

Profit maximization

A firm's profit equals its total revenue minus its total cost, all of which are functions of the firm's output. In symbols, $\pi(Q) = R(Q) - C(Q)$. A firm that wishes to maximize its profits may find the corresponding output by differentiating $\pi(Q)$ with respect to output and finding the output that equates the derivative to zero: $d\pi(Q)/dQ = dR(Q)/dQ - dC(Q)/dQ = MR - MC = 0$. That is, profit maximization requires that, if the firm chooses to produce anything at all, it should equate marginal revenue and marginal cost. In the specific case of competitive firms, this takes the form $P = MC$.

The second-order condition requires that $\frac{d^2 R(Q)}{dQ^2} - \frac{d^2 C(Q)}{dQ^2} < 0$, or that marginal cost cuts marginal revenue from below.

While equating marginal revenue with marginal cost provides a maximum value for the firm's profits, this maximum may be only a *local* maximum. Further, the value of profits at this local maximum need not even be positive. To see if $MR = MC$ provides a *global* maximum, we also need to check profits at an output of zero. If output is zero, then revenue and variable cost are also zero. The firm's profit equals $\pi(0) = 0 - (FC + 0) = -FC$; that is, the firm loses an amount equal to its fixed costs. Naturally, then, the firm would only select this option if its losses at the $MR = MC$ output were greater than its fixed costs. Let Q^* be the output that satisfies the $MR = MC$ rule. Total profits at this point will be $\pi(Q^*) = PQ^* - (FC + VC(Q^*))$. Profits at $Q = 0$ will be higher than at $Q = Q^*$ if and only if $\pi(0) = -FC > PQ^* - (FC + VC(Q^*)) = \pi(Q^*)$. Dividing both sides of this inequality by Q^* and rearranging, the firm maximizes profit at $Q = 0$ if and only if $P < \frac{VC(Q^*)}{Q^*}$, that is, if and only if the firm's price is less than its

average variable cost at the $MR = MC$ output. Since average variable cost is equal to marginal cost at minimum average variable cost, we can state the complete short-run profit-maximizing rule as follows: produce the output for which $MR = MC$, provided price is greater than minimum average variable cost; otherwise shut down.