both products might be attained by producing none of one and as much as possible of the other.)

The Management Science Group next identifies the information it needs to gather to conduct this study:

1. Available production capacity in each of the plants.
2. How much of the production capacity in each plant would be needed by each product.
3. Profitability of each product.

Concrete data are not available for any of these quantities, so estimates have to be made. Estimating these quantities requires enlisting the help of key personnel in other units of the company.

Bill Tasto's staff develops the estimates that involve production capacities. Specifically, the staff estimates that the production facilities in Plant 1 needed for the new kind of doors will be available approximately four hours per week. (The rest of the time Plant 1 will continue with current products.) The production facilities in Plant 2 will be available for the new kind of windows about 12 hours per week. The facilities needed for both products in Plant 3 will be available approximately 18 hours per week.

The amount of each plant's production capacity actually used by each product depends on its production rate. It is estimated that each door will require one hour of production time in Plant 1 and three hours in Plant 3. For each window, about two hours will be needed in Plant 2 and two hours in Plant 3.

By analyzing the cost data and the pricing decision, the Accounting Department estimates the profit from the two products. The projection is that the profit per unit will be $300 for the doors and $500 for the windows.

Table 2.1 summarizes the data now gathered.

The Management Science Group recognizes this as being a classic **product-mix problem.** Therefore, the next step is to develop a *mathematical model*—that is, a *linear programming model*—to represent the problem so that it can be solved mathematically. The next four sections focus on how to develop this model and then how to solve it to find the most profitable mix between the two products, assuming the estimates in Table 2.1 are accurate.

Review *Questions*

1. What is the market niche being filled by the Wyndor Glass Co.?
2. What were the two issues addressed by management?
3. The Management Science Group was asked to help analyze which of these issues?
4. How did this group define the key issue to be addressed?
5. What information did the group need to gather to conduct its study?

TABLE 2.1
Data for the Wyndor Glass Co. Product-Mix Problem

	Production Time Used for Each Unit Produced		
Plant	**Doors**	**Windows**	**Available per Week**
1	1 hour	0	4 hours
2	0	2 hours	12 hours
3	3 hours	2 hours	18 hours
Unit profit	$300	$500	

2.2 FORMULATING THE WYNDOR PROBLEM ON A SPREADSHEET

Spreadsheets provide a powerful and intuitive tool for displaying and analyzing many management problems. We now will focus on how to do this for the Wyndor problem with the popular spreadsheet package Microsoft Excel.[1]

Formulating a Spreadsheet Model for the Wyndor Problem

Excel Tip: Cell shading and borders can be added either by using the borders button and the fill color button in the Font Group of the Home tab (Excel 2007) or the formatting toolbar (earlier versions).

Figure 2.1 displays the Wyndor problem by transferring the data in Table 2.1 onto a spreadsheet. (Columns E and F are being reserved for later entries described below.) We will refer to the cells showing the data as **data cells.** To distinguish the data cells from other cells in the spreadsheet, they are shaded light blue. (In the textbook figures, the light blue shading appears as light gray.) The spreadsheet is made easier to interpret by using range names. The data cells in the Wyndor Glass Co. problem are given the range names UnitProfit (C4:D4), HoursUsedPerUnitProduced (C7:D9), and HoursAvailable (G7:G9). To enter a range name, first select the range of cells, then click in the name box on the left of the formula bar above the spreadsheet and type a name.

Excel Tip: See the margin notes in Section 1.2 for tips on adding range names.

Three questions need to be answered to begin the process of using the spreadsheet to formulate a mathematical model (in this case, a **linear programming model**) for the problem.

1. What are the *decisions* to be made?
2. What are the *constraints* on these decisions?
3. What is the overall *measure of performance* for these decisions?

These are the three key questions to be addressed in formulating any spreadsheet model.

The preceding section described how Wyndor's Management Science Group spent considerable time with Bill Tasto, vice president for manufacturing, to clarify management's view of their problem. These discussions provided the following answers to these questions.

1. The decisions to be made are the *production rates* (number of units produced per week) for the two new products.
2. The constraints on these decisions are that the number of hours of production time used per week by the two products in the respective plants cannot exceed the number of hours available.
3. The overall measure of performance for these decisions is the *total profit* per week from the two products.

Some students find it helpful to organize their thoughts by verbally writing out their answers to the three key questions before beginning to formulate the spreadsheet model.

Figure 2.2 shows how these answers can be incorporated into the spreadsheet. Based on the first answer, the *production rates* of the two products are placed in cells C12 and D12 to locate them in the columns for these products just under the data cells. Since we don't know yet what these production rates should be, they are just entered as zeroes in Figure 2.2. (Actually, any

FIGURE 2.1
The initial spreadsheet for the Wyndor problem after transferring the data in Table 2.1 into data cells.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			**Doors**	**Windows**			
4		Unit Profit	$300	$500			
5							Hours
6			Hours Used per Unit Produced				Available
7		Plant 1	1	0			4
8		Plant 2	0	2			12
9		Plant 3	3	2			18

[1] Other spreadsheet packages with similar capabilities also are available, and the basic ideas presented here are still applicable.

FIGURE 2.2
The complete spreadsheet for the Wyndor problem with an initial trial solution (both production rates equal to zero) entered into the changing cells (C12 and D12).

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			**Doors**	**Windows**			
4		Unit Profit	$300	$500			
5					Hours		Hours
6			Hours Used per Unit Produced		Used		Available
7		Plant 1	1	0	0	≤	4
8		Plant 2	0	2	0	≤	12
9		Plant 3	3	2	0	≤	18
10							
11			**Doors**	**Windows**			**Total Profit**
12		Units Produced	0	0			$0

trial solution can be entered, although *negative* production rates should be excluded since they are impossible.) Later, these numbers will be changed while seeking the best mix of production rates. Therefore, these cells containing the decisions to be made are called **changing cells** (or *adjustable cells*). To highlight the changing cells, they are shaded bright yellow with a light border. (In the textbook figures, the bright yellow appears as gray.) The changing cells are given the range name UnitsProduced (C12:D12).

The changing cells contain the decisions to be made.

Using the second answer, the total number of hours of production time used per week by the two products in the respective plants is entered in cells E7, E8, and E9, just to the right of the corresponding data cells. The total number of production hours depends on the production rates of the two products, so this total is zero when the production rates are zero. With positive production rates, the total number of production hours used per week in a plant is the sum of the production hours used per week by the respective products. The production hours used by a product is the number of hours needed for *each* unit of the product *times* the number of units being produced. Therefore, when positive numbers are entered in cells C12 and D12 for the number of doors and windows to produce per week, the data in cells C7:D9 are used to calculate the total production hours per week as follows:

Production hours in Plant 1 = 1(# of doors) + 0(# of windows)

Production hours in Plant 2 = 0(# of doors) + 2(# of windows)

Production hours in Plant 3 = 3(# of doors) + 2(# of windows)

(The colon in C7:D9 is Excel shorthand for the *range from* C7 *to* D9; that is, the entire block of cells in column C or D and in row 7, 8, or 9.) Consequently, the Excel equations for the three cells in column E are

E7 = C7*C12 + D7*D12

E8 = C8*C12 + D8*D12

E9 = C9*C12 + D9*D12

where each asterisk denotes multiplication. Since each of these cells provides output that depends on the changing cells (C12 and D12), they are called **output cells.**

Output cells show quantities that are calculated from the changing cells.

Notice that each of the equations for the output cells involves the sum of two products. There is a function in Excel called SUMPRODUCT that will sum up the product of each of the individual terms in two different ranges of cells when the two ranges have the same number of rows and the same number of columns. Each product being summed is the product of a term in the first range and the term in the corresponding location in the second range. For example, consider the two ranges, C7:D7 and C12:D12, so that each range has one row and two columns. In this case, SUMPRODUCT (C7:D7, C12:D12) takes each of the individual terms in the range C7:D7, multiplies them by the corresponding term in the range C12:D12, and then sums up these individual products, just as shown in the first equation above. Applying the range

The SUMPRODUCT function is used extensively in linear programming spreadsheet models.

name for UnitsProduced (C12:D12), the formula becomes SUMPRODUCT(C7:D7, UnitsProduced). Although optional with such short equations, this function is especially handy as a shortcut for entering longer equations.

The formulas in the output cells E7:E9 are very similar. Rather than typing each of these formulas separately into the three cells, it is quicker (and less prone to typos) to type the formula just once in E7 and then copy the formula down into cells E8 and E9. To do this, first enter the formula =SUMPRODUCT(C7:D7, UnitsProduced) in cell E7. Then select cell E7 and drag the fill handle (the small box on the lower right corner of the cell cursor) down through cells E8 and E9.

When using the fill handle, it is important to understand the difference between relative and absolute references. In the formula in cell E7, the reference to cells C7:D7 is based upon the relative position to the cell containing the formula. In this case, this means the two cells in the same row and immediately to the left. This is known as a **relative reference.** When this formula is copied to new cells using the fill handle, the reference is automatically adjusted to refer to the new cell(s) at the same relative location (the two cells in the same row and immediately to the left). The formula in E8 becomes =SUMPRODUCT(C8:D8, UnitsProduced) and the formula in E9 becomes =SUMPRODUCT(C9:D9, UnitsProduced). This is exactly what we want, since we always want the hours used at a given plant to be based upon the hours used per unit produced at that same plant (the two cells in the same row and immediately to the left).

In contrast, the reference to the UnitsProduced in E7 is called an **absolute reference.** These references do not change when they are filled into other cells but instead always refer to the same absolute cell locations.

You can make the column absolute and the row relative (or vice versa) by putting a $ sign in front of only the letter (or number) of the cell reference.

To make a relative reference, simply enter the cell address (e.g., C7:D7). References referred to by a range name are treated as absolute references. Another way to make an absolute reference to a range of cells is to put $ signs in front of the letter and number of the cell reference (e.g., $C$12:$D$12). See Appendix B for more details about relative and absolute referencing and copying formulas.

Excel tip: After entering a cell reference, repeatedly pressing the F4 key (or command-T on a Mac) will rotate among the four possibilities of relative and absolute references (e.g., C12, $C$12, C$12, $C12).

Next, ≤ signs are entered in cells F7, F8, and F9 to indicate that each total value to their left cannot be allowed to exceed the corresponding number in column G. The spreadsheet still will allow you to enter trial solutions that violate the ≤ signs. However, these ≤ signs serve as a reminder that such trial solutions need to be rejected if no changes are made in the numbers in column G.

Finally, since the answer to the third question is that the overall measure of performance is the total profit from the two products, this profit (per week) is entered in cell G12. Much like the numbers in column E, it is the sum of products. Since cells C4 and D4 give the profit from *each* door and window produced, the total profit per week from these products is

On the computer ≤ (or ≥) is often represented as <= (or >=), since there is no ≤ (or ≥) key on the keyboard. One easy way to enter a ≤ (or ≥) in a spreadsheet is to type < (or >) with underlining turned on.

$$\text{Profit} = \$300(\text{\# of doors}) + \$500(\text{\# of windows})$$

Hence, the equation for cell G12 is

$$\text{G12} = \text{SUMPRODUCT(C4:D4, C12:D12)}$$

Utilizing range names of TotalProfit (G12), UnitProfit (C4:D4), and UnitsProduced (C12:D12), this equation becomes

$$\text{TotalProfit} = \text{SUMPRODUCT(UnitProfit, UnitsProduced)}$$

This is a good example of the benefit of using range names for making the resulting equation easier to interpret.

TotalProfit (G12) is a special kind of output cell. It is the particular cell that is being targeted to be made as large as possible when making decisions regarding production rates. Therefore, TotalProfit (G12) is referred to as the **target cell** (or *objective cell*). The target cell is shaded orange with a heavy border. (In the textbook figures, the orange appears as gray and is distinguished from the changing cells by its heavy border.)

The target cell contains the overall measure of performance for the decisions in the changing cells.

The bottom of Figure 2.3 summarizes all the formulas that need to be entered in the Hours Used column and in the Total Profit cell. Also shown is a summary of the range names (in alphabetical order) and the corresponding cell addresses.

This completes the formulation of the spreadsheet model for the Wyndor problem.

FIGURE 2.3
The spreadsheet model for the Wyndor problem, including the formulas for the target cell TotalProfit (G12) and the other output cells in column E, where the objective is to maximize the target cell.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			**Doors**	**Windows**			
4		Unit Profit	$300	$500			
5					Hours		Hours
6			Hours Used per Unit Produced		Used		Available
7		Plant 1	1	0	0	≤	4
8		Plant 2	0	2	0	≤	12
9		Plant 3	3	2	0	≤	18
10							
11			**Doors**	**Windows**			**Total Profit**
12		Units Produced	0	0			$0

Range Name	Cell
HoursAvailable	G7:G9
HoursUsed	E7:E9
HoursUsedPerUnitProduced	C7:D9
TotalProfit	G12
UnitProfit	C4:D4
UnitsProduced	C12:D12

	E
5	Hours
6	Used
7	=SUMPRODUCT(C7:D7, UnitsProduced)
8	=SUMPRODUCT(C8:D8, UnitsProduced)
9	=SUMPRODUCT(C9:D9, UnitsProduced)

	G
11	Total Profit
12	=SUMPRODUCT(UnitProfit, UnitsProduced)

With this formulation, it becomes easy to analyze any trial solution for the production rates. Each time production rates are entered in cells C12 and D12, Excel immediately calculates the output cells for hours used and total profit. For example, Figure 2.4 shows the spreadsheet when the production rates are set at four doors per week and three windows per week. Cell G12 shows that this yields a total profit of $2,700 per week. Also note that E7 = G7, E8 < G8, and E9 = G9, so the ≤ signs in column F are all satisfied. Thus, this trial solution is *feasible.* However, it would *not* be feasible to further increase both production rates, since this would cause E7 > G7 and E9 > G9.

Does this trial solution provide the best mix of production rates? Not necessarily. It might be possible to further increase the total profit by simultaneously increasing one production

FIGURE 2.4
The spreadsheet for the Wyndor problem with a new trial solution entered into the changing cells, UnitsProduced (C12:D12).

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			**Doors**	**Windows**			
4		Unit Profit	$300	$500			
5					Hours		Hours
6			Hours Used per Unit Produced		Used		Available
7		Plant 1	1	0	4	≤	4
8		Plant 2	0	2	6	≤	12
9		Plant 3	3	2	18	≤	18
10							
11			**Doors**	**Windows**			**Total Profit**
12		Units Produced	4	3			$2,700

rate and decreasing the other. However, it is not necessary to continue using trial and error to explore such possibilities. We shall describe in Section 2.5 how the Excel Solver can be used to quickly find the best (optimal) solution.

This Spreadsheet Model Is a Linear Programming Model

The spreadsheet model displayed in Figure 2.3 is an example of a *linear programming* model. The reason is that it possesses all the following characteristics.

Characteristics of a Linear Programming Model on a Spreadsheet

1. Decisions need to be made on the levels of a number of activities, so *changing cells* are used to display these levels. (The two activities for the Wyndor problem are the production of the two new products, so the changing cells display the number of units produced per week for each of these products.)
2. These activity levels can have any value (including fractional values) that satisfy a number of constraints. (The production rates for Wyndor's new products are restricted only by the constraints on the number of hours of production time available in the three plants.)
3. Each **constraint** describes a restriction on the feasible values for the levels of the activities, where a constraint commonly is displayed by having an output cell on the left, a mathematical sign (≤, ≥, or =) in the middle, and a data cell on the right. (Wyndor's three constraints involving hours available in the plants are displayed in Figures 2.2–2.4 by having output cells in column E, ≤ signs in column F, and data cells in column G.)
4. The decisions on activity levels are to be based on an overall measure of performance, which is entered in the *target cell*. The objective is to either *maximize* the target cell or *minimize* the target cell, depending on the nature of the measure of performance. (Wyndor's overall measure of performance is the total profit per week from the two new products, so this measure has been entered in the target cell G12, where the objective is to maximize this target cell.)
5. The Excel equation for each *output cell* (including the target cell) can be expressed as a SUMPRODUCT function,[2] where each term in the sum is the product of a *data cell* and a *changing cell*. (The bottom of Figure 2.3 shows how a SUMPRODUCT function is used for each output cell for the Wyndor problem.)

Characteristics 2 and 5 are key ones for differentiating a linear programming model from other kinds of mathematical models that can be formulated on a spreadsheet.

Characteristic 2 rules out situations where the activity levels need to have *integer* values. For example, such a situation would arise in the Wyndor problem if the decisions to be made were the *total* numbers of doors and windows to produce (which must be integers) rather than the numbers per week (which can have fractional values since a door or window can be started in one week and completed in the next week). When the activity levels do need to have integer values, a similar kind of model (called an *integer programming* model) is used instead by making a small adjustment on the spreadsheet, as will be illustrated in Section 3.2.

Characteristic 5 prohibits those cases where the Excel equation for an output cell cannot be expressed as a SUMPRODUCT function. To illustrate such a case, suppose that the weekly profit from producing Wyndor's new windows can be *more* than doubled by doubling the production rate because of economies in marketing larger amounts. This would mean that the Excel equation for the target cell would need to be more complicated than a SUMPRODUCT function. Consideration of how to formulate such models will be deferred to Chapter 8.

Summary of the Formulation Procedure

The procedure used to formulate a linear programming model on a spreadsheet for the Wyndor problem can be adapted to many other problems as well. Here is a summary of the steps involved in the procedure.

1. Gather the data for the problem (such as summarized in Table 2.1 for the Wyndor problem).
2. Enter the data into *data cells* on a spreadsheet.

[2] There also are some special situations where a SUM function can be used instead because all the numbers that would have gone into the corresponding data cells are 1's.

3. Identify the decisions to be made on the levels of activities and designate *changing cells* for displaying these decisions.
4. Identify the constraints on these decisions and introduce *output cells* as needed to specify these constraints.
5. Choose the overall measure of performance to be entered into the *target cell.*
6. Use a SUMPRODUCT function to enter the appropriate value into each output cell (including the target cell).

This procedure does not spell out the details of how to set up the spreadsheet. There generally are alternative ways of doing this rather than a single "right" way. One of the great strengths of spreadsheets is their flexibility for dealing with a wide variety of problems.

Review *Questions*

1. What are the three questions that need to be answered to begin the process of formulating a linear programming model on a spreadsheet?
2. What are the roles for the data cells, the changing cells, the output cells, and the target cell when formulating such a model?
3. What is the form of the Excel equation for each output cell (including the target cell) when formulating such a model?

2.3 THE MATHEMATICAL MODEL IN THE SPREADSHEET

A linear programming model can be formulated either as a spreadsheet model or as an algebraic model.

There are two widely used methods for formulating a linear programming model. One is to formulate it directly on a spreadsheet, as described in the preceding section. The other is to use algebra to present the model. The two versions of the model are equivalent. The only difference is whether the language of spreadsheets or the language of algebra is used to describe the model. Both versions have their advantages, and it can be helpful to be bilingual. For example, the two versions lead to different, but complementary, ways of analyzing problems like the Wyndor problem (as discussed in the next two sections). Since this book emphasizes the spreadsheet approach, we will only briefly describe the algebraic approach.

Formulating the Wyndor Model Algebraically

The reasoning for the algebraic approach is similar to that for the spreadsheet approach. In fact, except for making entries on a spreadsheet, the initial steps are just as described in the preceding section for the Wyndor problem.

1. Gather the relevant data (Table 2.1 in Section 2.1).
2. Identify the decisions to be made (the production rates for the two new products).
3. Identify the constraints on these decisions (the production time used in the respective plants cannot exceed the amount available).
4. Identify the overall measure of performance for these decisions (the total profit from the two products).
5. Convert the verbal description of the constraints and measure of performance into quantitative expressions in terms of the data and decisions (see below).

Table 2.1 indicates that the number of hours of production time available per week for the two new products in the respective plants are 4, 12, and 18. Using the data in this table for the number of hours used per door or window produced then leads to the following quantitative expressions for the constraints:

$$\begin{aligned}
&\text{Plant 1:} && (\text{\# of doors}) && \leq 4\\
&\text{Plant 2:} && 2(\text{\# of windows}) && \leq 12\\
&\text{Plant 3:} && 3(\text{\# of doors}) + 2(\text{\# of windows}) && \leq 18
\end{aligned}$$

In addition, negative production rates are impossible, so two other constraints on the decisions are

$$(\text{\# of doors}) \geq 0 \qquad (\text{\# of windows}) \geq 0$$

The overall measure of performance has been identified as the total profit from the two products. Since Table 2.1 gives the unit profits for doors and windows as $300 and $500, respectively, the expression obtained in the preceding section for the total profit per week from these products is

$$\text{Profit} = \$300(\text{\# of doors}) + \$500(\text{\# of windows})$$

The objective is to make the decisions (number of doors and number of windows) so as to maximize this profit, subject to satisfying all the constraints identified above.

To state this objective in a compact algebraic model, we introduce algebraic symbols to represent the measure of performance and the decisions. Let

P = Profit (total profit per week from the two products, in dollars)

D = # of doors (number of the special new doors to be produced per week)

W = # of windows (number of the special new windows to be produced per week)

Substituting these symbols into the above expressions for the constraints and the measure of performance (and dropping the dollar signs in the latter expression), the linear programming model for the Wyndor problem now can be written in algebraic form as shown below.

Algebraic Model

Choose the values of D and W so as to maximize

$$P = 300D + 500W$$

subject to satisfying all the following constraints:

$$\begin{aligned} D \quad\quad\quad &\leq 4 \\ 2W &\leq 12 \\ 3D + 2W &\leq 18 \end{aligned}$$

and

$$D \geq 0 \qquad W \geq 0$$

Terminology for Linear Programming Models

Much of the terminology of algebraic models also is sometimes used with spreadsheet models. Here are the key terms for both kinds of models in the context of the Wyndor problem.

1. D and W (or C12 and D12 in Figure 2.3) are the **decision variables.**
2. $300D + 500W$ [or SUMPRODUCT (UnitProfit, UnitsProduced)] is the **objective function.**
3. P (or G12) is the *value of the objective function* (or *objective value* for short).
4. $D \geq 0$ and $W \geq 0$ (or C12 ≥ 0 and D12 ≥ 0) are called the **nonnegativity constraints** (or *nonnegativity conditions*).
5. The other constraints are referred to as **functional constraints** (or *structural constraints*).
6. The **parameters** of the model are the constants in the algebraic model (the numbers in the data cells).
7. *Any* choice of values for the decision variables (regardless of how desirable or undesirable the choice) is called a **solution** for the model.
8. A **feasible solution** is one that satisfies all the constraints, whereas an **infeasible solution** violates at least one constraint.
9. The *best* feasible solution, the one that maximizes P (or G12), is called the **optimal solution.**

Comparisons

Management scientists often use algebraic models, but managers generally prefer spreadsheet models.

So what are the relative advantages of algebraic models and spreadsheet models? An algebraic model provides a very concise and explicit statement of the problem. Sophisticated software packages that can solve huge problems generally are based on algebraic models because of both their compactness and their ease of use in rescaling the size of a problem. Management science practitioners with an extensive mathematical background find algebraic models very useful. For others, however, spreadsheet models are far more intuitive. Many very intelligent people (including many managers and business students) find algebraic models overly abstract. Spreadsheets lift this "algebraic curtain." Both managers and business students training to be managers generally live with spreadsheets, not algebraic models. Therefore, the emphasis throughout this book is on spreadsheet models.

Review *Questions*

1. When formulating a linear programming model, what are the initial steps that are the same with either a spreadsheet formulation or an algebraic formulation?
2. When formulating a linear programming model algebraically, algebraic symbols need to be introduced to represent which kinds of quantities in the model?
3. What are decision variables for a linear programming model? The objective function? Nonnegativity constraints? Functional constraints?
4. What is meant by a feasible solution for the model? An optimal solution?

2.4 THE GRAPHICAL METHOD FOR SOLVING TWO-VARIABLE PROBLEMS

graphical method
The graphical method provides helpful intuition about linear programming.

Linear programming problems having only two decision variables, like the Wyndor problem, can be solved by a **graphical method.**

Although this method cannot be used to solve problems with more than two decision variables (and most linear programming problems have far more than two), it still is well worth learning. The procedure provides geometric intuition about linear programming and what it is trying to achieve. This intuition is helpful in analyzing larger problems that cannot be solved directly by the graphical method.

It is more convenient to apply the graphical method to the *algebraic version* of the linear programming model rather than the spreadsheet version. We shall briefly illustrate the method by using the algebraic model obtained for the Wyndor problem in the preceding section. (A far more detailed description of the graphical method, including its application to the Wyndor problem, is provided in the supplement to this chapter on the CD-ROM.) For this purpose, keep in mind that

D = Production rate for the special new doors (the number in changing cell C12 of the spreadsheet)

W = Production rate for the special new windows (the number in changing cell D12 of the spreadsheet)

The key to the graphical method is the fact that possible solutions can be displayed as points on a two-dimensional graph that has a horizontal axis giving the value of D and a vertical axis giving the value of W. Figure 2.5 shows some sample points.

Notation: Either $(D, W) = (2, 3)$ or just (2, 3) refers to the solution where $D = 2$ and $W = 3$, as well as to the corresponding point in the graph. Similarly, $(D, W) = (4, 6)$ means $D = 4$ and $W = 6$, whereas the origin (0, 0) means $D = 0$ and $W = 0$.

To find the optimal solution (the best feasible solution), we first need to display graphically where the feasible solutions are. To do this, we must consider each constraint, identify the solutions graphically that are permitted by that constraint, and then combine this information to identify the solutions permitted by all the constraints. The solutions permitted by all the constraints are the feasible solutions and the portion of the two-dimensional graph where the feasible solutions lie is referred to as the **feasible region.**

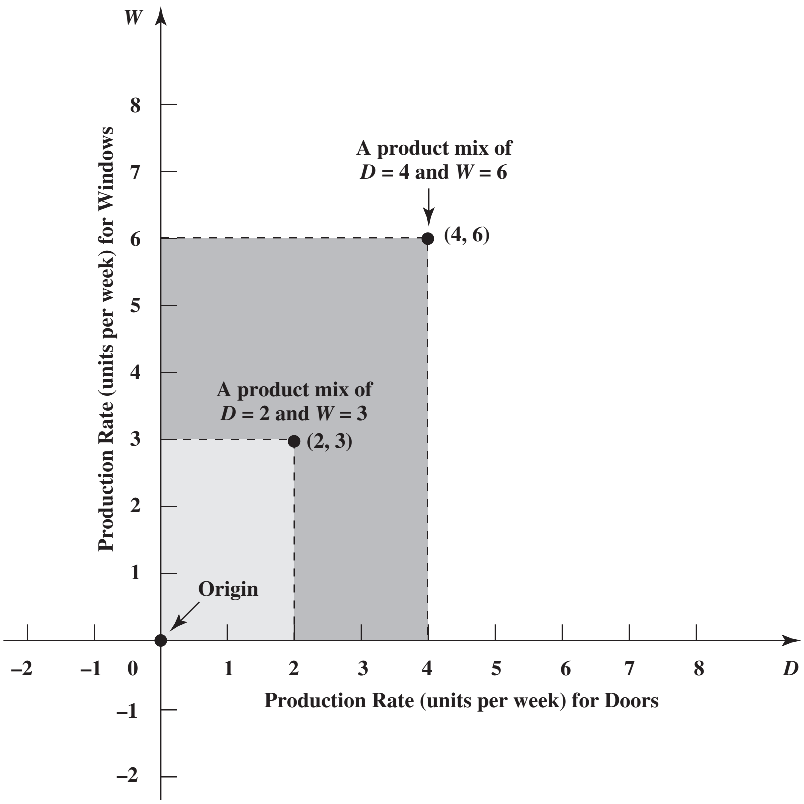


FIGURE 2.5
Graph showing the points $(D, W) = (2, 3)$ and $(D, W) = (4, 6)$ for the Wyndor Glass Co. product-mix problem.

The shaded region in Figure 2.6 shows the feasible region for the Wyndor problem. We now will outline how this feasible region was identified by considering the five constraints one at a time.

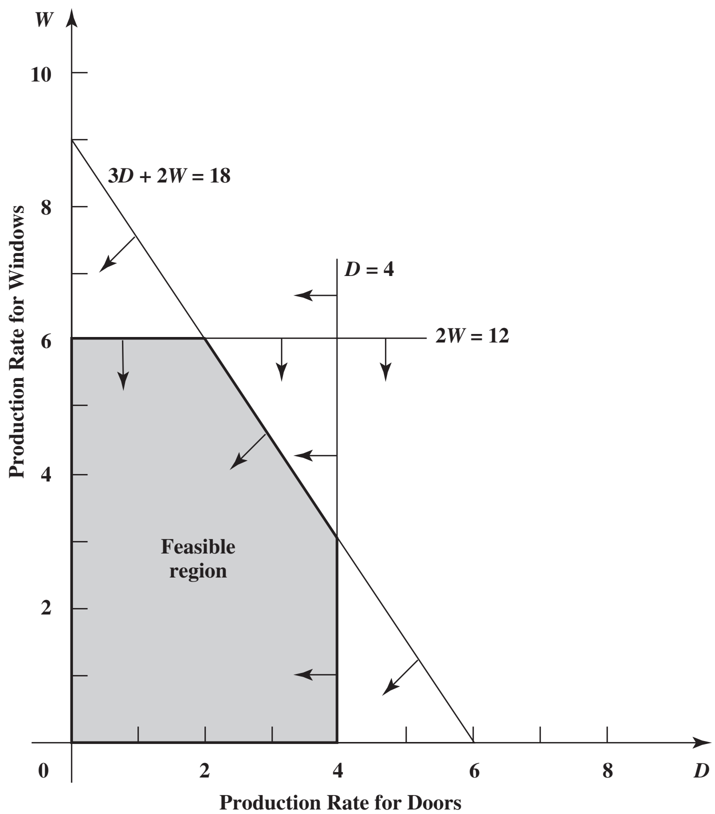


FIGURE 2.6
Graph showing how the feasible region is formed by the constraint boundary lines, where the arrows indicate which side of each line is permitted by the corresponding constraint.

To begin, the constraint $D \geq 0$ implies that consideration must be limited to points that lie on or to the right of the W axis. Similarly, the constraint $W \geq 0$ restricts consideration to the points on or above the D axis.

Next, consider the first functional constraint, $D \leq 4$, which limits the usage of Plant 1 for producing the special new doors to a maximum of four hours per week. The solutions permitted by this constraint are those that lie on, or to the left of, the vertical line that intercepts the D axis at $D = 4$, as indicated by the arrows pointing to the left from this line in Figure 2.6.

For any constraint with an inequality sign, its constraint boundary equation is obtained by replacing the inequality sign by an equality sign.

The second functional constraint, $2W \leq 12$, has a similar effect, except now the boundary of its permissible region is given by a *horizontal* line with the equation, $2W = 12$ (or $W = 6$), as indicated by the arrows pointing downward from this line in Figure 2.6. The line forming the boundary of what is permitted by a constraint is sometimes referred to as a **constraint boundary line,** and its equation may be called a **constraint boundary equation.** Frequently, a *constraint boundary line* is identified by its equation.

For each of the first two functional constraints, $D \leq 4$ and $2W \leq 12$, note that the equation for the constraint boundary line ($D = 4$ and $2W = 12$, respectively) is obtained by replacing the inequality sign with an equality sign. For *any* constraint with an inequality sign (whether a functional constraint or a nonnegativity constraint), the general rule for obtaining its constraint boundary equation is to substitute an equality sign for the inequality sign.

We now need to consider one more functional constraint, $3D + 2W \leq 18$. Its constraint boundary equation

$$3D + 2W = 18$$

includes both variables, so the boundary line it represents is neither a vertical line nor a horizontal line. Therefore, the boundary line must intercept (cross through) both axes somewhere. But where?

> When a constraint boundary line is neither a vertical line nor a horizontal line, the line *intercepts* the D axis at the point on the line where $W = 0$. Similarly, the line *intercepts* the W axis at the point on the line where $D = 0$.

Hence, the constraint boundary line $3D + 2W = 18$ intercepts the D axis at the point where $W = 0$.

When $W = 0$, $3D + 2W = 18$ becomes $3D = 18$
so the intercept with the D axis is at $D = 6$

The location of a slanting constraint boundary line is found by identifying where it intercepts each of the two axes.

Similarly, the line intercepts the W axis where $D = 0$.

When $D = 0$, $3D + 2W = 18$ becomes $2W = 18$
so the intercept with the D axis is at $W = 9$

Consequently, the constraint boundary line is the line that passes through these two intercept points, as shown in Figure 2.6.

Checking whether (0, 0) satisfies a constraint indicates which side of the constraint boundary line satisfies the constraint.

As indicated by the arrows emanating from this line in Figure 2.6, the solutions permitted by the constraint $3D + 2W \leq 18$ are those that lie on the *origin* side of the constraint boundary line $3D + 2W = 18$. The easiest way to verify this is to check whether the origin itself, $(D, W) = (0, 0)$, satisfies the constraint.[3] If it does, then the permissible region lies on the side of the constraint boundary line where the origin is. Otherwise, it lies on the other side. In this case,

$$3(0) + 2(0) = 0$$

so $(D, W) = (0, 0)$ satisfies

$$3D + 2W \leq 18$$

(In fact, the origin satisfies *any* constraint with a ≤ sign and a positive right-hand side.)

A feasible solution for a linear programming problem must satisfy *all* the constraints *simultaneously*. The arrows in Figure 2.6 indicate that the nonnegative solutions permitted by

[3] The one case where using the origin to help determine the permissible region does *not* work is if the constraint boundary line passes through the origin. In this case, any other point *not* lying on this line can be used just like the origin.

feasible region
The points in the feasible region are those that satisfy *every* constraint.

each of these constraints lie on the side of the constraint boundary line where the origin is (or on the line itself). Therefore, the *feasible solutions* are those that lie nearer to the origin than *all three* constraint boundary lines (or on the line nearest the origin).

Having identified the feasible region, the final step is to find which of these feasible solutions is the best one—the *optimal solution*. For the Wyndor problem, the objective happens to be to *maximize* the total profit per week from the two products (denoted by P). Therefore, we want to find the feasible solution (D, W) that makes the value of the objective function

$$P = 300D + 500W$$

as large as possible.

To accomplish this, we need to be able to locate all the points (D, W) on the graph that give a specified value of the objective function. For example, consider a value of $P = 1{,}500$ for the objective function. Which points (D, W) give $300D + 500W = 1{,}500$?

This equation is the equation of a *line.* Just as when plotting constraint boundary lines, the location of this line is found by identifying its intercepts with the two axes. When $W = 0$, this equation yields $D = 5$, and similarly, $W = 3$ when $D = 0$, so these are the two intercepts, as shown by the bottom slanting line passing through the feasible region in Figure 2.7.

$P = 1{,}500$ is just one sample value of the objective function. For any other specified value of P, the points (D, W) that give this value of P also lie on a line called an *objective function line.*

An **objective function line** is a line whose points all have the same value of the objective function.

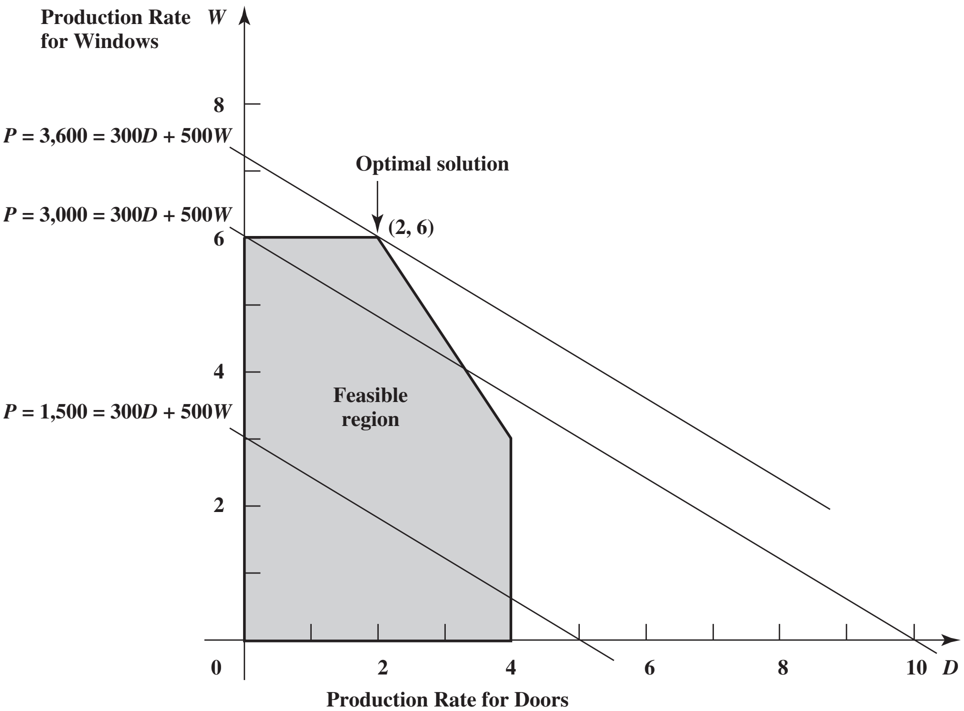
For the bottom objective function line in Figure 2.7, the points on this line that lie in the feasible region provide alternate ways of achieving an objective function value of $P = 1{,}500$. Can we do better? Let us try doubling the value of P to $P = 3{,}000$. The corresponding objective function line

$$300D + 500W = 3{,}000$$

is shown as the middle line in Figure 2.7. (Ignore the top line for the moment.) Once again, this line includes points in the feasible region, so $P = 3{,}000$ is achievable.

Let us pause to note two interesting features of these objective function lines for $P = 1{,}500$ and $P = 3{,}000$. First, these lines are *parallel.* Second, *doubling* the value of P from 1,500 to 3,000 also *doubles* the value of W at which the line intercepts the W axis from $W = 3$ to $W = 6$. These features are no coincidence, as indicated by the following properties.

FIGURE 2.7
Graph showing three objective function lines for the Wyndor Glass Co. product-mix problem, where the top one passes through the optimal solution.



Key Properties of Objective Function Lines: All objective function lines for the same problem are *parallel.* Furthermore, the value of W at which an objective function line intercepts the W axis is *proportional* to the value of P.

These key properties of objective function lines suggest the strategy to follow to find the optimal solution. We already have tried $P = 1{,}500$ and $P = 3{,}000$ in Figure 2.7 and found that their objective function lines include points in the feasible region. Increasing P again will generate another parallel objective function line farther from the origin. The objective function line of special interest is the one farthest from the origin that still includes a point in the feasible region. This is the third objective function line in Figure 2.7. The point on this line that is in the feasible region, $(D, W) = (2, 6)$, is the optimal solution since no other feasible solution has a larger value of P.

Optimal Solution

$D = 2$ (Produce 2 special new doors per week)

$W = 6$ (Produce 6 special new windows per week)

These values of D and W can be substituted into the objective function to find the value of P.

$$P = 300D + 500W = 300(2) + 500(6) = 3{,}600$$

Check out this module in the Interactive Management Science Modules to learn more about the graphical method.

The Interactive Management Science Modules (available at **www.mhhe.com/hillier3e** or in your CD-ROM) includes a module that is designed to help increase your understanding of the graphical method. This module, called *Graphical Linear Programming and Sensitivity Analysis,* enables you to immediately see the constraint boundary lines and objective function lines that result from any linear programming model with two decision variables. You also can see how the objective function lines lead you to the optimal solution. Another key feature of the module is the ease with which you can perform what-if analysis.

Summary of the Graphical Method

The graphical method can be used to solve any linear programming problem having only two decision variables. The method uses the following steps:

1. Draw the constraint boundary line for each functional constraint. Use the origin (or any point not on the line) to determine which side of the line is permitted by the constraint.
2. Find the feasible region by determining where all constraints are satisfied simultaneously.
3. Determine the slope of one objective function line. All other objective function lines will have the same slope.
4. Move a straight edge with this slope through the feasible region in the direction of improving values of the objective function. Stop at the last instant that the straight edge still passes through a point in the feasible region. This line given by the straight edge is the optimal objective function line.
5. A feasible point on the optimal objective function line is an optimal solution.

Review *Questions*

1. The graphical method can be used to solve linear programming problems with how many decision variables?
2. What do the axes represent when applying the graphical method to the Wyndor problem?
3. What is a constraint boundary line? A constraint boundary equation?
4. What is the easiest way of determining which side of a constraint boundary line is permitted by the constraint?

2.5 USING EXCEL TO SOLVE LINEAR PROGRAMMING PROBLEMS

The graphical method is very useful for gaining geometric intuition about linear programming, but its practical use is severely limited by only being able to solve tiny problems with two decision variables. Another procedure that will solve linear programming problems of any

Excel Tip: If you select cells by clicking on them, they will first appear in the dialogue box with their cell addresses and with dollar signs (e.g., \$C\$9:\$D\$9). You can ignore the dollar signs. Solver eventually will replace both the cell addresses and the dollar signs with the corresponding range name (if a range name has been defined for the given cell addresses), but only after either adding a constraint or closing and reopening the Solver dialogue box.

reasonable size is needed. Fortunately, Excel includes a tool called **Solver** that will do this once the spreadsheet model has been formulated as described in Section 2.2. (A more powerful version of Solver, called *Premium Solver for Education* also is available in your MS Courseware.) To access Solver the first time, you need to install it by going to Excel's Add-in menu and adding Solver, after which you will find it in the Tools menu.

Figure 2.3 in Section 2.2 shows the spreadsheet model for the Wyndor problem. The values of the decision variables (the production rates for the two products) are in the *changing cells,* UnitsProduced (C12:D12), and the value of the objective function (the total profit per week from the two products) is in the *target cell,* TotalProfit (G12). To get started, an arbitrary trial solution has been entered by placing zeroes in the changing cells. The Solver will then change these to the optimal values after solving the problem.

This procedure is started by choosing Solver on the Data tab (for Excel 2007) or in the Tools menu (for earlier versions of Excel). The Solver dialogue box is shown in Figure 2.8.

Before the Solver can start its work, it needs to know exactly where each component of the model is located on the spreadsheet. You have the choice of typing the range names, typing in the cell addresses, or clicking on the cells in the spreadsheet. Figure 2.8 shows the result of using the first choice, so TotalProfit (rather than G12) has been entered for the target cell and UnitsProduced (rather than the range C12:D12) has been entered for the changing cells. Since the goal is to maximize the target cell, Max also has been selected.

Next, the cells containing the functional constraints need to be specified. This is done by clicking on the Add button on the Solver dialogue box. This brings up the Add Constraint dialogue box shown in Figure 2.9. The $\leq$ signs in cells F7, F8, and F9 of Figure 2.3 are a reminder that the cells in HoursUsed (E7:E9) all need to be less than or equal to the corresponding cells in HoursAvailable (G7:G9). These constraints are specified for the Solver by entering HoursUsed (or E7:E9) on the left-hand side of the Add Constraint dialogue box and HoursAvailable (or G7:G9) on the right-hand side. For the sign between these two sides, there is a menu to choose between <= , =, or >= , so <= has been chosen. This choice is needed

FIGURE 2.8
The Solver dialogue box after specifying which cells in Figure 2.3 are the target cell and the changing cells, plus indicating that the target cell is to be maximized.

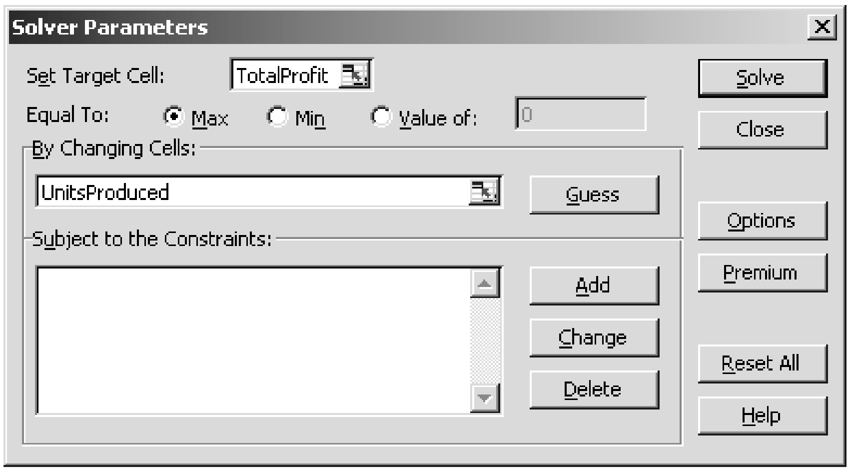
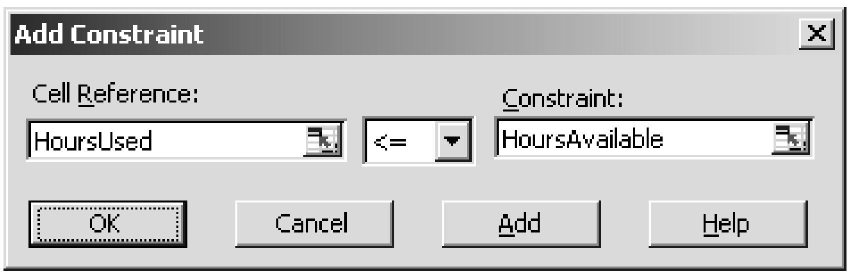


FIGURE 2.9
The Add Constraint dialogue box after specifying that cells E7, E8, and E9 in Figure 2.3 are required to be less than or equal to cells G7, G8, and G9, respectively.



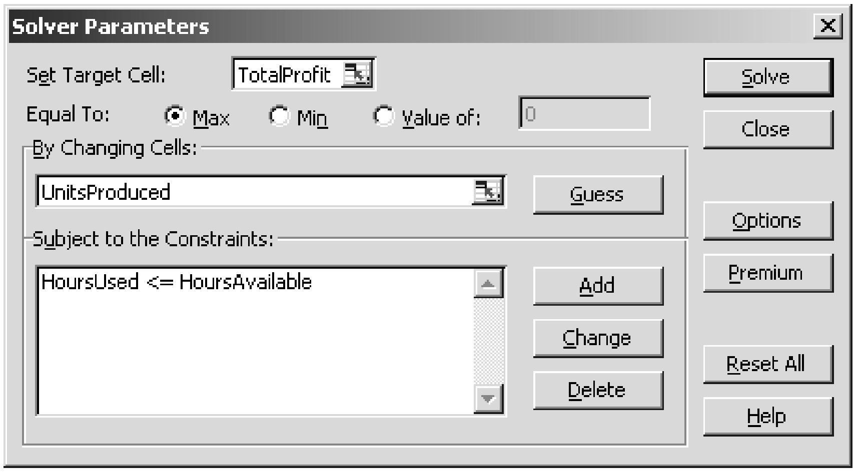


FIGURE 2.10
The Solver dialogue box after specifying the entire model in terms of the spreadsheet.

The Add Constraint dialogue box is used to specify all the functional constraints.

even though ≤ signs were previously entered in column F of the spreadsheet because the Solver only uses the constraints that are specified with the Add Constraint dialogue box.

If there were more functional constraints to add, you would click on Add to bring up a new Add Constraint dialogue box. However, since there are no more in this example, the next step is to click on OK to go back to the Solver dialogue box.

The Solver dialogue box now summarizes the complete model (see Figure 2.10) in terms of the spreadsheet in Figure 2.3. However, before asking Solver to solve the model, one more step should be taken. Clicking on the Options button brings up the dialogue box shown in Figure 2.11. This box allows you to specify a number of options about how the problem will be solved. The most important of these are the Assume Linear Model option and the Assume Non-Negative option. Be sure that both options are checked as shown in the figure. This tells Solver that the problem is a *linear* programming problem and that nonnegativity constraints are needed for the changing cells to reject negative production rates. Regarding the other options, accepting the default values shown in the figure usually is fine for small problems. Clicking on the OK button then returns you to the Solver dialogue box.

The Assume Linear Model and Assume Non-Negative options specify that the problem is a linear programming problem with nonnegativity constraints.

Now you are ready to click on Solve in the Solver dialogue box, which will start the solving of the problem in the background. After a few seconds (for a small problem), Solver will then indicate the results. Typically, it will indicate that it has found an optimal solution, as specified in the

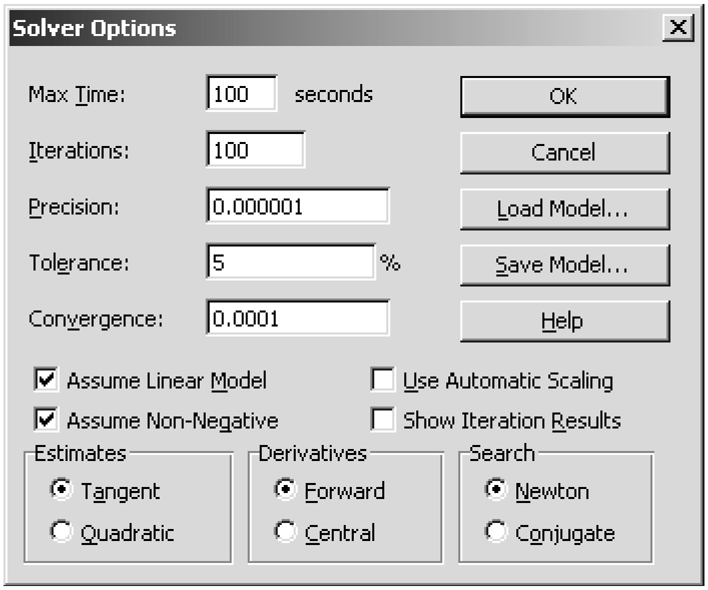
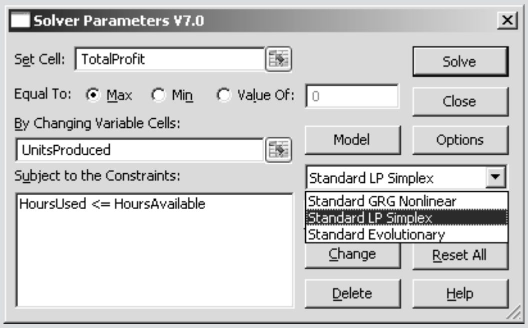


FIGURE 2.11
The Solver Options dialogue box after checking the Assume Linear Model and Assume Non-Negative options to indicate that we wish to solve a linear programming model that has nonnegativity constraints.

Software on the CD-ROM: Premium Solver for Education

Frontline Systems, the original developer of the standard Solver included with Excel, also has developed Premium versions of Solver that provide additional functionality. One such version (Premium Solver for Education) is available in your MS Courseware. Once it is installed, it is invoked by choosing Premium Solver from the Add-Ins tab (for Excel 2007) or the Tools menu (for earlier versions of Excel). This brings up the dialogue box shown below for a typical example.



Premium Solver for Education is more robust than the standard Solver in the sense that it sometimes will accurately solve difficult problems where the standard Solver fails. In addition to this advantage, the other key advantage of Premium Solver for Education is that it includes three different search techniques chosen in a dropdown menu. The choices are Standard GRG Nonlinear, Standard LP Simplex, and Standard Evolutionary. The first choice (Standard GRG Nonlinear) is basically identical to using the standard Solver *without* the "Assume Linear Model" option selected. The second choice (Standard Simplex LP) is basically equivalent to using the standard Solver *with* the "Assume Linear Model" option selected. The final choice (Standard Evolutionary) employs the Evolutionary Solver that will be discussed in Chapter 8. This choice is not available with the standard Solver.

Even with the Premium Solver for Education installed, the standard Excel Solver can still be used in the usual way by choosing Solver on the Data tab (for Excel 2007) or the Tools menu (for earlier versions of Excel). We encourage you to install and try the Premium Solver for Education as well.

Solver Tip: The message "Solver could not find a feasible solution" means that there are no solutions that satisfy all the constraints. The message "The Set Cell values do not converge" means that Solver could not find a best solution, because better solutions always are available (e.g., if the constraints do not prevent infinite profit). The message "The conditions for Assume Linear Model are not satisfied" means that the Assume Linear Model checkbox was checked, but the model is not linear.

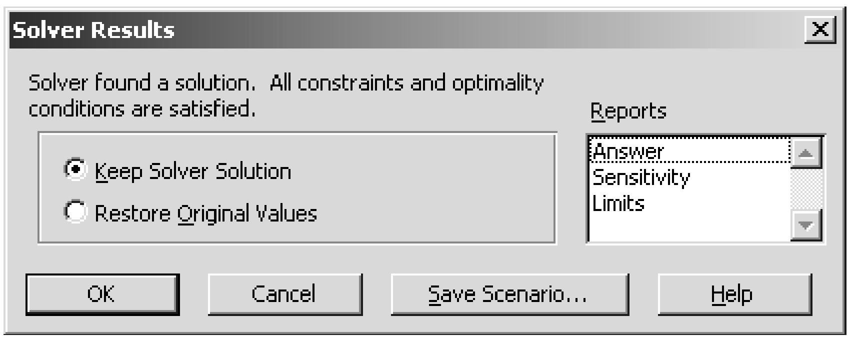
Solver Results dialogue box shown in Figure 2.12. If the model has no feasible solutions or no optimal solution, the dialogue box will indicate that instead by stating that "Solver could not find a feasible solution" or that "The Set Cell values do not converge." (Section 14.1 will describe how these possibilities can occur.) The dialogue box also presents the option of generating various reports. One of these (the Sensitivity Report) will be discussed in detail in Chapter 5.

After solving the model, the Solver replaces the original numbers in the changing cells with the optimal numbers, as shown in Figure 2.13. Thus, the optimal solution is to produce two doors per week and six windows per week, just as was found by the graphical method in the preceding section. The spreadsheet also indicates the corresponding number in the target cell (a total profit of $3,600 per week), as well as the numbers in the output cells HoursUsed (E7:E9).

At this point, you might want to check what would happen to the optimal solution if any of the numbers in the data cells were to be changed to other possible values. This is easy to do because Solver saves all the addresses for the target cell, changing cells, constraints, and so on when you save the file. All you need to do is make the changes you want in the data cells and then click on Solve in the Solver dialogue box again. (Chapter 5 will focus on this kind of *what-if analysis,* including how to use the Solver's Sensitivity Report to expedite the analysis.)

To assist you with experimenting with these kinds of changes, your MS Courseware includes Excel files for this chapter (as for others) that provide a complete formulation and

FIGURE 2.12
The Solver Results dialogue box that indicates that an optimal solution has been found.



solution of the examples here (the Wyndor problem and the one in the next section) in a spreadsheet format. We encourage you to “play” with these examples to see what happens with different data, different solutions, and so forth. You might also find these spreadsheets useful as templates for homework problems.

Review ***Questions***

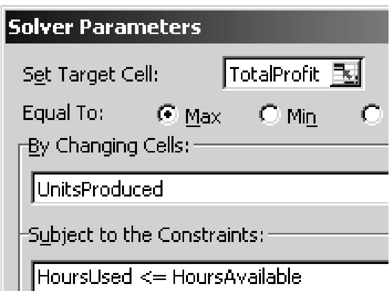
1. Which dialogue box is used to enter the addresses for the target cell and the changing cells?
2. Which dialogue box is used to specify the functional constraints for the model?
3. With the Solver Options dialogue box, which options normally need to be chosen to solve a linear programming model?

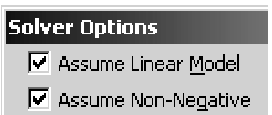
2.6 A MINIMIZATION EXAMPLE—THE PROFIT & GAMBIT CO. ADVERTISING-MIX PROBLEM

The analysis of the Wyndor Glass Co. case study in Sections 2.2 and 2.5 illustrated how to formulate and solve one type of linear programming model on a spreadsheet. The same general approach can be applied to many other problems as well. The great flexibility of linear programming and spreadsheets provides a variety of options for how to adapt the

FIGURE 2.13
The spreadsheet obtained after solving the Wyndor problem.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			**Doors**	**Windows**			
4		Unit Profit	$300	$500			
5					Hours		Hours
6			Hours Used per Unit Produced		Used		Available
7		Plant 1	1	0	2	≤	4
8		Plant 2	0	2	12	≤	12
9		Plant 3	3	2	18	≤	18
10							
11			**Doors**	**Windows**			**Total Profit**
12		Units Produced	2	6			$3,600





	E
5	Hours
6	Used
7	=SUMPRODUCT(C7:D7, UnitsProduced)
8	=SUMPRODUCT(C8:D8, UnitsProduced)
9	=SUMPRODUCT(C9:D9, UnitsProduced)

	G
11	Total Profit
12	=SUMPRODUCT(UnitProfit, UnitsProduced)

Range Name	Cell
HoursAvailable	G7:G9
HoursUsed	E7:E9
HoursUsedPerUnitProduced	C7:D9
TotalProfit	G12
UnitProfit	C4:D4
UnitsProduced	C12:D12

formulation of the spreadsheet model to fit each new problem. Our next example illustrates some options not used for the Wyndor problem.

Planning an Advertising Campaign

The Profit & Gambit Co. produces cleaning products for home use. This is a highly competitive market, and the company continually struggles to increase its small market share. Management has decided to undertake a major new advertising campaign that will focus on the following three key products:

- A spray prewash stain remover.
- A liquid laundry detergent.
- A powder laundry detergent.

This campaign will use both television and the print media. A commercial has been developed to run on national television that will feature the liquid detergent. The advertisement for the print media will promote all three products and will include cents-off coupons that consumers can use to purchase the products at reduced prices. The general goal is to increase the sales of each of these products (but especially the liquid detergent) over the next year by a significant percentage over the past year. Specifically, management has set the following goals for the campaign:

- Sales of the stain remover should increase by at least 3 percent.
- Sales of the liquid detergent should increase by at least 18 percent.
- Sales of the powder detergent should increase by at least 4 percent.

Table 2.2 shows the estimated increase in sales for each *unit* of advertising in the respective outlets.[4] (A *unit* is a standard block of advertising that Profit & Gambit commonly purchases, but other amounts also are allowed.) The reason for −1 percent for the powder detergent in the Television column is that the TV commercial featuring the new liquid detergent will take away some sales from the powder detergent. The bottom row of the table shows the cost per unit of advertising for each of the two outlets.

Management's objective is to determine how much to advertise in each medium to meet the sales goals at a minimum total cost.

Formulating a Spreadsheet Model for This Problem

The procedure summarized at the end of Section 2.2 can be used to formulate the spreadsheet model for this problem. Each step of the procedure is repeated below, followed by a description of how it is performed here.

1. Gather the data for the problem. This has been done as presented in Table 2.2.
2. Enter the data into *data cells* on a spreadsheet. The top half of Figure 2.14 shows this spreadsheet. The data cells are in columns C and D (rows 4 and 8 to 10), as well as in cells G8:G10. Note how this particular formatting of the spreadsheet has facilitated a direct transfer of the data from Table 2.2.

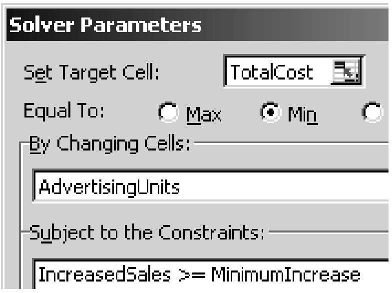
TABLE 2.2
Data for the Profit & Gambit Co. Advertising-Mix Problem

	Increase in Sales per Unit of Advertising		
Product	**Television**	**Print Media**	**Minimum Required Increase**
Stain remover	0%	1%	3%
Liquid detergent	3	2	18
Powder detergent	−1	4	4
Unit cost	$1 million	$2 million	

[4] A simplifying assumption is being made that each additional unit of advertising in a particular outlet will yield the same increase in sales regardless of how much advertising already is being done. This becomes a poor assumption when the levels of advertising under consideration can reach a saturation level (as in Case 8.1), but is a reasonable approximation for the small levels of advertising being considered in this problem.

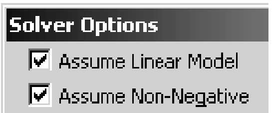
FIGURE 2.14
The spreadsheet model for the Profit & Gambit problem, including the formulas for the target cell TotalCost (G14) and the other output cells in column E, as well as the specifications needed to set up the Solver. The changing cells, AdvertisingUnits (C14:D14), show the optimal solution obtained by the Solver.

	A	B	C	D	E	F	G
1	**Profit & Gambit Co. Advertising-Mix Problem**						
2							
3			**Television**	**Print Media**			
4		Unit Cost ($millions)	1	2			
5							
6					Increased		Minimum
7			Increase in Sales per Unit of Advertising		Sales		Increase
8		Stain Remover	0%	1%	3%	≥	3%
9		Liquid Detergent	3%	2%	18%	≥	18%
10		Powder Detergent	-1%	4%	8%	≥	4%
11							
12							**Total Cost**
13			**Television**	**Print Media**			**($millions)**
14		Advertising Units	4	3			10



	E
6	Increased
7	Sales
8	=SUMPRODUCT(C8:D8, AdvertisingUnits)
9	=SUMPRODUCT(C9:D9, AdvertisingUnits)
10	=SUMPRODUCT(C10:D10, AdvertisingUnits)

	G
12	Total Cost
13	($millions)
14	=SUMPRODUCT(UnitCost, AdvertisingUnits)



Range Name	Cells
AdvertisingUnits	C14: D14
IncreasedSales	E8: E10
IncreasedSalesPerUnitAdvertising	C8: D10
MinimumIncrease	G8: G10
TotalCost	G14
UnitCost	C4: D4

3. Identify the decisions to be made on the levels of activities and designate *changing cells* for making these decisions. In this case, the activities of concern are *advertising on television* and *advertising in the print media,* so the *levels* of these activities refer to the *amount* of advertising in these media. Therefore, the decisions to be made are

Decision 1: TV = Number of units of advertising on television

Decision 2: PM = Number of units of advertising in the print media

The two gray cells with light borders in Figure 2.14—C14 and D14—have been designated as the changing cells to hold these numbers:

TV → cell C14   PM → cell D14

with AdvertisingUnits as the range name for these cells. (See the bottom of Figure 2.14 for a list of all the range names.) These are natural locations for the changing cells, since each one is in the column for the corresponding advertising medium. To get started, an arbitrary trial solution (such as all zeroes) is entered into these cells. (Figure 2.14 shows the optimal solution after having already applied the Solver.)

4. Identify the constraints on these decisions and introduce *output cells* as needed to specify these constraints. The three constraints imposed by management are the goals for the increased sales for the respective products, as shown in the rightmost column of Table 2.2. These constraints are

Unlike the Wyndor problem, we need to use ≥ signs for these constraints.

Stain remover:	Total increase in sales ≥ 3%
Liquid detergent:	Total increase in sales ≥ 18%
Powder detergent:	Total increase in sales ≥ 4%

The second and third columns of Table 2.2 indicate that the *total* increases in sales from both forms of advertising are

Total for stain remover	= 1% of PM
Total for liquid detergent	= 3% of TV + 2% of PM
Total for powder detergent	= −1% of TV + 4% of PM

Consequently, since rows 8, 9, and 10 in the spreadsheet are being used to provide information about the three products, cells E8, E9, and E10 are introduced as output cells to show the total increase in sales for the respective products. In addition, ≥ signs have been entered in column F to remind us that the increased sales need to be at least as large as the numbers in column G. (The use of ≥ signs here rather than ≤ signs is one key difference from the spreadsheet model for the Wyndor problem in Figure 2.3.)

Unlike the Wyndor problem, the objective now is to minimize the target cell.

5. Choose the overall measure of performance to be entered into the *target cell.* Management's stated objective is to determine how much to advertise in each medium to meet the sales goals at a *minimum total cost.* Therefore, the *total cost* of the advertising is entered in the target cell TotalCost(G14). G14 is a natural location for this cell since it is in the same row as the changing cells. The bottom row of Table 2.2 indicates that the number going into this cell is

$$\text{Cost} = (\$1 \text{ million})\ \text{TV} + (\$2 \text{ million})\ \text{PM} \rightarrow \text{cell G14}$$

6. Use a SUMPRODUCT function to enter the appropriate value into each output cell (including the target cell). Based on the above expressions for cost and total increases in sales, the SUMPRODUCT functions needed here for the output cells are those shown under the right side of the spreadsheet in Figure 2.14. Note that each of these functions involves the relevant data cells and the changing cells, AdvertisingUnits(C14, D14).

   This spreadsheet model is a linear programming model, since it possesses all the characteristics of such models enumerated in Section 2.2.

Applying the Solver to This Model

The procedure for using the Excel Solver to obtain an optimal solution for this model is basically the same as described in Section 2.5. The key part of the Solver dialogue box is shown below the left-hand side of the spreadsheet in Figure 2.14. In addition to specifying the target cell and changing cells, the constraints that IncreasedSales ≥ MinimumIncrease have been specified in this box by using the Add Constraint dialogue box. Since the objective is to *minimize* total cost, Min also has been selected. (This is in contrast to the choice of Max for the Wyndor problem.)

The lower left-hand side of Figure 2.14 shows the options selected after clicking on the Options button in the Solver dialogue box. The Assume Linear Model option specifies that the model is a linear programming model. The Assume Non-Negative option specifies that the changing cells need nonnegativity constraints because negative values of advertising levels are not possible alternatives.

After clicking on Solve in the Solver dialogue box, the optimal solution shown in the changing cells of the spreadsheet in Figure 2.14 is obtained.

An Application Vignette

Samsung Electronics Corp., Ltd. (SEC), is a leading merchant of dynamic and static random access memory devices and other advanced digital integrated circuits. Its site at Kiheung, South Korea (probably the largest semiconductor fabrication site in the world), fabricates more than 300,000 silicon wafers per month and employs over 10,000 people.

Cycle time is the industry's term for the elapsed time from the release of a batch of blank silicon wafers into the fabrication process until completion of the devices that are fabricated on those wafers. Reducing cycle times is an ongoing goal since it both decreases costs and enables offering shorter lead times to potential customers, a real key to maintaining or increasing market share in a very competitive industry.

Three factors present particularly major challenges when striving to reduce cycle times. One is that the product mix changes continually. Another is that the company often needs to make substantial changes in the fab-out schedule inside the target cycle time as it revises forecasts of customer demand. The third is that the machines of a general type are not homogeneous so only a small number of machines are qualified to perform each device step.

A management science team developed a huge linear programming model with tens of thousands of decision variables and functional constraints to cope with these challenges. The objective function involved minimizing backorders and finished-goods inventory.

The ongoing implementation of this model enabled the company to reduce manufacturing cycle times to fabricate dynamic random access memory devices from more than 80 days to less than 30 days. This tremendous improvement and the resulting reduction in both manufacturing costs and sale prices enabled Samsung to capture an additional $200 million in annual sales revenue.

Source: R. C. Leachman, J. Kang, and Y. Lin, "SLIM: Short Cycle Time and Low Inventory in Manufacturing at Samsung Electronics," *Interfaces* 32, no.1 (January–February 2002), pp. 61–77.

Optimal Solution

C14 = 4 (Undertake 4 units of advertising on television)

D14 = 3 (Undertake 3 units of advertising in the print media)

The target cell indicates that the total cost of this advertising plan would be $10 million.

The Mathematical Model in the Spreadsheet

When performing step 5 of the procedure for formulating a spreadsheet model, the total cost of advertising was determined to be

$$\text{Cost} = \text{TV} + 2\text{ PM} \quad \text{(in millions of dollars)}$$

where the objective is to choose the values of TV (number of units of advertising on television) and PM (number of units of advertising in the print media) so as to minimize this cost. Step 4 identified three functional constraints:

$$\begin{aligned} &\text{Stain remover:} && 1\% \text{ of PM} \geq 3\% \\ &\text{Liquid detergent:} && 3\% \text{ of TV} + 2\% \text{ of PM} \geq 18\% \\ &\text{Powder detergent:} && -1\% \text{ of TV} + 4\% \text{ of PM} \geq 4\% \end{aligned}$$

Choosing the Assume Non-Negative option with the Solver recognized that TV and PM cannot be negative. Therefore, after dropping the percentage signs from the functional constraints, the complete mathematical model in the spreadsheet can be stated in the following succinct form.

$$\text{Minimize Cost} = \text{TV} + 2\text{ PM} \quad \text{(in millions of dollars)}$$

subject to

$$\begin{aligned} &\text{Stain remover increased sales:} && \text{PM} \geq 3 \\ &\text{Liquid detergent increased sales:} && 3\text{ TV} + 2\text{ PM} \geq 18 \\ &\text{Powder detergent increased sales:} && -\text{TV} + 4\text{ PM} \geq 4 \end{aligned}$$

and

$$\text{TV} \geq 0 \qquad \text{PM} \geq 0$$

Implicit in this statement is "Choose the values of TV and PM so as to . . . ." The term "subject to" is shorthand for "Choose these values *subject to* the requirement that the values satisfy all the following constraints."

This model is the *algebraic* version of the *linear programming* model in the spreadsheet. Note how the parameters (constants) of this algebraic model come directly from the numbers in Table 2.2. In fact, the entire model could have been formulated directly from this table.

The differences between this algebraic model and the one obtained for the Wyndor problem in Section 2.3 lead to some interesting changes in how the graphical method is applied to solve the model. To further expand your geometric intuition about linear programming, we briefly describe this application of the graphical method next.

Since this linear programming model has only two decision variables, it can be solved by the graphical method described in Section 2.4. The method needs to be adapted in two ways to fit this particular problem. First, because all the functional constraints now have a $\geq$ sign with a positive right-hand side, after obtaining the constraint boundary lines in the usual way, the arrows indicating which side of each line satisfies that constraint now all point *away* from the origin. Second, the method is adapted to *minimization* by moving the objective function lines in the direction that *reduces* Cost and then stopping at the last instant that an objective function line still passes through a point in the feasible region, where such a point then is an optimal solution. The supplement to this chapter includes a description of how the graphical method is applied to the Profit & Gambit problem in this way.

Review ***Questions***

1. What kind of product is produced by the Profit & Gambit Co.?
2. Which advertising media are being considered for the three products under consideration?
3. What is management's objective for the problem being addressed?
4. What was the rationale for the placement of the target cell and the changing cells in the spreadsheet model?
5. The algebraic form of the linear programming model for this problem differs from that for the Wyndor Glass Co. problem in which two major ways?

2.7 LINEAR PROGRAMMING FROM A BROADER PERSPECTIVE

Linear programming is an invaluable aid to managerial decision making in all kinds of companies throughout the world. The emergence of powerful spreadsheet packages has helped to further spread the use of this technique. The ease of formulating and solving small linear programming models on a spreadsheet now enables some managers with a very modest background in management science to do this themselves on their own desktop.

Many linear programming studies are major projects involving decisions on the levels of many hundreds or thousands of activities. For such studies, sophisticated software packages that go beyond spreadsheets generally are used for both the formulation and solution processes. These studies normally are conducted by technically trained teams of management scientists, sometimes called operations research analysts, at the instigation of management. Management needs to keep in touch with the management science team to ensure that the study reflects management's objectives and needs. However, management generally does not get involved with the technical details of the study.

Consequently, there is little reason for a manager to know the details of how linear programming models are solved beyond the rudiments of using the Excel Solver. (Even most management science teams will use commercial software packages for solving their models on a computer rather than developing their own software.) Similarly, a manager does not need to know the technical details of how to formulate complex models, how to validate such a model, how to interact with the computer when formulating and solving a large model, how to efficiently perform what-if analysis with such a model, and so forth. Therefore, these technical details are de-emphasized in this book. A student who becomes interested in conducting technical analyses as part of a management science team should plan to take additional, more technically oriented courses in management science.

So what does an enlightened manager need to know about linear programming? A manager needs to have a good intuitive feeling for what linear programming is. One objective of this chapter is to begin to develop that intuition. That's the purpose of studying the graphical method for solving two-variable problems. It is rare to have a *real* linear programming problem with as few as two decision variables. Therefore, the graphical method has essentially no practical value for solving real problems. However, it has great value for conveying the basic notion that linear programming involves pushing up against constraint boundaries and moving objective function values in a favorable direction as far as possible. You also will see in Chapter 14 that this approach provides considerable geometric insight into how to analyze larger models by other methods.

A manager must also have an appreciation for the relevance and power of linear programming to encourage its use where appropriate. For *future* managers using this book, this appreciation is being promoted by describing *real* applications of linear programming and the resulting impact, as well as by including (in miniature form) various realistic examples and case studies that illustrate what can be done.

Certainly a manager must be able to recognize situations where linear programming is applicable. We focus on developing this skill in Chapter 3, where you will learn how to recognize the *identifying features* for each of the major types of linear programming problems (and their mixtures).

In addition, a manager should recognize situations where linear programming should *not* be applied. Chapter 8 will help to develop this skill by examining certain underlying assumptions of linear programming and the circumstances that violate these assumptions. That chapter also describes other approaches that *can* be applied where linear programming should not.

A manager needs to be able to distinguish between competent and shoddy studies using linear programming (or any other management science technique). Therefore, another goal of the upcoming chapters is to demystify the overall process involved in conducting a management science study, all the way from first studying a problem to final implementation of the managerial decisions based on the study. This is one purpose of the case studies throughout the book.

Finally, a manager must understand how to interpret the results of a linear programming study. He or she especially needs to understand what kinds of information can be obtained through *what-if analysis,* as well as the implications of such information for managerial decision making. Chapter 5 focuses on these issues.

Review ***Questions***

1. Does management generally get heavily involved with the technical details of a linear programming study?
2. What is the purpose of studying the graphical method for solving problems with two decision variables when essentially all real linear programming problems have more than two?
3. List the things that an enlightened manager should know about linear programming.

2.8 Summary

Linear programming is a powerful technique for aiding managerial decision making for certain kinds of problems. The basic approach is to formulate a mathematical model called a linear programming model to represent the problem and then to analyze this model. Any linear programming model includes decision variables to represent the decisions to be made, constraints to represent the restrictions on the feasible values of these decision variables, and an objective function that expresses the overall measure of performance for the problem.

Spreadsheets provide a flexible and intuitive way of formulating and solving a linear programming model. The data are entered into data cells. Changing cells display the values of the decision variables, and a target cell shows the value of the objective function. Output cells are used to help specify the constraints. After formulating the model on the spreadsheet and specifying it further with the Solver dialogue box, the Solver is used to quickly find an optimal solution.

The graphical method can be used to solve a linear programming model having just two decision variables. This method provides considerable insight into the nature of linear programming models and optimal solutions.

Glossary

absolute reference A reference to a cell (or a column or a row) with a fixed address, as indicated either by using a range name or by placing a $ sign in front of the letter and number of the cell reference. (Section 2.2), 24

changing cells The cells in the spreadsheet that show the values of the decision variables. (Section 2.2), 23

constraint A restriction on the feasible values of the decision variables. (Sections 2.2 and 2.3), 26

constraint boundary equation The equation for the constraint boundary line. (Section 2.4), 31

constraint boundary line For linear programming problems with two decision variables, the line forming the boundary of the solutions that are permitted by the constraint. (Section 2.4), 31

data cells The cells in the spreadsheet that show the data of the problem. (Section 2.2), 22

decision variable An algebraic variable that represents a decision regarding the level of a particular activity. The value of the decision variable appears in a changing cell on the spreadsheet. (Section 2.3), 28

feasible region The geometric region that consists of all the feasible solutions. (Section 2.4), 29

feasible solution A solution that simultaneously satisfies all the constraints in the linear programming model. (Section 2.3), 28

functional constraint A constraint with a function of the decision variables on the left-hand side. All constraints in a linear programming model that are not nonnegativity constraints are called functional constraints. (Section 2.3), 28

graphical method A method for solving linear programming problems with two decision variables on a two-dimensional graph. (Section 2.4), 29

infeasible solution A solution that violates at least one of the constraints in the linear programming model. (Section 2.3), 28

linear programming model The mathematical model that represents a linear programming problem. (Sections 2.2 and 2.3), 22

nonnegativity constraint A constraint that expresses the restriction that a particular decision variable must be nonnegative (greater than or equal to zero). (Section 2.3), 28

relative reference A reference to a cell whose address is based upon its position relative to the cell containing the formula. (Section 2.2), 24

objective function The part of a linear programming model that expresses what needs to be either maximized or minimized, depending on the objective for the problem. The value of the objective function appears in the target cell on the spreadsheet. (Section 2.3), 28

objective function line For a linear programming problem with two decision variables, a line whose points all have the same value of the objective function. (Section 2.4), 32

optimal solution The best feasible solution according to the objective function. (Section 2.3), 28

output cells The cells in the spreadsheet that provide output that depends on the changing cells. These cells frequently are used to help specify constraints. (Section 2.2), 23

parameter The parameters of a linear programming model are the constants (coefficients or right-hand sides) in the functional constraints and the objective function. Each parameter represents a quantity (e.g., the amount available of a resource) that is of importance for the analysis of the problem. (Section 2.3), 28

product-mix problem A type of linear programming problem where the objective is to find the most profitable mix of production levels for the products under consideration. (Section 2.1), 21

solution Any single assignment of values to the decision variables, regardless of whether the assignment is a good one or even a feasible one. (Section 2.3), 28

Solver The spreadsheet tool that is used to specify the model in the spreadsheet and then to obtain an optimal solution for that model. (Section 2.5), 34

target cell The cell in the spreadsheet that shows the overall measure of performance of the decisions. (Section 2.2), 24

Learning Aids for This Chapter in Your MS Courseware

Chapter 2 Excel Files:

Wyndor Example
Profit & Gambit Example

Interactive Management Science Modules:

Module for Graphical Linear Programming and Sensitivity Analysis

Excel Add-ins:

Premium Solver for Education

Supplement to Chapter 2 on the CD-ROM:

More About the Graphical Method for Linear Programming

Solved Problem (See the CD-ROM for the Solution)

2.S1. Conducting a Marketing Survey

The marketing group for a cell phone manufacturer plans to conduct a telephone survey to determine consumer attitudes toward a new cell phone that is currently under development. In order to have a sufficient sample size to conduct the analysis, they need to contact at least 100 young males (under age 40), 150 older males (over age 40), 120 young females (under age 40), and 200 older females (over age 40). It costs $1 to make a daytime phone call and $1.50 to make an evening phone call (because of higher labor costs). This cost is incurred whether or not anyone answers the phone. The table below shows the likelihood of a given customer type answering each phone call. Assume the survey is conducted with whoever first answers the phone. Also, because of limited evening staffing, at most one-third of phone calls placed can be evening phone calls. How should the marketing group conduct the telephone survey so as to meet the sample size requirements at the lowest possible cost?

Who Answers?	Daytime Calls	Evening Calls
Young male	10%	20%
Older male	15%	30%
Young female	20%	20%
Older female	35%	25%
No answer	20%	5%

Problems

We have inserted the symbol E* (for Excel) to the left of e1ch problem or part where Excel should be used. An asterisk on the problem number indicates that at least a partial answer is given in the back of the book.

2.1 Reconsider the Wyndor Glass Co. case study introduced in Section 2.1. Suppose that the estimates of the unit profits for the two new products now have been revised to $600 for the doors and $300 for the windows.

E* *a.* Formulate and solve the revised linear programming model for this problem on a spreadsheet.

b. Formulate this same model algebraically.

c. Use the graphical method to solve this revised model.

2.2 Reconsider the Wyndor Glass Co. case study introduced in Section 2.1. Suppose that Bill Tasto (Wyndor's vice president for manufacturing) now has found a way to provide a little additional production time in Plant 2 to the new products.

a. Use the graphical method to find the new optimal solution and the resulting total profit if *one* additional hour per week is provided.

b. Repeat part *a* if *two* additional hours per week are provided instead.

c. Repeat part *a* if *three* additional hours per week are provided instead.

d. Use these results to determine how much each additional hour per week would be worth in terms of increasing the total profit from the two new products.

E*2.3 Use the Excel Solver to do Problem 2.2.

2.4 The following table summarizes the key facts about two products, A and B, and the resources, Q, R, and S, required to produce them.

	Resource Usage per Unit Produced		
Resource	Product A	Product B	Amount of Resource Available
Q	2	1	2
R	1	2	2
S	3	3	4
Profit/unit	$3,000	$2,000	

All the assumptions of linear programming hold.

E* *a.* Formulate and solve a linear programming model for this problem on a spreadsheet.

b. Formulate this same model algebraically.

2.5* This is your lucky day. You have just won a $10,000 prize. You are setting aside $4,000 for taxes and partying expenses, but you have decided to invest the other $6,000. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a *full* partner in the first friend's venture would require an investment of $5,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be $4,500. The corresponding figures for the second friend's venture are \$4,000 and 500 hours, with an estimated profit to you of \$4,500. However, both friends are flexible and would allow you to come in at any *fraction* of a full partnership you would like. If you choose a fraction of a full partnership, all the above figures given for a full partnership (money investment, time investment, and your profit) would be multiplied by this same fraction.

Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.

a. Describe the analogy between this problem and the Wyndor Glass Co. problem discussed in Section 2.1. Then construct and fill in a table like Table 2.1 for this problem, identifying both the activities and the resources.

b. Identify verbally the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions.

c. Convert these verbal descriptions of the constraints and the measure of performance into quantitative expressions in terms of the data and decisions.

E* *d.* Formulate a spreadsheet model for this problem. Identify the data cells, the changing cells, and the target cell. Also show the Excel equation for each output cell expressed as a SUMPRODUCT function. Then use the Excel Solver to solve this model.

e. Indicate why this spreadsheet model is a linear programming model.

f. Formulate this same model algebraically.

g. Identify the decision variables, objective function, nonnegativity constraints, functional constraints, and parameters in both the algebraic version and spreadsheet version of the model.

h. Use the graphical method by hand to solve this model. What is your total estimated profit?

i. Use the Graphical Linear Programming and Sensitivity Analysis module in your Interactive Management Science Modules to apply the graphical method to this model.

2.6 You are given the following linear programming model in algebraic form, where x_1 and x_2 are the decision variables and Z is the value of the overall measure of performance.

$$\text{Maximize} \qquad Z = x_1 + 2x_2$$

subject to

Constraint on resource 1: $x_1 + x_2 \leq 5$ (amount available)
Constraint on resource 2: $x_1 + 3x_2 \leq 9$ (amount available)

and

$$x_1 \geq 0 \qquad x_2 \geq 0$$

a. Identify the objective function, the functional constraints, and the nonnegativity constraints in this model.

E* *b.* Incorporate this model into a spreadsheet.

c. Is $(x_1, x_2) = (3, 1)$ a feasible solution?

d. Is $(x_1, x_2) = (1, 3)$ a feasible solution?

E* *e.* Use the Excel Solver to solve this model.

2.7 You are given the following linear programming model in algebraic form, where x_1 and x_2 are the decision variables and Z is the value of the overall measure of performance.

$$\text{Maximize} \qquad Z = 3x_1 + 2x_2$$

subject to

Constraint on resource 1: $3x_1 + x_2 \leq 9$ (amount available)
Constraint on resource 2: $x_1 + 2x_2 \leq 8$ (amount available)

and

$$x_1 \geq 0 \qquad x_2 \geq 0$$

a. Identify the objective function, the functional constraints, and the nonnegativity constraints in this model.

E* *b.* Incorporate this model into a spreadsheet.

c. Is $(x_1, x_2) = (2, 1)$ a feasible solution?

d. Is $(x_1, x_2) = (2, 3)$ a feasible solution?

e. Is $(x_1, x_2) = (0, 5)$ a feasible solution?

E* *f.* Use the Excel Solver to solve this model.

2.8 The Whitt Window Company is a company with only three employees that makes two different kinds of handcrafted windows: a wood-framed and an aluminum framed window. They earn \$60 profit for each wood-framed window and \$30 profit for each aluminum-framed window. Doug makes the wood frames and can make 6 per day. Linda makes the aluminum frames and can make 4 per day. Bob forms and cuts the glass and can make 48 square feet of glass per day. Each wood-framed window uses 6 square feet of glass and each aluminum-framed window uses 8 square feet of glass.

The company wishes to determine how many windows of each type to produce per day to maximize total profit.

a. Describe the analogy between this problem and the Wyndor Glass Co. problem discussed in Section 2.1. Then construct and fill in a table like Table 2.1 for this problem, identifying both the activities and the resources.

b. Identify verbally the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions.

c. Convert these verbal descriptions of the constraints and the measure of performance into quantitative expressions in terms of the data and decisions.

E* *d.* Formulate a spreadsheet model for this problem. Identify the data cells, the changing cells, and the target cell. Also show the Excel equation for each output cell expressed as a SUMPRODUCT function. Then use the Excel Solver to solve this model.

e. Indicate why this spreadsheet model is a linear programming model.

f. Formulate this same model algebraically.

g. Identify the decision variables, objective function, nonnegativity constraints, functional constraints, and parameters in both the algebraic version and spreadsheet version of the model.

h. Use the graphical method to solve this model.

i. A new competitor in town has started making wood-framed windows as well. This may force the company to lower the price it charges and so lower the profit made for each wood-framed window. How would the optimal solution change (if at all) if the profit per wood-framed window decreases from $60 to $40? From $60 to $20?

j. Doug is considering lowering his working hours, which would decrease the number of wood frames he makes per day. How would the optimal solution change if he only makes 5 wood frames per day?

2.9 The Apex Television Company has to decide on the number of 27" and 20" sets to be produced at one of its factories. Market research indicates that at most 40 of the 27" sets and 10 of the 20" sets can be sold per month. The maximum number of work-hours available is 500 per month. A 27" set requires 20 work-hours and a 20" set requires 10 work-hours. Each 27" set sold produces a profit of $120 and each 20" set produces a profit of $80. A wholesaler has agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by the market research.

E* *a.* Formulate and solve a linear programming model for this problem on a spreadsheet.

b. Formulate this same model algebraically.

c. Solve this model by using the Graphical Linear Programming and Sensitivity Analysis module in your Interactive Management Science Modules to apply the graphical method.

2.10 The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, one unit of frame parts and two units of electrical components are required. For each unit of product 2, three units of frame parts and two units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1, and each unit of product 2, up to 60 units, gives a profit of $2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.

a. Identify verbally the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions.

b. Convert these verbal descriptions of the constraints and the measure of performance into quantitative expressions in terms of the data and decisions.

E* *c.* Formulate and solve a linear programming model for this problem on a spreadsheet.

d. Formulate this same model algebraically.

e. Solve this model by using the Graphical Linear Programming and Sensitivity Analysis module in your Interactive Management Science Modules to apply the graphical method. What is the resulting total profit?

2.11 The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is $5 per unit on special risk insurance and $2 per unit on mortgages.

Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

	Work-Hours per Unit		
Department	**Special Risk**	**Mortgage**	**Work-Hours Available**
Underwriting	3	2	2,400
Administration	0	1	800
Claims	2	0	1,200

a. Identify verbally the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions.

b. Convert these verbal descriptions of the constraints and the measure of performance into quantitative expressions in terms of the data and decisions.

E* *c.* Formulate and solve a linear programming model for this problem on a spreadsheet.

d. Formulate this same model algebraically.

2.12.* You are given the following linear programming model in algebraic form, with x_1 and x_2 as the decision variables and constraints on the usage of four resources:

$$\text{Maximize} \quad \text{Profit} = 2x_1 + x_2$$

subject to

$$x_2 \le 10 \quad \text{(resource 1)}$$
$$2x_1 + 5x_2 \le 60 \quad \text{(resource 2)}$$
$$x_1 + x_2 \le 18 \quad \text{(resource 3)}$$
$$3x_1 + x_2 \le 44 \quad \text{(resource 4)}$$

and

$$x_1 \ge 0 \qquad x_2 \ge 0$$

a. Use the graphical method to solve this model.

E* *b.* Incorporate this model into a spreadsheet and then use the Excel Solver to solve this model.

2.13 Because of your knowledge of management science, your boss has asked you to analyze a product mix problem involving two products and two resources. The model is shown below in algebraic form, where x_1 and x_2 are the production rates for the two products and P is the total profit.

$$\text{Maximize} \quad P = 3x_1 + 2x_2$$

subject to

$$x_1 + x_2 \leq 8 \quad \text{(resource 1)}$$
$$2x_1 + x_2 \leq 10 \quad \text{(resource 2)}$$

and

$$x_1 \geq 0 \quad x_2 \geq 0$$

a. Use the graphical method to solve this model.

E* *b.* Incorporate this model into a spreadsheet and then use the Excel Solver to solve this model.

2.14 Weenies and Buns is a food processing plant that manufactures hot dogs and hot dog buns. They grind their own flour for the hot dog buns at a maximum rate of 200 pounds per week. Each hot dog bun requires 0.1 pound of flour. They currently have a contract with Pigland, Inc., which specifies that a delivery of 800 pounds of pork product is delivered every Monday. Each hot dog requires 1/4 pound of pork product. All the other ingredients in the hot dogs and hot dog buns are in plentiful supply. Finally, the labor force at Weenies and Buns consists of five employees working full time (40 hours per week each). Each hot dog requires three minutes of labor, and each hot dog bun requires two minutes of labor. Each hot dog yields a profit of $0.20, and each bun yields a profit of $0.10.

Weenies and Buns would like to know how many hot dogs and how many hot dog buns they should produce each week so as to achieve the highest possible profit.

a. Identify verbally the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions.

b. Convert these verbal descriptions of the constraints and the measure of performance into quantitative expressions in terms of the data and decisions.

E* *c.* Formulate and solve a linear programming model for this problem on a spreadsheet.

d. Formulate this same model algebraically.

e. Use the graphical method to solve this model. Decide yourself whether you would prefer to do this by hand or by using the Graphical Linear Programming and Sensitivity Analysis module in your Interactive Management Science Modules.

2.15 The Oak Works is a family-owned business that makes handcrafted dining room tables and chairs. They obtain the oak from a local tree farm, which ships them 2,500 pounds of oak each month. Each table uses 50 pounds of oak while each chair uses 25 pounds of oak. The family builds all the furniture itself and has 480 hours of labor available each month. Each table or chair requires six hours of labor. Each table nets Oak Works $400 in profit, while each chair nets $100 in profit. Since chairs are often sold with the tables, they want to produce *at least* twice as many chairs as tables.

The Oak Works would like to decide how many tables and chairs to produce so as to maximize profit.

a. Formulate and solve a linear programming model for this problem on a spreadsheet.

b. Formulate this same model algebraically.

2.16 Nutri-Jenny is a weight-management center. It produces a wide variety of frozen entrees for consumption by its clients. The entrees are strictly monitored for nutritional content to ensure that the clients are eating a balanced diet. One new entree will be a "beef sirloin tips dinner." It will consist of beef tips and gravy, plus some combination of peas, carrots, and a dinner roll. Nutri-Jenny would like to determine what quantity of each item to include in the entree to meet the nutritional requirements, while costing as little as possible. The nutritional information for each item and its cost are given in the following table.

Item	Calories (per oz.)	Calories from Fat (per oz.)	Vitamin A (IU per oz.)	Vitamin C (mg per oz.)	Protein (gr. per oz.)	Cost (per oz.)
Beef tips	54	19	0	0	8	40¢
Gravy	20	15	0	1	0	35¢
Peas	15	0	15	3	1	15¢
Carrots	8	0	350	1	1	18¢
Dinner roll	40	10	0	0	1	10¢

The nutritional requirements for the entree are as follows: (1) it must have between 280 and 320 calories, (2) calories from fat should be no more than 30 percent of the total number of calories, and (3) it must have at least 600 IUs of vitamin A, 10 milligrams of vitamin C, and 30 grams of protein. Furthermore, for practical reasons, it must include at least 2 ounces of beef, and it must have at least half an ounce of gravy per ounce of beef.

E* *a.* Formulate and solve a linear programming model for this problem on a spreadsheet.

b. Formulate this same model algebraically.

2.17 Ralph Edmund loves steaks and potatoes. Therefore, he has decided to go on a steady diet of only these two foods (plus some liquids and vitamin supplements) for all his meals. Ralph realizes that this isn't the healthiest diet, so he wants to make sure that he eats the right quantities of the two foods to satisfy some key nutritional requirements. He has obtained the following nutritional and cost information:

Grams of Ingredient per Serving			
Ingredient	**Steak**	**Potatoes**	**Daily Requirement (grams)**
Carbohydrates	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≤ 60
Cost per serving	$4	$2	

Ralph wishes to determine the number of daily servings (may be fractional) of steak and potatoes that will meet these requirements at a minimum cost.

a. Identify verbally the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions.

b. Convert these verbal descriptions of the constraints and the measure of performance into quantitative expressions in terms of the data and decisions.

c. Formulate and solve a linear programming model for this problem on a spreadsheet.

d. Formulate this same model algebraically.

e. Use the graphical method by hand to solve this model.

f. Use the Graphical Linear Programming and Sensitivity Analysis module in your Interactive Management Science Modules to apply the graphical method to this model.

2.18 Dwight is an elementary school teacher who also raises pigs for supplemental income. He is trying to decide what to feed his pigs. He is considering using a combination of pig feeds available from local suppliers. He would like to feed the pigs at minimum cost while also making sure each pig receives an adequate supply of calories and vitamins. The cost, calorie content, and vitamin content of each feed is given in the table below.

Contents	Feed Type A	Feed Type B
Calories (per pound)	800	1,000
Vitamins (per pound)	140 units	70 units
Cost (per pound)	$0.40	$0.80

Each pig requires at least 8,000 calories per day and at least 700 units of vitamins. A further constraint is that no more than 1/3 of the diet (by weight) can consist of Feed Type A, since it contains an ingredient that is toxic if consumed in too large a quantity.

a. Identify verbally the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions.

b. Convert these verbal descriptions of the constraints and the measure of performance into quantitative expressions in terms of the data and decisions.

E* *c.* Formulate and solve a linear programming model for this problem on a spreadsheet.

d. Formulate this same model algebraically.

2.19 Reconsider the Profit & Gambit Co. problem described in Section 2.6. Suppose that the estimated data given in Table 2.2 now have been changed as shown in the table that accompanies this problem.

E* *a.* Formulate and solve a linear programming model on a spreadsheet for this revised version of the problem.

b. Formulate this same model algebraically.

c. Use the graphical method to solve this model.

d. What were the key changes in the data that caused your answer for the optimal solution to change from the one for the original version of the problem?

	Increase in Sales per Unit of Advertising		
Product	**Television**	**Print Media**	**Minimum Required Increase**
Stain remover	0%	1.5%	3%
Liquid detergent	3	4	18
Powder detergent	−1	2	4
Unit cost	$1 million	$2 million	

e. Write a paragraph to the management of the Profit & Gambit Co. presenting your conclusions from the above parts. Include the potential effect of further refining the key data in the above table. Also point out the leverage that your results might provide to management in negotiating a decrease in the unit cost for either of the advertising media.

2.20 You are given the following linear programming model in algebraic form, with x_1 and x_2 as the decision variables:

$$\text{Minimize} \quad \text{Cost} = 40x_1 + 50x_2$$

subject to

$$\text{Constraint 1:} \quad 2x_1 + 3x_2 \geq 30$$
$$\text{Constraint 2:} \quad x_1 + x_2 \geq 12$$
$$\text{Constraint 3:} \quad 2x_1 + x_2 \geq 20$$

and

$$x_1 \geq 0 \qquad x_2 \geq 0$$

a. Use the graphical method to solve this model.

b. How does the optimal solution change if the objective function is changed to Cost = $40x_1 + 70x_2$?

c. How does the optimal solution change if the third functional constraint is changed to $2x_1 + x_2 \geq 15$?

E* *d.* Now incorporate the original model into a spreadsheet and use the Excel Solver to solve this model.

E* *e.* Use Excel to do parts *b* and *c.*

2.21 The Learning Center runs a day camp for 6–10 year olds during the summer. Its manager, Elizabeth Reed, is trying to reduce the center's operating costs to avoid having to raise the tuition fee. Elizabeth is currently planning what to feed the children for lunch. She would like to keep costs to a minimum, but also wants to make sure she is meeting the nutritional requirements of the children. She has already decided to go with peanut butter and jelly sandwiches, and some combination of apples, milk, and/or cranberry juice. The nutritional content of each food choice and its cost are given in the table that accompanies this problem.

Food Item	Calories from Fat	Total Calories	Vitamin C (mg)	Fiber (g)	Cost (¢)
Bread (1 slice)	15	80	0	4	6
Peanut butter (1 tbsp)	80	100	0	0	5
Jelly (1 tbsp)	0	70	4	3	8
Apple	0	90	6	10	35
Milk (1 cup)	60	120	2	0	20
Cranberry juice (1 cup)	0	110	80	1	40

The nutritional requirements are as follows. Each child should receive between 300 and 500 calories, but no more than 30 percent of these calories should come from fat. Each child should receive at least 60 milligrams (mg) of vitamin C and at least 10 grams (g) of fiber.

To ensure tasty sandwiches, Elizabeth wants each child to have a minimum of 2 slices of bread, 1 tablespoon (tbsp) of peanut butter, and 1 tbsp of jelly, along with at least 1 cup of liquid (milk and/or cranberry juice).

Elizabeth would like to select the food choices that would minimize cost while meeting all these requirements.

E* *a.* Formulate and solve a linear programming model for this problem on a spreadsheet.

b. Formulate this same model algebraically.

Case 2-1

Auto Assembly

Automobile Alliance, a large automobile manufacturing company, organizes the vehicles it manufactures into three families: a family of trucks, a family of small cars, and a family of midsized and luxury cars. One plant outside Detroit, Michigan, assembles two models from the family of midsized and luxury cars. The first model, the Family Thrillseeker, is a four-door sedan with vinyl seats, plastic interior, standard features, and excellent gas mileage. It is marketed as a smart buy for middle-class families with tight budgets, and each Family Thrillseeker sold generates a modest profit of $3,600 for the company. The second model, the Classy Cruiser, is a two-door luxury sedan with leather seats, wooden interior, custom features, and navigational capabilities. It is marketed as a privilege of affluence for upper-middle-class families, and each Classy Cruiser sold generates a healthy profit of $5,400 for the company.

Rachel Rosencrantz, the manager of the assembly plant, is currently deciding the production schedule for the next month. Specifically, she must decide how many Family Thrillseekers and how many Classy Cruisers to assemble in the plant to maximize profit for the company. She knows that the plant possesses a capacity of 48,000 labor-hours during the month. She also knows that it takes six labor-hours to assemble one Family Thrillseeker and 10.5 labor-hours to assemble one Classy Cruiser.

Because the plant is simply an assembly plant, the parts required to assemble the two models are not produced at the plant. Instead, they are shipped from other plants around the Michigan area to the assembly plant. For example, tires, steering wheels, windows, seats, and doors all arrive from various supplier plants. For the next month, Rachel knows that she will only be able to obtain 20,000 doors from the door supplier. A recent labor strike forced the shutdown of that particular supplier plant for several days, and that plant will not be able to meet its production schedule for the next month. Both the Family Thrillseeker and the Classy Cruiser use the same door part.

In addition, a recent company forecast of the monthly demands for different automobile models suggests that the demand for the Classy Cruiser is limited to 3,500 cars. There is no limit on the demand for the Family Thrillseeker within the capacity limits of the assembly plant.

a. Formulate and solve a linear programming model to determine the number of Family Thrillseekers and the number of Classy Cruisers that should be assembled.

Before she makes her final production decisions, Rachel plans to explore the following questions independently, except where otherwise indicated.

b. The marketing department knows that it can pursue a targeted $500,000 advertising campaign that will raise the demand for the Classy Cruiser next month by 20 percent. Should the campaign be undertaken?

c. Rachel knows that she can increase next month's plant capacity by using overtime labor. She can increase the plant's labor-hour capacity by 25 percent. With the new assembly plant capacity, how many Family Thrillseekers and how many Classy Cruisers should be assembled?

d. Rachel knows that overtime labor does not come without an extra cost. What is the maximum amount she should be willing to pay for all overtime labor beyond the cost of this labor at regular-time rates? Express your answer as a lump sum.

e. Rachel explores the option of using both the targeted advertising campaign and the overtime labor hours. The advertising campaign raises the demand for the Classy Cruiser by 20 percent, and the overtime labor increases the plant's labor-hour capacity by 25 percent. How many Family Thrillseekers and how many Classy Cruisers should be assembled using the advertising campaign and overtime labor-hours if the profit from each Classy Cruiser sold continues to be 50 percent more than for each Family Thrillseeker sold?

f. Knowing that the advertising campaign costs $500,000 and the maximum usage of overtime labor hours costs $1,600,000 beyond regular time rates, is the solution found in part *e* a wise decision compared to the solution found in part *a*?

g. Automobile Alliance has determined that dealerships are actually heavily discounting the price of the Family Thrillseekers to move them off the lot. Because of a profit-sharing agreement with its dealers, the company is not making a profit of $3,600 on the Family Thrillseeker but instead is making a profit of $2,800. Determine the number of Family Thrillseekers and the number of Classy Cruisers that should be assembled given this new discounted profit.

h. The company has discovered quality problems with the Family Thrillseeker by randomly testing Thrillseekers at the end of the assembly line. Inspectors have discovered that in over 60 percent of the cases, two of the four doors on a Thrillseeker do not seal properly. Because the percentage of defective Thrillseekers determined by the random testing is so high, the floor foreman has decided to perform quality control tests on every Thrillseeker at the end of the line. Because of the added tests, the time it takes to assemble one Family Thrillseeker has increased from 6 hours to 7.5 hours. Determine the number of units of each model that should be assembled given the new assembly time for the Family Thrillseeker.

i. The board of directors of Automobile Alliance wishes to capture a larger share of the luxury sedan market and therefore would like to meet the full demand for Classy Cruisers. They ask Rachel to determine by how much the profit of her assembly plant would decrease as compared to the profit found in part *a*. They then ask her to meet the full demand for Classy Cruisers if the decrease in profit is not more than $2,000,000.

j. Rachel now makes her final decision by combining all the new considerations described in parts *f, g,* and *h*. What are her final decisions on whether to undertake the advertising campaign, whether to use overtime labor, the number of Family Thrillseekers to assemble, and the number of Classy Cruisers to assemble?

Case 2-2

Cutting Cafeteria Costs

A cafeteria at All-State University has one special dish it serves like clockwork every Thursday at noon. This supposedly tasty dish is a casserole that contains sautéed onions, boiled sliced potatoes, green beans, and cream of mushroom soup. Unfortunately, students fail to see the special quality of this dish, and they loathingly refer to it as the Killer Casserole. The students reluctantly eat the casserole, however, because the cafeteria provides only a limited selection of dishes for Thursday's lunch (namely, the casserole).

Maria Gonzalez, the cafeteria manager, is looking to cut costs for the coming year, and she believes that one sure way to cut costs is to buy less expensive and perhaps lower quality ingredients. Because the casserole is a weekly staple of the cafeteria menu, she concludes that if she can cut costs on the ingredients purchased for the casserole, she can significantly reduce overall cafeteria operating costs. She therefore decides to invest time in determining how to minimize the costs of the casserole while maintaining nutritional and taste requirements.

Maria focuses on reducing the costs of the two main ingredients in the casserole, the potatoes and green beans. These two ingredients are responsible for the greatest costs, nutritional content, and taste of the dish.

Maria buys the potatoes and green beans from a wholesaler each week. Potatoes cost $0.40 per pound (lb), and green beans cost $1.00 per lb.

All-State University has established nutritional requirements that each main dish of the cafeteria must meet. Specifically, the dish must contain 180 grams (g) of protein, 80 milligrams (mg) of iron, and 1,050 mg of vitamin C. (There are 454 g in one lb and 1,000 mg in one g.) For simplicity when planning, Maria assumes that only the potatoes and green beans contribute to the nutritional content of the casserole.

Because Maria works at a cutting-edge technological university, she has been exposed to the numerous resources on the World Wide Web. She decides to surf the Web to find the nutritional content of potatoes and green beans. Her research yields the following nutritional information about the two ingredients:

	Potatoes	Green Beans
Protein	1.5 g per 100 g	5.67 g per 10 ounces
Iron	0.3 mg per 100 g	3.402 mg per 10 ounces
Vitamin C	12 mg per 100 g	28.35 mg per 10 ounces

(There are 28.35 g in one ounce.)

Edson Branner, the cafeteria cook who is surprisingly concerned about taste, informs Maria that an edible casserole must contain at least a six-to-five ratio in the weight of potatoes to green beans.

Given the number of students who eat in the cafeteria, Maria knows that she must purchase enough potatoes and green beans to prepare a minimum of 10 kilograms (kg) of casserole each week. (There are 1,000 g in one kg.) Again, for simplicity in planning, she assumes that only the potatoes and green beans determine the amount of casserole that can be prepared. Maria does not establish an upper limit on the amount of casserole to prepare since she knows all leftovers can be served for many days thereafter or can be used creatively in preparing other dishes.

a. Determine the amount of potatoes and green beans Maria should purchase each week for the casserole to minimize the ingredient costs while meeting nutritional, taste, and demand requirements.

Before she makes her final decision, Maria plans to explore the following questions independently, except where otherwise indicated.

b. Maria is not very concerned about the taste of the casserole; she is only concerned about meeting nutritional requirements and cutting costs. She therefore forces Edson to change the recipe to allow only for at least a one-to-two ratio in the weight of potatoes to green beans. Given the new recipe, determine the amount of potatoes and green beans Maria should purchase each week.

c. Maria decides to lower the iron requirement to 65 mg since she determines that the other ingredients, such as the onions and cream of mushroom soup, also provide iron. Determine the amount of potatoes and green beans Maria should purchase each week given this new iron requirement.

d. Maria learns that the wholesaler has a surplus of green beans and is therefore selling the green beans for a lower price of $0.50 per lb. Using the same iron requirement from part *c* and the new price of green beans, determine the amount of potatoes and green beans Maria should purchase each week.

e. Maria decides that she wants to purchase lima beans instead of green beans since lima beans are less expensive and provide a greater amount of protein and iron than green beans. Maria again wields her absolute power and forces Edson to change the recipe to include lima beans instead of green beans. Maria knows she can purchase lima beans for $0.60 per lb from the wholesaler. She also knows that lima beans contain 22.68 g of protein and 6.804 mg of iron per 10 ounces of lima beans and no vitamin C. Using the new cost and nutritional content of lima beans, determine the amount of potatoes and lima beans Maria should purchase each week to minimize the ingredient costs while meeting nutritional, taste, and demand requirements. The nutritional requirements include the reduced iron requirement from part *c*.

f. Will Edson be happy with the solution in part *e?* Why or why not?

g. An All-State student task force meets during Body Awareness Week and determines that All-State University's nutritional requirements for iron are too lax and that those for vitamin C are too stringent. The task force urges the university to adopt a policy that requires each serving of an entrée to contain at least 120 mg of iron and at least 500 mg of vitamin C. Using potatoes and lima beans as the ingredients for the dish and using the new nutritional requirements, determine the amount of potatoes and lima beans Maria should purchase each week.

Case 2-3

Staffing a Call Center

California Children's Hospital has been receiving numerous customer complaints because of its confusing, decentralized appointment and registration process. When customers want to make appointments or register child patients, they must contact the clinic or department they plan to visit. Several problems exist with this current strategy. Parents do not always know the most appropriate clinic or department they must visit to address their children's ailments. They therefore spend a significant amount of time on the phone being transferred from clinic to clinic until they reach the most appropriate clinic for their needs. The hospital also does not publish the phone numbers of all clinics and departments, and parents must therefore invest a large amount of time in detective work to track down the correct phone number. Finally, the various clinics and departments do not communicate with each other. For example, when a doctor schedules a referral with a colleague located in another department or clinic, that department or clinic almost never receives word of the referral. The parent must contact the correct department or clinic and provide the needed referral information.

In efforts to reengineer and improve its appointment and registration process, the children's hospital has decided to centralize the process by establishing one call center devoted exclusively to appointments and registration. The hospital is currently in the middle of the planning stages for the call center. Lenny Davis, the hospital manager, plans to operate the call center from 7 AM to 9 PM during the weekdays.

Several months ago, the hospital hired an ambitious management consulting firm, Creative Chaos Consultants, to forecast the number of calls the call center would receive each hour of the day. Since all appointment and registration-related calls would be received by the call center, the consultants decided that they could forecast the calls at the call center by totaling the number of appointment and registration-related calls received by all clinics and departments. The team members visited all the clinics and departments, where they diligently recorded every call relating to appointments and registration. They then totaled these calls and altered the totals to account for calls missed during data collection. They also altered totals to account for repeat calls that occurred when the same parent called the hospital many times because of the confusion surrounding the decentralized process. Creative Chaos Consultants determined the average number of calls the call center should expect during each hour of a weekday. The following table provides the forecasts.

Work Shift	Average Number of Calls
7 AM to 9 AM	40 calls per hour
9 AM to 11 AM	85 calls per hour
11 AM to 1 PM	70 calls per hour
1 PM to 3 PM	95 calls per hour
3 PM to 5 PM	80 calls per hour
5 PM to 7 PM	35 calls per hour
7 PM to 9 PM	10 calls per hour

After the consultants submitted these forecasts, Lenny became interested in the percentage of calls from Spanish speakers since the hospital services many Spanish patients. Lenny knows that he has to hire some operators who speak Spanish to handle these calls. The consultants performed further data collection and determined that, on average, 20 percent of the calls were from Spanish speakers.

Given these call forecasts, Lenny must now decide how to staff the call center during each two-hour shift of a weekday. During the forecasting project, Creative Chaos Consultants closely observed the operators working at the individual clinics and departments and determined the number of calls operators process per hour. The consultants informed Lenny that an operator is able to process an average of six calls per hour. Lenny also knows that he has both full-time and part-time workers available to staff the call center. A full-time employee works eight hours per day, but because of paperwork that must also be completed, the employee spends only four hours per day on the phone. To balance the schedule, the employee alternates the two-hour shifts between answering phones and completing paperwork. Full-time employees can start their day either by answering phones or by completing paperwork on the first shift. The full-time employees speak either Spanish or English, but none of them are bilingual. Both Spanish-speaking and English-speaking employees are paid $10 per hour for work before 5 PM and $12 per hour for work after 5 PM. The full-time employees can begin work at the beginning of the 7 AM to 9 AM shift, 9 AM to 11 AM shift, 11 AM to 1 PM shift, or 1 PM to 3 PM shift. The part-time employees work for four hours, only answer calls, and only speak English. They can start work at the beginning of the 3 PM to 5 PM shift or the 5 PM to 7 PM shift, and, like the full-time employees, they are paid $10 per hour for work before 5 PM and $12 per hour for work after 5 PM.

For the following analysis, consider only the labor cost for the time employees spend answering phones. The cost for paperwork time is charged to other cost centers.

a. How many Spanish-speaking operators and how many English-speaking operators does the hospital need to staff the call center during each two-hour shift of the day in order to answer all calls? Please provide an integer number since half a human operator makes no sense.

b. Lenny needs to determine how many full-time employees who speak Spanish, full-time employees who speak English, and part-time employees he should hire to begin on each shift. Creative Chaos Consultants advises him that linear programming can be used to do this in such a way as to minimize operating costs while answering all calls. Formulate a linear programming model of this problem.

c. Obtain an optimal solution for the linear programming model formulated in part *b* to guide Lenny's decision.

d. Because many full-time workers do not want to work late into the evening, Lenny can find only one qualified English-speaking operator willing to begin work at 1 pm. Given this new constraint, how many full-time English-speaking operators, full-time Spanish-speaking operators, and part-time operators should Lenny hire for each shift to minimize operating costs while answering all calls?

e. Lenny now has decided to investigate the option of hiring bilingual operators instead of monolingual operators. If all the operators are bilingual, how many operators should be working during each two-hour shift to answer all phone calls? As in part *a,* please provide an integer answer.

f. If all employees are bilingual, how many full-time and part-time employees should Lenny hire to begin on each shift to minimize operating costs while answering all calls? As in part *b,* formulate a linear programming model to guide Lenny's decision.

g. What is the maximum percentage increase in the hourly wage rate that Lenny can pay bilingual employees over monolingual employees without increasing the total operating costs?

h. What other features of the call center should Lenny explore to improve service or minimize operating costs?

Source: This case is based on an actual project completed by a team of master's students in what is now the Department of Management Science and Engineering at Stanford University.