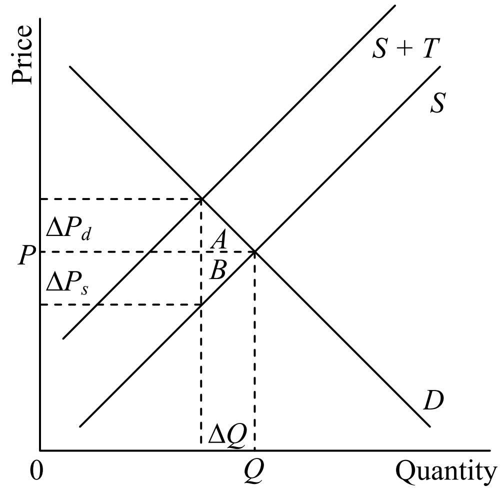
Elasticity and the efficiency loss of a tax



Consider the adjacent graph, which shows the impact of a unit tax of $\$T$ in a competitive market. Initially, the equilibrium price is P, and the equilibrium quantity is Q. The imposition of the tax causes the equilibrium quantity to fall by ΔQ, and the price to consumers increases by ΔP_d while the price to sellers falls by ΔP_s. The **efficiency loss** of the tax is given by the sum of the two triangles labeled A and B on the diagram. For convenience, call the efficiency loss Z, so $Z = A + B$. As stated in the text, the size of this loss increases with the elasticities of either supply or demand. This note will develop a formula for the size of the efficiency loss so that we may show the dependency of this area on the two elasticities.

We begin by noting that the area of a triangle is one half the base times the height. In this example, each triangle has base equal to ΔQ. Triangle A has height ΔP_d, while triangle B has height ΔP_s. The efficiency loss is therefore $Z = A + B = ½\Delta Q\Delta P_d + ½\Delta Q\Delta P_s = ½\Delta Q(\Delta P_d + \Delta P_s) = ½\Delta QT$. This last equality follows because the tax, T, must be the difference between the price paid by consumers and the price received by sellers. The size of the loss clearly depends on T, but we are left with some uncertainty because we do not yet know the size of ΔQ. We suspect that it relates to the elasticities of supply and demand, so that is our next step.

Recall that the elasticity of demand, E_d, can be written as $E_d = \dfrac{\Delta Q / Q}{\Delta P_d / P} = \dfrac{\Delta Q}{\Delta P_d}\dfrac{P}{Q}$. Suppose we solve this for ΔP_d as follows: $\Delta P_d = \dfrac{P}{Q}\dfrac{\Delta Q}{E_d}$. Likewise, we could find that $\Delta P_s = \dfrac{P}{Q}\dfrac{\Delta Q}{E_s}$. We know that the total change in the two prices, $\Delta P_d + \Delta P_s$, is equal to the tax, so $T = \dfrac{P}{Q}\dfrac{\Delta Q}{E_d} + \dfrac{P}{Q}\dfrac{\Delta Q}{E_s}$. If we multiply and divide the first term in this sum by E_s and the second term by E_d, we get a common denominator and can add the two terms to get $T = \dfrac{P\Delta QE_s + P\Delta QE_d}{QE_dE_s} = \dfrac{P\Delta Q}{Q}\left(\dfrac{E_s + E_d}{E_dE_s}\right)$.

Now what we need is ΔQ, so we solve this last expression in terms of ΔQ to get $\Delta Q = \dfrac{TQ}{P}\left(\dfrac{E_dE_s}{E_d + E_s}\right)$. As suspected, the change in quantity depends on the size of the tax and the two elasticities. We can now plug this value of ΔQ into our formula for the efficiency loss, $Z = ½\Delta QT = \dfrac{½T^2Q}{P}\left(\dfrac{E_dE_s}{E_d + E_s}\right)$.

Notice that the size of this loss increases with the *square* of the tax. That is, doubling the size of the tax will increase the size of the efficiency loss by a factor of four. To find how the elasticities affect Z, we can take the partial derivatives of Z with respect to E_d and E_s to find $\dfrac{\partial Z}{\partial E_d} = \dfrac{½T^2Q}{P}\left(\dfrac{E_s}{E_d + E_s}\right)^2$ and $\dfrac{\partial Z}{\partial E_s} = \dfrac{½T^2Q}{P}\left(\dfrac{E_d}{E_d + E_s}\right)^2$. Clearly these are both positive, so that all else constant, the more elastic is either demand or supply, the greater the size of the efficiency loss.