The total-revenue test

The text suggests a test for demand elasticity based on the relationship between price and elasticity. The basis for this test is a direct mathematical link between price, revenue, and elasticity.

What we require for our test is a relationship between the change in revenue and the change in price that brought it about. As hoped, this relationship will depend on the elasticity of demand.

Before we find this relationship, we require a slight modification of our elasticity formula. We begin by writing the demand relationship as $Q = f(P)$. We know that the elasticity of demand is given by $E_d = -\frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$. (The minus sign accounts for the fact that demand elasticities are usually stated as positive values, even though $\frac{\Delta Q}{\Delta P}$ is negative.) Taking the limit as ΔP approaches 0, the elasticity becomes $E_d = -\frac{dQ}{dP} \times \frac{P}{Q}$. Note that the first term, $dQ/dP = f'(P)$, is the (inverse of the) slope of the demand relationship. That is, elasticity of demand can be written as $E_d = -f'(P) \times \frac{P}{Q}$.

To continue, we note that revenue, R, is defined as price times quantity: $R = PQ$. If we substitute $f(P)$ for Q, then $R = Pf(P)$. The relationship we require is dR/dP. Using the multiplication rule for differentiation, $dR/dP = f'(P) \text{ x } P + f(P)$. However, $f(P)$ is just Q: $dR/dP = f'(P) \text{ x } P + Q$. By both multiplying and dividing by Q, this can be rewritten as $dR/dP = Q(f'(P) \times \frac{P}{Q} + 1)$. From our earlier result, we can now substitute the elasticity of demand for $f'(P) \times \frac{P}{Q}$ to obtain $dR/dP = Q(-E_d + 1) = Q(1 - E_d)$.

The total-revenue test is a straightforward application of this result: $dR/dP = Q(1 - E_d)$. If demand is elastic, $E_d > 1$, then $dR/dP < 0$: price and total revenue move in opposite directions. If demand is inelastic, $E_d < 1$, then $dR/dP > 0$: price and total revenue move in the same direction. Finally, if demand is unit elastic, $E_d = 1$, then $dR/dP = 0$: an increase in price leaves total revenue unchanged.