Equilibrium price and quantity

Demand and supply curves are graphical representations of the relationships between price and quantity, all else equal. Sometimes it is useful to characterize these relationships as straight lines, rather than the curved lines as in Figure 3.6 from the text. Once we know the equations for these relationships, it is a straightforward exercise in algebra to determine the equilibrium price and quantity.

The general equation for a linear (straight-line) demand curve is $P = a - bQ_D$. (You will note this conforms with the economics convention of placing the price on the vertical axis and the quantity demanded on the horizontal axis, rather than our usual interpretation of quantity as a function of price. This convention can be traced to Alfred Marshall.) Mathematically, a is the vertical axis intercept and $-b$ is the slope. Economically, if price goes as high as \$$a$, consumers will demand zero units; b is the rate at which the price must fall for the quantity demanded to increase by one unit. Clearly, a must be positive, and the minus sign on b indicates that quantity demanded and the price are inversely related.

The general equation for a linear supply curve is $P = c + dQ_S$, where c is the vertical axis intercept of the supply curve and d is its slope. Economically, producers will offer zero units for sale if the price drops as low as \$$c$, and d is the rate at which the price must rise for suppliers to produce and offer for sale one more unit. As with demand, the intercept c is positive, and the positive sign on d indicates a direct relationship between price and quantity demanded.

To find the equilibrium price, we simply add the condition that quantity demanded equals quantity supplied in equilibrium: $Q_D = Q_S = Q_E$. Substituting Q_E into the demand and supply equations and noting that there is just one equilibrium price, we can set the two equations equal to each other: $a - bQ_E = c + dQ_E$ and solve for Q_E. First, add bQ_E and subtract c on both sides to obtain $a - c = bQ_E + dQ_E$. If we factor out the quantity from the right side, then divide through both sides by $(b + d)$, we obtain the final result: $Q_E = \dfrac{a-c}{b+d}$.

As long as the greatest price consumers willingly pay (a) exceeds the lowest price at which producers will offer some output (c), the equilibrium quantity will be positive: $\dfrac{a-c}{b+d} > 0$.

If we insert this value for the equilibrium quantity into either the demand or supply equation, we can determine the equilibrium price: $P_E = \dfrac{ad+bc}{b+d}$. Since a, b, c, and d are all positive, equilibrium price will also be positive.

Consider the following numerical example:

$$P = 20 - 0.6Q_D$$
$$P = 8 + 0.4\,Q_S$$
$$Q_D = Q_S = Q_E$$

In this market, $a = 20$, $b = 0.6$, $c = 8$, and $d = 0.4$. Inserting these values into our general solutions yields an equilibrium quantity of $Q_E = \dfrac{20-8}{0.6+0.4} = 12$, and an equilibrium price of $P_E = \dfrac{20\text{x}0.4+8\text{x}0.6}{0.6+0.4} = \$12.80$.