The case for equality

Suppose we have two individuals with the same utility function, defined over their incomes: $U_A = U(I_A)$ and $U_B = U(I_B)$. This utility function is increasing in income, but is subject to diminishing marginal utility. That is, $U'(I) > 0$ and $U''(I) < 0$. There is a fixed amount of income, I, to be distributed between persons A and B and we wish to distribute this income between them so as to maximize the combined utility $U = U_A + U_B$.

To begin, we note that B's income is the amount left over after A gets I_A. That is, $I_B = I - I_A$. With this substitution, we wish to maximize $U = U(I_A) + U(I - I_A)$ with respect to I_A. Take the first derivative, and equate to zero: $\frac{\mathrm{d}U}{\mathrm{d}I_A} = U'(I_A) - U'(I - I_A) = 0$. As $U''(I)$ is strictly negative, this condition can hold only if $I_A = I - I_A$, or $I_A = \frac{1}{2}I$. That is, maximum combined utility is achieved only when each individual gets exactly half the combined available income.