## GENERAL EQUILIBRIUM and Market efficiency 0

Barbers earn more today than they did 50 years ago, not because they cut hair any faster than they did then but because productivity has grown so rapidly in the other occupations they could have chosen. By the same token, $8 \frac{1}{2}^{\prime \prime} \times 11^{\prime \prime}$ paper now sells in much greater quantities, not because we have discovered a cheaper way to produce it but because so many more people now own their own printers and copying machines. And we know that when a frost kills half the coffee crop in Brazil, the price of tea grown in Darjeeling usually rises substantially.

In the preceding chapters we saw occasional glimpses of the rich linkages between markets in the real world. But for the most part, we ignored these linkages in favor of what economists call partial equilibrium analysis-the study of how individual markets function in isolation. One of our tasks in this chapter is to investigate the properties of an interconnected system of markets. This is called general equilibrium analysis, and its focus is to make explicit the links that exist between individual markets. It takes into account, for example, the fact that inputs supplied to one market are unavailable for any other and that an increase in demand in one market implies a reduction in demand in others.

## CHAPTER PREVIEW

We will begin with one of the simplest forms of general equilibrium analysis, a pure exchange economy with only two consumers and two goods. We will see that for any given initial allocation of the two goods between the two consumers,
general equilibrium analysis the study of how conditions in each market in a set of related markets affect equilibrium outcomes in other markets in that set.
a competitive exchange process always exhausts all possible mutually beneficial gains from trade.

Next we will add the possibility of production, again using one of the simplest possible models, one with only two inputs whose total supply is fixed. We will see that here too competitive trading exploits all mutually beneficial gains from exchange.

We will then add the possibility of international trade, assuming that prices are given externally in world markets. We will see that even though trade leaves domestic production possibilities unchanged, its immediate effect is to increase the value of goods available for domestic consumption.

From trade, we will move to the question of how taxes affect the allocation of resources. We will conclude with a brief discussion of factors that interfere with the efficient allocation of resources.

## A SIMPLE EXCHANGE ECONOMY

Imagine a simple economy in which there are only two consumers-Ann and Billand two goods, food and clothing. Food and clothing are not produced in this economy. Rather, they arrive in fixed quantities in each time period, just like manna from heaven. To help fix ideas, suppose there is a total of 100 units of food each time period and a total of 200 units of clothing. An allocation is defined as an assignment of these total amounts between Ann and Bill. An example is the allocation in which Ann receives 70 units of clothing and 75 units of food, with the remaining 130 units of clothing and 25 units of food going to Bill. In general, if Ann receives $F_{A}$ units of food and $C_{A}$ units of clothing, then Bill will get $100-F_{A}$ units of food and $200-C_{A}$ units of clothing. The amounts of the two goods with which Ann and Bill begin each time period are called their initial endowments.

In the next section we'll have more to say about where these initial endowments come from, but for now let's take them as externally determined. The question before us here is "What will Ann and Bill do with their initial endowments?" One possibility is that they might simply consume them, but only in rare circumstances will that be the best option available. To see why, it is helpful to begin by portraying the initial endowments diagrammatically. Consider again the case in which Ann receives 70 units of clothing and 75 units of food, with the remaining 130 units of clothing and 25 units of food going to Bill. From earlier chapters, we know how to represent these initial endowments as bundles in two separate food-clothing diagrams. The same allocation can also be represented as a point in a single rectangular diagramnamely, point $R$ in Figure 18W.1. The height of the rectangle corresponds to the total

FIGURE I8W.I
An Edgeworth

## Exchange Box

A's quantity of food at any point is measured by how far the point lies above $O^{A}$. A's clothing is measured by how far the point lies to the right of $O^{A}$. B's clothing is measured leftward from $O^{B}$, and his food downward from $O^{B}$. At any point within the Edgeworth box, the individual quantities of food and clothing sum to the total amounts available.

amount of food available per time period, 100 units. Its width is equal to the total amount of clothing, 200 units. $\mathrm{O}^{A}$ is the origin for Ann, and the left and bottom sides of the rectangle are the axes that measure her quantities of food and clothing, respectively. $O^{B}$ is the origin for Bill, and movements to the left from $O^{B}$ correspond to increases in his amount of clothing. Downward movements from $O^{B}$ correspond to increases in Bill's amount of food.

Because of the special way the rectangle is constructed, every point that lies within it corresponds to an allocation that exactly exhausts the total quantities of food and clothing available. Thus, point $R$ is 70 units to the right of $O^{A}$ and 130 units to the left of $O^{B}$, which means 70 units of clothing for Ann and 130 units for Bill, for a total of 200. $R$ also lies 75 units above $O^{A}$ and 25 units below $O^{B}$, which means 75 units of food for Ann and 25 for Bill, for a total of 100. The rectangular diagram in Figure 18W. 1 is often referred to as an Edgeworth exchange box, after the British economist Francis Y. Edgeworth, who introduced it.

## EXERCISE 18W.I Suppose point $S$ in Figure 18 W .I lies 25 units above $O^{A}$ and $\mathbf{2 5}$ units to the right of $O^{A}$. Verify that Bill's initial endowment at $S$ is $\mathbf{7 5}$ units of food and 175 units of clothing.

If Ann and Bill have the initial endowments represented by $R$, what will they do with them? Their possibilities are either to consume what they already have or to engage in exchange with one another. Exchange is purely voluntary, so trades can take place only if they make both parties better off.

Our criterion for saying an exchange makes someone better off is very simple: It must place him on a higher indifference curve. In the Edgeworth box in Figure 18W.2, Ann's indifference map has the conventional orientation, while Bill's is rotated $180^{\circ}$. Thus the curves labeled $I_{A 1}, I_{A 2}$, and $I_{A 3}$ are representative curves from Ann's indifference map, while $I_{B 1}, I_{B 2}$, and $I_{B 3}$ play the corresponding role for Bill. Ann's satisfaction increases as we move to the northeast in the box; Bill's as we move to the southwest.

Because we assume that preference orderings are complete, we know that each party will have an indifference curve passing through the initial endowment point $R$. In Figure 18W. 2 these curves are labeled $I_{A 2}$ and $I_{B 2}$. Note that Ann's MRS between food and clothing at $R$ (that is, the slope of her indifference curve) is much

Edgeworth exchange box a diagram used to analyze the general equilibrium of an exchange economy.


FIGURE I8W. 2
Gains from Exchange
By moving from $R$ to $T$, each party attains a higher indifference curve.

FIGURE I8W. 3
Further Gains from

## Exchange

Any point in the shaded region lies on a higher indifference curve for both parties than the ones that pass through $T$.
larger than Bill's (where the MRS for Bill is measured with respect to his own food and clothing axes). Suppose, for example, that Ann requires 2 units of food in order to be willing to part with a unit of clothing, while Bill requires only $\frac{1}{2}$ unit of food to make the same exchange. Both parties will then be better off if Ann gives Bill a unit of food in exchange for a unit of clothing. Indeed, any point in the lens-shaped shaded region in Figure 18 W .2 is one for which each party lies on a higher indifference curve than at $R$. Point $T$, at which Ann has 65 units of food and 85 units of clothing, is one such point. The two parties can move from $R$ to $T$ by having Ann give Bill 10 units of food in exchange for 15 units of clothing.

But the movement from $R$ to $T$ does not exhaust all possible gains from exchange. Note in Figure 18W. 3 that there is an additional, albeit smaller, lens-shaped region enclosed by the indifference curves that pass through $T$ by both parties.
would necessarily enclose yet another lens-shaped region in which further gains from exchange would be possible.) Note that at $M$ the marginal rates of substitution of Ann and Bill are exactly the same. It was a difference in these rates that provided the original basis for exchange, and once they are the same, all voluntary trading will cease.

One allocation is said to be Pareto preferred or Pareto superior to another if at least one party prefers it and the other party likes it at least as well. Allocations like the one at $M$ are called Pareto optimal. A Pareto-optimal allocation is one for which there is no other feasible reallocation that is preferred by one party and liked at least equally well by the other party. The concept of Pareto optimality was introduced by the nineteenth-century Italian economist Vilfredo Pareto. Pareto-optimal allocations are essentially ones from which further mutually beneficial moves are impossible.

> EXERCISE I8W. 2 Suppose Ann has an initial allocation of 50 units of food and 100 units of clothing in Figure 18W.4. She regards food and clothing as perfect, I-for-I substitutes. Bill regards them as perfect, I-for-I complements, always wanting to consume I unit of clothing for every unit of food. Describe the set of allocations that are Pareto preferred to the initial allocation.

In any Edgeworth exchange box, there will be not one but an infinite number of mutual tangencies, as illustrated in Figure 18W.5. The locus of these tangencies is called the contract curve, a name that was chosen because it describes where all final, voluntary contracts between rational, well-informed persons must lie. Put another way, the contract curve identifies all the efficient ways of dividing the two goods between the two consumers.
pareto superior allocation an allocation that at least one individual prefers and others like at least as well.
pareto optimal the term used to describe situations in which it is impossible to make one person better off without making at least some others worse off.
contract curve a curve along which all final, voluntary contracts must lie.


FIGURE I8W. 5
The Contract Curve
The locus of mutual tangencies in the Edgeworth exchange box is called the contract curve. Any point that does not lie on the contract curve cannot be the final outcome of a voluntary exchange because both parties will always prefer a move from that point in the direction of the contract curve.

Where Ann and Bill end up on the contract curve naturally depends on the initial endowments with which they start. Suppose they start with the one labeled $F$ in Figure 18 W .6 . We can then say that they will end up somewhere on the contract curve between points $U$ and $V$. Given that they are starting from $F$, the best possible outcome from Ann's point of view is to end up at $V$. Bill, of course, would most prefer $U$. Whether they end up closer to $U$ or to $V$ depends on the relative bargaining skills of the two traders. Had they instead started at the allocation $G$, they would have ended up between $W$ and $Z$ on the contract curve.

FIGURE I8W. 6 Initial Endowments Constrain Final Outcomes
Starting from $F$, traders will move to a point on the contract curve between $U$ and $V$. They will land closer to $V$ the better Ann's bargaining skills are relative to Bill's. If they start at $G$, they will end up between $W$ and $Z$ on the contract curve.


The uses and limitations of the two Pareto criteria-Pareto preferred and Pareto optimal-can be seen by an examination of some of the points in Figure 18W.6. Note, for example, that both $W$ and $Z$ are Pareto preferred to the original allocation $G$. This follows because $W$ is better than $G$ from Bill's point of view and no worse from Ann's; and similarly, $Z$ is better from Ann's point of view and no worse from Bill's. Note that both points are also Pareto optimal. The two Pareto criteria are essentially relative in nature. Thus, when we say that $U$ is Pareto preferred to $F$, or even when we say that $U$ is Pareto optimal, we are not saying that $U$ is good in any absolute sense. On the contrary, Ann is hardly likely to find $U$ very attractive, and it is certainly much worse, from her standpoint, than an allocation like $G$, which is neither Pareto optimal nor even Pareto preferred to $U$. If Ann is starving to death in a tattered coat at $U$, she will not take much comfort in being told that $U$ is Pareto optimal.

The Pareto criteria thus have force only in relation to the allocation with which the two players begin. Rather than remain at an initial allocation, both will always agree to move to one that is Pareto preferred and, indeed, to keep on moving until they reach one that is Pareto optimal.

In the simple, two-person economy described above, exchange took place through a process of personal bargaining. In market economies, by contrast, most exchanges have a much more impersonal character. People have given endowments and face given prices, and then decide how much of the various goods and services they want to buy and sell. We can introduce market-type exchange into our simple economy by the simple expedient of assuming that there is a third person who plays the role of an auctioneer. His function is to keep adjusting relative prices until the quantities demanded of each good match the quantities supplied.

Suppose Ann and Bill start with the allocation at $E$ in Figure 18 W .7 , in which each has 50 units of food and 100 units of clothing. Suppose also that the ratio of food to clothing prices announced by the auctioneer is $P_{C 0} / P_{F 0}=1$, meaning that food and clothing both sell for the same price. When the prices of the two goods are the same, the auctioneer stands ready to exchange 1 unit of clothing for 1 unit of food. (More generally, he will exchange clothing for food at the rate of $P_{C 0} / P_{F 0}$ units of food for each unit of clothing.) Note that with the given initial endowments, this rate of exchange uniquely determines the budget constraints for both Ann and Bill. We know that $E$ has to be a point on each person's budget constraint because each has the option of simply consuming all of his or her initial endowment. But suppose that Ann wants to sell some food and use the proceeds to buy more clothing. She can do this by moving downward from $E$ along the line labeled $H H^{\prime}$. Alternatively, if she wants to sell clothing to buy more food, she can move upward along $H H^{\prime}$. The same $H H^{\prime}$, seen from Bill's point of view, constitutes the budget constraint for Bill.


Given their budget constraints and preferences, Ann and Bill face a simple choice problem of the sort we discussed in Chapter 3. The optimal bundle for Ann on the budget $H H^{\prime}$ is the one labeled $A_{0}^{*}$ in Figure 18W.7, in which she consumes 30 units of food and 120 units of clothing. The corresponding bundle for Bill is labeled $B_{0}^{*}$, and it too contains 30 units of food and 120 units of clothing. Note that by choosing $A_{0}^{*}$, Ann indicates that she wants to sell 20 units of her initial endowment of food in order to buy 20 units of additional clothing. Similarly, by choosing $B_{0}^{*}$, Bill indicates that he too wants to sell 20 units of food and buy 20 more units of clothing.

This creates a problem, however. There are only 200 units of clothing to begin with, and the initial endowments of clothing at $E$ add to precisely that amount. It is thus mathematically impossible for each person to have more clothing. By the same token, it is not possible for each person to sell food. The auctioneer in this exercise is a figment, a hypothetical person who calls out relative prices in the hope of stimulating mutually beneficial exchange. He acts as a middleman, arranging for clothing to be exchanged for an equivalent value of food. But if everyone wants to sell food and buy clothing, there is no such exchange he can arrange.

At the price ratio $P_{C 0} / P_{F 0}=1$, there is excess demand for clothing and excess supply of food. At this price ratio the markets are not in general equilibrium. The solution to this problem is straightforward: The auctioneer simply calls out a new price ratio in which the price of clothing relative to food is higher than before. If there is still excess demand for clothing, he calls out a still higher price ratio, and so on, until the excess demand in each market is exactly zero. ${ }^{1}$ Starting with the allocation at $E$, the price ratio $\left(P_{\mathrm{C}} / P_{F}\right)^{*}$ that produces general equilibrium is shown in Figure 18 W .8 . On the budget line through $E$ with slope $\left(P_{C} / P_{F}\right)^{*}$, the highest attainable indifference curves for Ann and Bill are tangent. In order to move from $E$ to the bundle $A^{*}$, Ann must purchase exactly the quantity of food (12 units) that Bill wishes to sell. And for Bill to move from $E$ to the bundle $B^{*}$, he must purchase exactly the quantity of clothing (10 units) that Ann wishes to sell. In this illustration, excess demands for both products are exactly equal to zero at the price ratio $\left(P_{C} / P_{F}\right)^{*}=\frac{6}{5}$.

[^0]FIGURE 18W. 7 A Disequilibrium Relative Price Ratio
At the price ratio $P_{c 0} / P_{F 0}=1$, both Ann and Bill want to sell 20 units of food and buy 20 more units of clothing. But in general equilibrium, the amount sold by one party must equal the amount bought by the other. Both the food and clothing markets are out of equilibrium here.

FIGURE I8W. 8

General Equilibrium
A simple exchange economy is in equilibrium when excess demands for both products are exactly equal to zero. At the price ratio $\left(P_{C} / P_{F}\right)^{*}=\frac{6}{5}$, Ann wants to buy 12 units of food, which is exactly the amount Bill wants to sell; also, Ann wants to sell 10 units of clothing, which is exactly the amount Bill wants to buy.


## Only Relative Prices Are Determined

From the information given in our simple exchange model, note that we are able to determine only the ratio of clothing to food prices, not the actual value of individual prices. If, for example, $P_{C}=6$ and $P_{F}=5$ produce a budget constraint with the slope shown in Figure 18 W .8 , then so will the prices $P_{C}=12$ and $P_{F}=10$, or indeed any other pair of prices whose ratio is $\frac{6}{5}$. Doubling or halving all prices will double or halve the dollar value of each consumer's initial endowment. In real terms, such price movements leave budget constraints unchanged.

## The Invisible Hand Theorem

We are now in a position to consider one of the most celebrated claims in intellectual history, namely, Adam Smith's theorem of the invisible hand. In the context of our simple exchange economy, the theorem can be stated as follows:

An equilibrium produced by competitive markets will exhaust all possible gains from exchange.

The invisible hand theorem is also known as the first theorem of welfare econom$i c s$, and an alternative way of stating it is that equilibrium in competitive markets is Pareto optimal. To see why this must be so, recall that at the general equilibrium allocation, the optimizing indifference curves are tangent to one another. The possible allocations that Ann regards as better than the equilibrium allocation all lie beyond her budget constraint, and the same is true for Bill. And since the two budget constraints coincide in the Edgeworth box, this means that there is no allocation that both prefer to the equilibrium allocation, which is just another way of saying that the equilibrium allocation is Pareto optimal.

The invisible hand theorem tells us that every competitive equilibrium allocationlike $D$ in Figure 18W.9-is efficient. But suppose you are a social critic and don't like that particular allocation; you feel that Bill gets too much of each good and Ann too little. The problem, in your view, was that the initial endowment point- $J$ in the diagram-is unjustly favorable to Bill. Suppose there is some other allocation on the contract curve-such as $E$-that you find much more equitable. Is there a set


FIGURE I8W. 9 Sustaining Efficient Allocations
If indifference curves are convex, any efficient allocation can be sustained through a suitable choice of initial endowments and relative prices. To sustain $E$, for example, we announce a relative price ratio equal to the slope of $H H^{\prime}$, the mutual tangent to $I_{A 2}$ and $I_{B 2}$, and give consumers an initial endowment bundle that lies anywhere on $\mathrm{HH}^{\prime}$, such as M .
of initial endowments and relative prices for which $E$ will be a competitive equilibrium? The second theorem of welfare economics says that, under relatively unrestrictive conditions:

Any allocation on the contract curve can be sustained as a competitive equilibrium.
The basic condition that assures this result is that consumer indifference curves be convex when viewed from the origin. We know that an allocation like $E$, or any other efficient allocation, lies at a point of tangency between indifference curves. In Figure 18 W .9 , the locus $H H^{\prime}$ is the mutual tangent between $I_{A 2}$ and $I_{B 2}$. If the difference curves are convex, any initial endowment along $H H^{\prime}$-such as $M$-will lead to a competitive equilibrium at $E$. If we redistribute the initial endowments from $J$ to $M$, and announce a price ratio equal to the slope of $H H^{\prime}$, Ann and Bill will then be led by the invisible hand to $E$. Indeed, any point along the contract curve can be reached in this fashion by a suitable choice of initial endowments and relative prices.

In the context of this simple, two-good, two-person exchange economy, it may not seem like a major accomplishment to be able to sustain all efficient allocations in the manner described by the second welfare theorem. After all, if we are free to redistribute initial endowments, why not simply redistribute them so as to achieve the desired final outcome directly? Why even bother with the intermediate steps of announcing prices and allowing people to make trades? If we are free to move from $J$ to $M$ in Figure 18W.9, then we ought to be able to move directly to $E$ and cut out the intervening steps.

The difficulty in practice is that the social institutions responsible for redistributing income have little idea of the shapes and locations of individual consumer indifference curves. People know their own preferences much better than governments do. And for an initial endowment of given value, they will generally achieve a much better result if they are free to make their own purchase decisions. The significance of the second welfare theorem is that the issue of equity in distribution is logically separable from the issue of efficiency in allocation. As the nineteenthcentury British economist John Stuart Mill saw clearly, society can redistribute incomes in accordance with whatever norms of justice it deems fitting, at the same time relying on market forces to assure that those incomes are spent to achieve the most good.

## EFFICIENCY IN PRODUCTION

In our simple exchange model, the total supply of each good was given externally. In practice, however, the product mix in the economy is the result of purposeful decisions about the allocation of productive inputs. Suppose we now add a productive sector to our exchange economy, one with two firms, each of which employs two inputs-capital $(K)$ and labor $(L)$-to produce either of two products, food $(F)$ or clothing ( $C$ ). Suppose firm $C$ produces clothing and firm $F$ produces food. In order to keep the model simple, suppose that the total quantities of the two inputs are fixed at $K=50$ and $L=100$, respectively. Suppose, finally, that the production processes employed by the two firms give rise to conventional, convexshaped isoquants.

Just as the Edgeworth exchange box provided a convenient way to summarize the conditions required for efficiency in consumption, a similar analysis serves an analogous purpose in the case of production. Figure 18 W .10 is called an Edgeworth production box. $O^{C}$ represents the origin of the clothing firm's isoquant map, $O^{F}$ the origin of the food firm's. Any point within the box represents an allocation of the total inputs to firm $C$ and firm $F$. Firm C's isoquants correspond to increasing quantities of clothing as we move to the northeast in the box; firm F's correspond to increasing quantities of food as we move to the southwest.

FIGURE 18W. 10
An Edgeworth Production Box
Firm C's quantity of capital at any point is measured by how far the point lies above $O^{C}$. Firm C's quantity of labor is measured by how far the point lies to the right of $O^{c}$. The corresponding values of firm F's inputs are measured downward and leftward, respectively, from $O^{F}$. At any point within the Edgeworth production box, the separate input allocations to the two firms add up to the total amounts available, $K=50$ for capital, $L=100$ for labor. The contract curve is the locus of tangencies between isoquants.


Suppose the initial allocation of inputs is at point $R$ in Figure 18W.10. We know that this allocation cannot be efficient because we can move to any point within the shaded lens-shaped region and obtain both more food and more clothing. As in the consumption case, the contract curve is the locus of efficient allocations, which here is the locus of tangencies between isoquants. Recalling from Chapter 9 that the slope of an isoquant at any point is called the marginal rate of technical substitution (MRTS) at that point, it is the ratio at which labor can be exchanged for capital without altering the total amount of output. Note that the MRTS between $K$ and $L$ must be the same for both firms at every point along the contract curve.

Suppose the equilibrium food and clothing prices are $P_{F}^{*}$ and $P_{C}^{*}$, respectively. Suppose also that the two firms hire labor and capital in perfectly competitive markets at the hourly rates of $w$ and $r$, respectively. If the firms maximize their profits, is there any reason to suppose that the resulting general equilibrium will satisfy the requirements of efficiency in production? That is, is there any reason to suppose that the MRTS between capital and labor will be the same for each firm? If both firms have conventional, convex-shaped isoquants, the answer is yes.

To see why, first note that a firm that maximizes its profits must also be minimizing its costs. Recall from Chapter 10 that the following conditions must be satisfied if the firms are minimizing costs:

$$
\begin{equation*}
\frac{\mathrm{MPL}_{C}}{\mathrm{MPK}_{C}}=\frac{w}{r} \tag{18W.1}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\mathrm{MPL}_{F}}{\mathrm{MPK}_{F}}=\frac{w}{r}, \tag{18W.2}
\end{equation*}
$$

where $\mathrm{MPL}_{C}$ and $\mathrm{MPK}_{C}$ are the marginal products of labor and capital in clothing production and $\mathrm{MPL}_{F}$ and $\mathrm{MPK}_{F}$ are the corresponding marginal products in food production. Recall, too, that the ratio of marginal products of the two inputs is equal to the marginal rate of technical substitution. Since both firms pay the same prices for labor and capital, Equations 18 W .1 and 18 W .2 tell us that the marginal rates of technical substitution for the two firms will be equal in competitive equilibrium. And this tells us, finally, that competitive general equilibrium is efficient, not only in the allocation of a given endowment of consumption goods, but also in the allocation of the factors used to produce those goods.

EXERCISE I8W. 3 For an economy like the one described above, suppose the price per unit of labor and the price per unit of capital are both equal to $\$ 4 / \mathrm{hr}$. Suppose also that in clothing production we have $M P L_{c} / M P K_{c}=2$ and that in food production we have MPL $/$ MPK $_{\mathrm{F}}=\frac{1}{2}$. Is this economy efficient in production? If not, how should it reallocate its inputs?

## EFFICIENCY IN PRODUCT MIX

An economy could be efficient in production and at the same time efficient in consumption and yet do a poor job of satisfying the wants of its members. This could happen if, for example, the economy for some reason devoted almost all its resources to producing clothing, almost none to food. The tiny quantity of food that resulted could be allocated efficiently. And the inputs could be allocated efficiently in the production of this lopsided product mix. But everyone would be happier if there were less clothing and more food. There is thus one additional efficiency criterion of concern, namely, whether the economy has an efficient mix of the two products.

To define an efficient product mix, it is helpful first to translate the contract curve from the Edgeworth production box into a production possibilities frontier, the set of all possible output combinations that can be produced with given quantities of capital and labor. Every point along the contract curve gives rise to specific quantities of clothing and food. Suppose $F_{C}(K, L)$ and $F_{F}(K, L)$ denote the production functions for clothing (firm $C$ ) and food (firm $F$ ), respectively. Point $O^{C}$ in the top panel in Figure 18W. 11 represents what happens when we
production possibilities frontier the set of all possible output combinations that can be produced with a given endowment of factor inputs.

FIGURE I8W.II
Generating the Production Possibilities

## Frontier

Each point on the contract curve in the Edgeworth production box (top panel) gives rise to specific quantities of food and clothing production. The food-clothing pairs that lie along the contract curve are plotted in the bottom panel, and their locus is called the production possibilities frontier. Movements to the northeast along the contract curve correspond to movements downward along the production possibilities frontier.

allocate all the inputs ( 50 units of capital, 100 units of labor) to food production and none to clothing. If $F_{F}(50,100)=275$, then the product mix to which this allocation gives rise has zero units of clothing and 275 units of food, and is shown by point $O^{C}$ in the bottom panel. Point $O^{F}$ in the top panel in Figure 18W. 11 represents what happens when we allocate all the inputs to clothing production and none to food. If $F_{C}(50,100)=575$, then the product mix to which this allocation gives rise has 575 units of clothing and zero units of food, and is shown by point $O^{F}$ in the bottom panel. The product mix corresponding to point $E$ in the top panel has $F_{C}(14,30)=200$ units of clothing and $F_{F}(36,70)=250$ units of food, and is shown by point $E$ in the bottom panel. Similarly, the product mix at $F$ in the top panel has $F_{C}(22,53)=400$ units of clothing and $F_{F}(28,47)=$ 200 units of food, and corresponds to $F$ in the bottom panel. Likewise, $G$ in the top panel has $F_{C}(38,76)=500$ units of clothing and $F_{F}(12,24)=100$ units of food, and corresponds to $G$ in the bottom panel. By plotting other correspondences in like fashion, we can generate the entire production possibilities frontier shown in the bottom panel.

EXERCISE I8W. 4 In the economy shown in Figure I8W.II, suppose that a technical change occurs in the clothing industry that makes any given combination of labor and capital yield twice as much clothing as before. Show the effect of this change on the production possibilities frontier.

As we move downward along the production possibilities frontier, we give up food for additional clothing. The slope of the production possibilities frontier at any point is called the marginal rate of transformation (MRT) at that point, and it measures the opportunity cost of clothing in terms of food. For the economy shown, the production possibilities frontier bows out from the origin, which means that the MRT increases as we move to the right. As long as both production functions have constant or decreasing returns to scale, the production possibilities frontier cannot bow in toward the origin.

In order for an economy to be efficient in terms of its product mix, it is necessary that the marginal rate of substitution for every consumer be equal to the marginal rate of transformation. To see why, consider a product mix for which some consumer's MRS is greater or less than the corresponding MRT. The product mix $Z$ in panel $a$ in Figure 18 W .12 , for instance, has an MRT of 1 , while Ann's consumption bundle at $W$ in panel $b$ shows that her MRS is 2 . This means that Ann is willing to give up 2 units of food in order to obtain an additional unit of clothing, but that an additional unit of clothing can be produced at a cost of only 1 unit of food. With the capital and labor saved by producing 2 fewer units of food for Ann, we can produce 2 additional units of clothing. We can give 1.5 units of this extra clothing to Ann and the remaining 0.5 unit to Bill, making both parties better off. It follows that the original product mix cannot have been efficient (where, again, efficient means Pareto optimal).


FIGURE I8W.I2

## An Inefficient

Product Mix
At the product mix $Z$ (panel
a) the MRT is smaller than

Ann's MRS at W (panel b). By producing 2 fewer units of food, we can produce 2 additional units of clothing. If we give 1.5 of these extra units to Ann and the remaining 0.5 unit to Bill, both parties will be better off. Efficiency requires that every consumer's MRS be exactly equal to the economy's MRT.
marginal rate of transformation (MRT) the rate at which one output can be exchanged for another at a point along the production possibilities frontier.

We are now in a position to ask, finally, whether a market in general competitive equilibrium will be efficient in terms of its product mix. Here, too, the answer turns out to be yes, provided the production possibilities frontier bows out from the origin. Let $P_{F}^{*}$ and $P_{C}^{*}$ again denote competitive equilibrium prices for food and clothing. As we have already seen in the case of the simple exchange economy, the MRS of every consumer in equilibrium will be equal to the ratio of these prices, $P_{C}^{*} / P_{F}^{*}$. What we must show is that the MRT will also be equal to $P_{C}^{*} / P_{F}^{*}$.

To do this, note first that the MRT at any point along the production possibilities frontier is equal to the ratio of the marginal cost of clothing $\left(\mathrm{MC}_{C}\right)$ to the marginal cost of food $\left(\mathrm{MC}_{F}\right)$. Suppose, for example, that $\mathrm{MC}_{C}$ at point $Z$ in Figure 18W. 13 is $\$ 100 /$ unit of clothing and that $\mathrm{MC}_{F}$ is $\$ 50 /$ unit of food. The marginal rate of transformation at $Z$ is $\Delta F / \Delta C$, the amount of food we have to give up to get an extra unit of clothing. Since $\mathrm{MC}_{C}$ is $\$ 100$, we need $\$ 100$ worth of extra labor and capital to produce an extra unit of clothing. And since $\mathrm{MC}_{\mathrm{F}}$ is $\$ 50$, we have to produce 2 fewer units of food in order to free up $\$ 100$ worth of labor and capital. MRT at $Z$ is therefore equal to 2 , which is exactly the ratio of $\mathrm{MC}_{C}$ to $\mathrm{MC}_{\mathrm{F}}$.

$$
\begin{equation*}
\mathrm{MRT}=\frac{\mathrm{MC}_{C}}{\mathrm{MC}_{F}} . \tag{18W.3}
\end{equation*}
$$

FIGURE I8W.I3
MRT Equals the Ratio of Marginal Costs
At Z, to produce an extra unit of clothing requires $\mathrm{MC}_{C}$ worth of labor and capital. Each unit less of food we produce at $Z$ frees up MC $_{F}$ worth of labor and capital. To get an extra unit of $C$, we must give up $M C_{C} / M C_{F}$ units of food, and so the marginal rate of transformation is equal to $M C_{C} / \mathrm{MC}_{F}$.


We also know that the equilibrium condition for competitive food and clothing producers is that product prices be equal to the corresponding values of marginal cost:

$$
\begin{equation*}
P_{F}^{*}=\mathrm{MC}_{F} \tag{18W.4}
\end{equation*}
$$

and

$$
\begin{equation*}
P_{C}^{*}=\mathrm{MC}_{\mathrm{C}} . \tag{18W.5}
\end{equation*}
$$

Dividing Equation 16.5 by Equation 16.4, we have

$$
\begin{equation*}
\frac{P_{C}^{*}}{P_{F}^{*}}=\frac{\mathrm{MC}_{C}}{\mathrm{MC}_{F}} \tag{18W.6}
\end{equation*}
$$

which establishes that the equilibrium product price ratio is indeed equal to the marginal rate of transformation.

To summarize, we have now established that an economy in competitive general equilibrium will, under certain conditions, be simultaneously efficient (Pareto optimal) in consumption, in production, and in the choice of product mix. As we have already seen, a society with a Pareto-optimal allocation of resources is not necessarily a good society. The final equilibrium in the marketplace depends very strongly on the distribution of initial endowments, and if this distribution isn't fair, we have no reason to expect the competitive equilibrium to be fair. Even so, it is truly remarkable to be able to claim, as Adam Smith did, that each person, merely
by pursuing his own interests, is "led by an invisible hand to promote an end which was no part of his intention"-namely, the exploitation of all gains from exchange possible under given initial endowments.

## GAINS FROM INTERNATIONAL TRADE

In our simple model of exchange and production, we saw why efficiency requires that every consumer's MRS be equal to the economy's MRT. This same requirement must be satisfied even for an economy that is free to engage in foreign trade. To illustrate, consider an economy just like the one we discussed, and suppose that its competitive general equilibrium in the absence of international trade occurs at point $V$ in Figure 18W.14. Now suppose that country opens its borders to international trade. If the country is small relative to the rest of the world, output prices will no longer be determined in its own internal markets, but in the much larger international markets. Suppose, in particular, that world prices for food and clothing are $P_{F}^{w}$ and $P_{C}^{w}$, respectively. The best option available to this economy will no longer be to produce and consume at $V$. On the contrary, it should now produce at $Z$, the point on its production possibilities frontier at which the MRT is exactly equal to the international price ratio, $P_{C}^{w} / P_{F}^{w} . Z$ is the point that maximizes the value of its output in world markets. Having produced at $Z$, the country is then free to choose any point along its "international budget constraint," $B B^{\prime}$. Since the original competitive equilibrium point, $V$, lies within $B B^{\prime}$, we know that it is now possible for every person in the economy to have more of each good than before.


FIGURE I8W. 14

## Gains from

 International Trade Without international trade, the economy's competitive equilibrium was at $V$. With the possibility of buying or selling in world markets, the economy maximizes the total value of its output by producing at $Z$, where its MRT is equal to the international price ratio, $P_{c}^{w} / P_{f}^{w}$. Along $B B^{\prime}$, the international budget constraint, it then chooses the consumption allocation for which every consumer's MRS is equal to $P_{c}^{w} / P_{f}^{w}$. If this occurs at $T$, the country will export $C^{*}-C^{* *}$ units of clothing and import $F^{* *}-F^{*}$ units of food.Which of the infinitely many bundles along $B B^{\prime}$ should be chosen? The best outcome is the one for which $P_{C}^{w} / P_{F}^{w}$ is equal to every consumer's MRS. We know that without international trade the common value of MRS was equal to the MRT at $V$, which is smaller than the MRT at $Z$. Since there is more clothing and less food at $Z$ than at $V$, it follows that the MRS at $Z$ will be smaller than the MRT at $V$. This
means that people will be better off moving to the northwest from $Z$. Suppose $T$ is the combination of food and clothing that equates everyone's MRS to $P_{C}^{w} / P_{F}^{w}$. This economy will then do best by exporting $C^{*}-C^{* *}$ units of clothing and using the proceeds to import $F^{* *}-F^{*}$ units of food.

As noted, the fact that the international budget constraint contains the original competitive equilibrium point means that it is possible to make everyone better off than before. But the impersonal workings of international trading markets provide no guarantee that every single person will in fact be made better off by trade. In the illustration given, international trading possibilities led the economy to produce more clothing and less food than it used to. The effect will be to increase the demand for factors of production used in clothing production and to reduce the demand for those used in food production. If food production is relatively intensive in the use of labor and clothing production is relatively intensive in the use of capital, the shift in product mix would drive up the price of capital and drive down the price of labor. In this case, the primary beneficiaries from trade would be the owners of capital. People whose incomes come exclusively from the sale of their own labor would actually do worse than before, even though the value of total output is higher. What our general equilibrium analysis shows is that trade makes it possible to give everyone more of everything. It does not prove that everyone necessarily will get more.

EXAMPLE 18 W .I You are the president of a small island nation that has never engaged in trade with any other nation. You are considering the possibility of opening the economy to international trade. The chief economist of the island's only labor union, to which every worker belongs, tells you that free trade will reduce the real purchasing power of labor, and you have no reason to doubt him. You are determined to remain in office and need the union's support in order to do so. The union will never support a candidate whose policies adversely affect the welfare of its members. Does this mean you should keep the island closed to trade?
The answer is yes only if there is no way to redistribute the gains that trade will produce. Our general equilibrium analysis establishes that trade will increase the total value of output, which makes it possible for everyone to do better. If the alternative is for the island to remain closed, the owners of capital should readily agree to transfer some of their gains to labor. The only president who would fail to open the island's economy is one who is too lazy or unimaginative to negotiate an agreement under which every party ends up with more of everything than before.

Much has been written about the agonizing trade-off between equity and efficiency, the notion that greater distributional fairness requires some sacrifice in efficiency. The lesson in Example 18W. 1 is that when people are able to negotiate costlessly with one another, there is in fact no conflict between equity and efficiency. When the total size of the economic pie grows larger, it is always possible for everyone to have a larger slice than before. Efficiency is achieved when we have made the economic pie as large as possible. Having done that, we are then free to discuss what constitutes a fair division of the pie.

## TAXES IN GENERAL EQUILIBRIUM

Suppose we are back in our simple production economy without the added complication of international trading opportunities. The economy is in competitive general equilibrium at point $V$ in Figure 18 W .15 , where the marginal rate of transformation is equal to the competitive equilibrium product price ratio, $P_{C}^{*} / P_{F}^{*}$. Now suppose the government decides to raise revenue by taxing food at the rate of $t /$ dollar. Every time a producer sells a unit of food for $P_{F}^{*}$, she gets to keep only $(1-t) P_{F}^{*}$. How will such a tax affect resource allocation?


FIGURE I8W.I5
Taxes Affect Product Mix
A tax on food causes a shift away from food toward clothing consumption. If the original allocation was Pareto optimal, the new one will not be. The marginal rate of transformation will exceed the marginal rate of substitution. There will be too much clothing and too little food.

The immediate effect of the tax is to raise the relative price ratio, as seen by producers, from $P_{C}^{*} / P_{F}^{*}$ to $P_{C}^{* /} /(1-t) P_{F}^{*}$. Producers who were once content to produce at $V$ on the production possibilities frontier will now find that they can increase their profits (or reduce their losses) by producing more clothing and less food than before. Suppose that, in the end, the effect is to cause producers to relocate at point $Z$ along the production possibilities frontier. Recall that the MRT at $V$ was equal to the common value of MRS at $V$. Since $Z$ has more clothing and less food than $V$, the MRS at $Z$ will be smaller than at $V$. It follows that the MRT will be higher than the MRS at $Z$, which means that the economy will no longer have an efficient product mix. The original allocation at $V$ was Pareto optimal. The new allocation has too much clothing, too little food.

Note that a tax on food does not alter the fact that consumers will all have a common value of MRS in equilibrium. Nor does it alter the fact that producers will all have a common value of MRTS. Even with such a tax, the economy remains efficient in consumption and production. The real problem created by the tax is that it causes producers to see a different price ratio from the one seen by consumers. Consumption decisions are based on gross prices-that is, on prices inclusive of taxes. Production decisions, by contrast, are guided by net prices-the amount producers get to keep after the tax has been paid. When producers confront a different price ratio from the one that guides consumers, the MRS can never be equal to the MRT in equilibrium. By driving a wedge between the price ratios seen by producers and consumers, the tax leads to an inefficient product mix.

Subsidies, like taxes, upset the conditions required for efficiency. The problem with a taxed product is that it appears too cheap to its producer. By contrast, the problem with a subsidized product is that it appears too expensive. In general equilibrium, we get too much of the subsidized product and too little of the unsubsidized one.

The distortionary effects of taxes and subsidies identified by our simple general equilibrium analysis form the cornerstone of the so-called supply-side school of economic policy. As supply-siders are ever ready to testify, taxes almost always lead to some sort of inefficiency in the allocation of resources.

Does it then follow that the world would be better off if we simply abolished all taxes? Hardly, for in such a world there could be no goods or services provided by government, and as we will presently see, there are many valuable goods and services that are unlikely to be provided in any other way. The practical message of general equilibrium analysis is that care should be taken to design taxes that keep distortions to a minimum. Note that in our simple model, the problem would have been eliminated had we taxed not just food but also clothing at the same rate $t$. Relative prices would then have stayed the same, and producers and consumers would again be motivated by a consistent set of price signals.

In more realistic general equilibrium models, however, even a general commodity tax would have distortionary effects. A tax on all commodities is essentially the same as a tax on income, including the income earned from the sale of one's own labor. In our simple model, the supply of labor was assumed to be fixed, but in practice it may be sensitive to the real, after-tax wage rate. In a fuller model that included this relationship, a general commodity tax might thus lead to a distortion in decisions about the allocation of time between labor and leisure-for example, people might work too little and consume too much leisure.

From the standpoint of efficiency, a better tax would be a head tax (also called a lump-sum tax), one that is levied on each person irrespective of his or her labor supply decisions. The problem with this kind of tax is that many object to it on equity grounds. If we levied the same tax on every person, the burden of taxation would fall much more heavily on the poor than it does under our current system, which collects taxes roughly in proportion to individual income. On efficiency grounds, the very best tax of all is one levied on activities of which there would otherwise be too much. And as we will see below, there are many such activities-more than enough, perhaps, to raise all the tax revenue we need.

## OTHER SOURCES OF INEFFICIENCY

## Monopoly

Taxes are but one of many factors that stand in the way of achieving Pareto optimality in the allocation of resources. One other source of inefficiency is monopoly. The general equilibrium effects of monopoly are closely analogous to those of a commodity tax. Consider again our simple production economy with two goods, and suppose that food is produced by a monopolist, clothing by a price taker. The competitive producer selects an output level for which marginal cost is equal to the price of clothing; the monopolist, as we saw in Chapter 12, selects one for which marginal cost is equal to marginal revenue. Because price always exceeds marginal revenue along a downward-sloping demand curve, this means that price will exceed marginal cost for the monopolist.

From the standpoint of efficiency, this wedge between price and marginal cost functions exactly like a tax on the monopolist's product. The marginal rate of transformation, which is the ratio of the marginal cost of clothing to the marginal cost of food, will no longer be equal to the ratio of product prices. Producers will be responding to one set of incentives, consumers to another. The result is that too few of the economy's resources will be devoted to the production of food (the monopolized product) and too many to the production of clothing (the competitive product).

The general equilibrium analysis of the effect of monopoly adds an important dimension to our partial equilibrium analysis from Chapter 12. The partial analysis called our attention to the fact that there would be too little output produced by the monopolist. The general equilibrium analysis forcefully reminds us that there is another side of this coin, which is that the resources not used by the monopolist will be employed by the competitive sector of the economy. Thus, if monopoly output is too small, competitive output is too big. The additional competitive output does not undo the damage caused by monopoly, but it partially compensates for it. Viewed within the framework of general equilibrium analysis, the welfare costs of monopoly are thus smaller than they appeared from our partial equilibrium analysis.

## Externalities

Another source of inefficiency occurs when production or consumption activities involve benefits or costs that fall on people not directly involved in the activities. As discussed in Chapter 16, such benefits and costs are usually referred to as externalities. A standard example of a negative externality is the case of pollution, in which a
production activity results in emissions that adversely affect people other than those who consume the product. The planting of additional apple trees, whose blossoms augment the output of honey in nearby beehives, is an example of a positive externality. And so is the case of the beekeeper who adds bees to his hive, unmindful of the higher pollination rates they will produce in nearby apple orchards.

Externalities are both widespread and important. The problem they create for efficiency stems from the fact that, like taxes, they cause producers and consumers to respond to different sets of relative prices. When the orchard owner decides how many trees to plant, he looks only at the price of apples, not at the price of honey. By the same token, when the consumer decides how much honey to buy, he ignores the effects of his purchases on the quantity and price of apples.

In the case of negative externalities in production, the effect on efficiency is much the same as that of a subsidy. In deciding what quantity of the product to produce, the producer equates price and his own private marginal cost. The problem is that the negative externalities impose additional costs on others, and these are ignored by the producer. As with the subsidized product, we end up with too much of the product with negative externalities and too little of all other products. With positive externalities, the reverse occurs. We end up with too little of such products and too much of others.

## Taxes as a Solution to Externalities and Monopoly

As noted earlier, the best tax from an efficiency standpoint is one levied on an activity there would otherwise be too much of. This suggests that the welfare losses from monopoly can be mitigated by placing an excise tax on the good produced in the competitive sector. Properly chosen, such a tax could exactly offset the wedge that is created by the disparity between the monopolist's price and marginal cost.

In the case of negative externalities, the difficulty is that individuals regard the product as being cheaper than it really is from the standpoint of society as a whole. By taxing the product with negative externalities at a suitable rate, the efficiency loss can be undone. For products accompanied by positive externalities, the corresponding solution is a subsidy.

## Public Goods

One additional factor that stands in the way of achieving efficient allocations through private markets is the existence of public goods. As discussed in Chapter 17, a pure public good is one with two specific properties: (1) nondiminishability, which means that one person's use of the good does not diminish the amount of it available for others; and (2) nonexcludability, which means that it is either impossible or prohibitively costly to exclude people who do not pay from using the good. In the days before the invention of channel scramblers, broadcast television signals were an example of a pure public good. My tuning in to a movie on channel 11 , for example, does not make that movie less available to anyone else. And before the advent of scramblers and cable TV, there was no practical way to exclude anyone from making use of a television signal once it was broadcast. National defense is another example of a pure public good. The fact that Smith enjoys the benefits of national defense does not make those benefits any less available to Jones. And it is exceedingly difficult for the government to protect some of its citizens from foreign attack while denying the same protection to others.

There is no reason to presume that private markets will supply optimal quantities of pure public goods. Indeed, if it is impossible to exclude people from using the good, it might seem impossible for a profit-seeking firm to supply any quantity of it at all. But profit-seeking firms often show great ingenuity in devising schemes
for providing pure public goods. Commercial broadcast television, for example, covers its costs by charging advertisers for access to the audience it attracts with its programming. But even in these cases, there is no reason to suppose that the amount and kind of television programming we get under this arrangement is economically efficient.

The problem is less acute in the case of goods that have the nondiminishability but not the nonexcludability property. Once every household is wired for cable TV, for example, it will be possible to prevent people from watching any given program if they do not pay for it. But even here, there are likely to be inefficiencies. Once a TV program has been produced, it costs society nothing to let an extra person see it. If there is a positive price for watching the program, however, all those who value seeing it at less than that price will not tune in. It is inefficient to exclude these viewers, since their viewing the program would not diminish its usefulness for anyone else.

## - SUMMARY

- One of the simplest possible general equilibrium models is a pure exchange economy with only two consumers and two goods. For any given initial allocation of the two goods between the two consumers in this model, a competitive exchange process will always exhaust all possible mutually beneficial gains from trade. This result is known as the invisible hand theorem and is also called the first theorem of welfare economics.
- If consumers have convex indifference curves, any efficient allocation can be sustained as a competitive equilibrium. This result is known as the second theorem of welfare economics. Its significance is that it demonstrates that the issues of efficiency and distributional equity are logically distinct. Society can redistribute initial endowments according to accepted norms of distributive justice, and then rely on markets to assure that endowments are used efficiently.
- An economy is efficient in production if the marginal rate of technical substitution is the same for all producers. In
the input market, too, competitive trading exploits all mutually beneficial gains from exchange.
- Even though international trade leaves domestic production possibilities unchanged, its immediate effect is to increase the value of goods available for domestic consumption. With a suitable redistribution of initial endowments, a free-trade economy will always be Pareto superior to a non-free-trade economy.
- Taxes often interfere with efficient resource allocation, usually because they cause consumers and producers to respond to different price ratios. The practical significance of this result is to guide us in the search for taxes that minimize distortions. The best tax, from an efficiency standpoint, is one levied on an activity that would otherwise be pursued too intensively.
- Monopoly, externalities, and public goods are three other factors that interfere with the efficient allocation of resources.


## - QUESTIONS FOR RENIEW

1. Why does efficiency in consumption require the MRS of all consumers to be the same?
2. Distinguish among the terms "Pareto superior," "Pareto preferred," and "Pareto optimal."
3. Why might voters in a country choose a non-Paretooptimal allocation over another that is Pareto optimal?
4. How do the initial endowments constrain where we end up on the contract curve?
5. In general equilibrium, can there be excess demand for every good?
6. How might a social critic respond to the claim that governmental involvement in the economy is unjustified because of the invisible hand theorem?
7. Why is the slope of the production possibilities frontier equal to the ratio of marginal production costs?
8. How might a critic respond to the claim that taxes always make the allocation of resources less efficient?
9. Bert has an initial endowment consisting of 10 units of food and 10 units of clothing. Ernie's initial endowment consists of 10 units of food and 20 units of clothing. Represent these initial endowments in an Edgeworth exchange box.
10. Bert regards food and clothing as perfect 1 -for- 1 substitutes. Ernie regards them as perfect complements, always wanting to consume 3 units of clothing for every 2 units of food.
a. Describe the set of allocations that are Pareto preferred to the one given in Problem 1.
b. Describe the contract curve for that allocation.
c. What price ratio will be required to sustain an allocation on the contract curve?
11. How will your answers to Problem 2 differ if 5 units of Ernie's clothing endowment are given to Bert?
12. Consider a simple economy with two goods, food and clothing, and two consumers, $A$ and $B$. For a given initial endowment, when the ratio of food to clothing prices in an economy is $3 / 1$, $A$ wants to buy 6 units of clothing while $B$ wants to sell 2 units of food. Is $P_{F} / P_{C}=3$ an equilibrium price ratio? If so, explain why. If not, state in which direction it will tend to change.
13. How will your answer to Problem 4 change if $A$ wants to sell 3 units of clothing and $B$ wants to sell 2 units of food?
14. Suppose Sarah has an endowment of 2 units of $X$ and 4 units of $Y$ and has indifference curves that satisfy our four basic assumptions (see Chapter 3). Suppose Brian has an endowment of 4 units of $X$ and 2 units of $Y$, and has preferences given by the utility function $U(X, Y)=\min \{X, Y\}$, where

$$
\min (X, Y)= \begin{cases}X & \text { if } X \leq Y \\ Y & \text { if } Y \leq X\end{cases}
$$

On an Edgeworth box diagram, indicate the set of Pareto-superior bundles.
7. A simple economy produces two goods, food and clothing, with two inputs, capital and labor. Given the current allocation of capital and labor between the two industries, the marginal rate of technical substitution between capital and labor in food production is 4 , while the corresponding MRTS in clothing production is 2 . Is this economy efficient in production? If so, explain why. If not, describe a reallocation that will lead to a Pareto improvement.
8. Given the current allocation of productive inputs, the marginal rate of transformation of food for clothing in a simple two-good economy is equal to 2 . At the current allocation of consumption goods, each consumer's marginal rate of substitution between food and clothing is 1.5 . Is this economy efficient in terms of its product mix? If so, explain why. If not, describe a reallocation that will lead to a Pareto improvement.
9. Crusoe can make 5 units of food per day if he devotes all his time to food production. He can make 10 units of clothing if he spends the whole day at clothing production. If he divides his time between the two activities, his output of each good will be proportional to the time spent on each. The corresponding figures for Friday are 10 units of food and 15 units of clothing. Describe the production possibilities frontier for their economy.
10. If Crusoe and Friday regard food and clothing as perfect 1 -for- 1 substitutes, what should each produce?
11. Now suppose a trading ship visits the island each day and offers to buy or sell food and clothing at the prices $P_{F}=4, P_{C}=1$. How, if at all, will the presence of this ship alter the production and consumption decisions of Crusoe and Friday?
12. How will your answers to Problems 9, 10, and 11 differ if Friday's maximum production figures change to 20 units of food and 50 units of clothing?
13. There are two industries in a simple economy, each of which faces the same marginal cost of production. One of the industries is perfectly competitive, the other a pure
monopoly. Describe a reallocation of resources that will lead to a Pareto improvement for this economy.
14. Suppose capital and labor are perfect substitutes in production for clothing: 2 units of capital or 2 units of labor produce 1 unit of clothing. Suppose capital and labor are perfect complements in production for food: 1 unit of capital and 1 unit of labor produce 1 unit of food. Suppose the economy has an endowment of 100 units of capital and 200 units of labor. Describe the set of efficient allocations of the factors to the two sectors (determine the contract curve in an Edgeworth production box).
15. Construct the production possibilities frontier for the economy described in Problem 14. What is the opportunity cost of food in terms of clothing?
16. Construct the production possibilities frontier for an economy just like the one described in Problem 14, except that its endowment of capital is 200 units.

## - ansWers to IN-chapter exercises

18W.1. Bill's endowment of food $=100-$ Ann's endowment $=75$. Bill's endowment of clothing $=200-$ Ann's endowment $=175$.
18W.2. Let $M$ denote the initial allocation. Ann's indifference curve through $M$ is a straight line with slope $=-1$. Bill's indifference curve through $M$ is right-angled, as shown in the following diagram. The set of Pareto-superior allocations is indicated by the shaded triangle.


18W.3. Here, $P_{L} / P_{K}=1$, which is half as big as $\mathrm{MPL}_{C} / \mathrm{MPK}_{C}$ :

$$
\frac{P_{L}}{P_{K}}=\frac{1}{2} \frac{\mathrm{MPL}_{C}}{\mathrm{MPK}_{C}},
$$

from which it follows that

$$
\frac{\mathrm{MPK}_{C}}{P_{K}}=\frac{1}{2} \frac{\mathrm{MPL}_{C}}{P_{L}} .
$$

In words, this says that the last dollar spent on capital in clothing production produces only half as much extra output as does the last dollar spent on labor in clothing production. It follows that clothing producers can get more output for the same cost by hiring less capital and more labor. Parallel reasoning tells us that food producers can increase food production at no extra cost by hiring less labor and more capital. Only when these producers have reached a cost-minimizing input mix
characteristic of a competitive equilibrium will efficiency in production be achieved.

18W.4. On the new production possibilities frontier, the maximum quantity of food that can be produced is unchanged. At every level of food production, the corresponding amount of clothing that can be produced is exactly double the original amount.



[^0]:    ${ }^{1}$ In advanced courses, we show that a competitive equilibrium will exist in a simple exchange economy if the sum of all individual excess demands is a continuous function of relative prices. This will always happen whenever individual indifference curves have the conventional convex shape.

