LEARNING STAGE 1

The Fundamentals

LEARNING STAGE 1
The Fundamentals

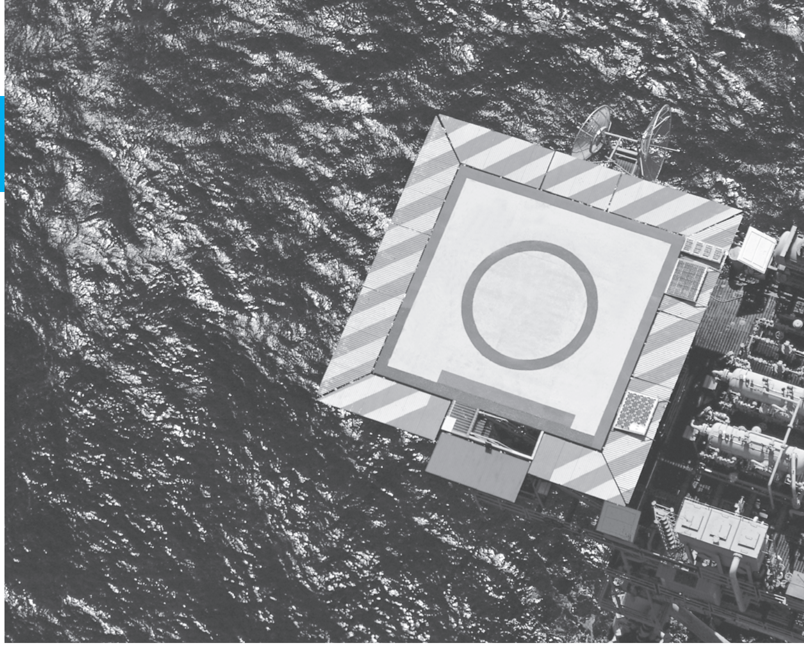
The fundamentals of engineering economy are introduced in these chapters. When you have completed stage 1, you will be able to understand and work problems that account for the **time value of money, cash flows** occurring at different times with different amounts, and **equivalence** at different interest rates. The techniques you master here form the basis of how an engineer in any discipline can take **economic value** into account in virtually any project environment.

The factors commonly used in all engineering economy computations are introduced and applied here. Combinations of these factors assist in moving monetary values forward and backward through time and at different interest rates. Also, after these chapters, you should be comfortable using many of the spreadsheet functions.

Many of the terms common to economic decision making are introduced in learning stage 1 and used in later chapters. A checkmark icon in the margin indicates that a new **concept or guideline** is introduced at this point.

CHAPTER 1

Foundations of Engineering Economy



LEARNING OUTCOMES

Purpose: Understand and apply fundamental concepts and use the terminology of engineering economics.

SECTION	TOPIC	LEARNING OUTCOME
1.1	Description and role	• Define engineering economics and describe its role in decision making.
1.2	Engineering economy study approach	• Understand and identify the steps in an engineering economy study.
1.3	Ethics and economics	• Identify areas in which economic decisions can present questionable ethics.
1.4	Interest rate	• Perform calculations for interest rates and rates of return.
1.5	Terms and symbols	• Identify and use engineering economic terminology and symbols.
1.6	Cash flows	• Understand cash flows and how to graphically represent them.
1.7	Economic equivalence	• Describe and calculate economic equivalence.
1.8	Simple and compound interest	• Calculate simple and compound interest amounts for one or more time periods.
1.9	MARR and opportunity cost	• State the meaning and role of Minimum Attractive Rate of Return (MARR) and opportunity costs.
1.10	Spreadsheet functions	• Identify and use some Excel functions commonly applied in engineering economics.

The need for engineering economy is primarily motivated by the work that engineers do in performing analyses, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of **making decisions**. These decisions involve the fundamental elements of **cash flows of money, time, and interest rates.** This chapter introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions.

1.1 Engineering Economics: Description and Role in Decision Making ●●●

Decisions are made routinely to choose one alternative over another by individuals in everyday life; by engineers on the job; by managers who supervise the activities of others; by corporate presidents who operate a business; and by government officials who work for the public good. Most decisions involve money, called **capital** or **capital funds**, which is usually limited in amount. The decision of where and how to invest this limited capital is motivated by a primary goal of **adding value** as future, anticipated results of the selected alternative are realized. Engineers play a vital role in capital investment decisions based upon their ability and experience to design, analyze, and synthesize. The factors upon which a decision is based are commonly a combination of economic and noneconomic elements. Engineering economy deals with the economic factors. By definition,

> Engineering economy involves formulating, estimating, and evaluating the expected economic outcomes of alternatives designed to accomplish a defined purpose. Mathematical techniques simplify the economic evaluation of alternatives.

Because the formulas and techniques used in engineering economics are applicable to all types of money matters, they are equally useful in business and government, as well as for individuals. Therefore, besides applications to projects in your future jobs, what you learn from this book and in this course may well offer you an economic analysis tool for making personal decisions such as car purchases, house purchases, major purchases on credit, e.g., furniture, appliances, and electronics.

Other terms that mean the same as *engineering economy* are *engineering economic analysis, capital allocation study, economic analysis,* and similar descriptors.

People make decisions; computers, mathematics, concepts, and guidelines assist people in their decision-making process. Since most decisions affect what will be done, the time frame of engineering economy is primarily the **future**. Therefore, the numbers used in engineering economy are **best estimates of what is expected to occur**. The estimates and the decision usually involve four essential elements:

Cash flows

Times of occurrence of cash flows

Interest rates for time value of money

Measure of economic worth for selecting an alternative

Since the estimates of cash flow amounts and timing are about the future, they will be somewhat different than what is actually observed, due to changing circumstances and unplanned events. In short, the variation between an amount or time estimated now and that observed in the future is caused by the stochastic (random) nature of all economic events. **Sensitivity analysis** is utilized to determine how a decision might change according to varying estimates, especially those expected to vary widely. Example 1.1 illustrates the fundamental nature of variation in estimates and how this variation may be included in the analysis at a very basic level.

EXAMPLE 1.1

An engineer is performing an analysis of warranty costs for drive train repairs within the first year of ownership of luxury cars purchased in the United States. He found the average cost (to the nearest dollar) to be $570 per repair from data taken over a 5-year period.

Year	2006	2007	2008	2009	2010
Average Cost, $/repair	525	430	619	650	625

What range of repair costs should the engineer use to ensure that the analysis is sensitive to changing warranty costs?

Solution

At first glance the range should be approximately –25% to +15% of the $570 average cost to include the low of $430 and high of $650. However, the last 3 years of costs are higher and more consistent with an average of $631. The observed values are approximately ±3% of this more recent average.

If the analysis is to use the most recent data and trends, a range of, say, ±5% of $630 is recommended. If, however, the analysis is to be more inclusive of historical data and trends, a range of, say, ±20% or ±25% of $570 is recommended.

The criterion used to select an alternative in engineering economy for a specific set of estimates is called a **measure of worth**. The measures developed and used in this text are

Present worth (PW)	Future worth (FW)	Annual worth (AW)
Rate of return (ROR)	Benefit/cost (B/C)	Capitalized cost (CC)
Payback period	Economic value added (EVA)	Cost Effectiveness

All these measures of worth account for the fact that money makes money over time. This is the concept of the **time value of money.**

Time value of money

It is a well-known fact that money **makes** money. The time value of money explains the change in the amount of money **over time** for funds that are owned (invested) or owed (borrowed). This is the most important concept in engineering economy.

The time value of money is very obvious in the world of economics. If we decide to invest capital (money) in a project today, we inherently expect to have more money in the future than we invested. If we borrow money today, in one form or another, we expect to return the original amount plus some additional amount of money.

Engineering economics is equally well suited for the future and for the **analysis of past cash flows** in order to determine if a specific criterion (measure of worth) was attained. For example, assume you invested $4975 exactly 3 years ago in 53 shares of IBM stock as traded on the New York Stock Exchange (NYSE) at $93.86 per share. You expect to make 8% per year appreciation, not considering any dividends that IBM may declare. A quick check of the share value shows it is currently worth $127.25 per share for a total of $6744.25. This increase in value represents a rate of return of 10.67% per year. (These type of calculations are explained later.) This past investment has well exceeded the 8% per year criterion over the last 3 years.

1.2 Performing an Engineering Economy Study

An engineering economy study involves many elements: problem identification, definition of the objective, cash flow estimation, financial analysis, and decision making. Implementing a structured procedure is the best approach to select the best solution to the problem.

The steps in an engineering economy study are as follows:

1. Identify and understand the problem; identify the objective of the project.
2. Collect relevant, available data and define viable solution alternatives.
3. Make realistic cash flow estimates.
4. Identify an economic measure of worth criterion for decision making.

5. Evaluate each alternative; consider noneconomic factors; use sensitivity analysis as needed.
6. Select the best alternative.
7. Implement the solution and monitor the results.

Technically, the last step is not part of the economy study, but it is, of course, a step needed to meet the project objective. There may be occasions when the best economic alternative requires more capital funds than are available, or significant noneconomic factors preclude the most economic alternative from being chosen. Accordingly, steps 5 and 6 may result in selection of an alternative different from the economically best one. Also, sometimes more than one project may be selected and implemented. This occurs when projects are independent of one another. In this case, steps 5 through 7 vary from those above. Figure 1–1 illustrates the steps above for one alternative. Descriptions of several of the elements in the steps are important to understand.

Problem Description and Objective Statement A succinct statement of the problem and primary objective(s) is very important to the formation of an alternative solution. As an illustration, assume the problem is that a coal-fueled power plant must be shut down by 2015 due to the production of excessive sulfur dioxide. The objectives may be to generate the forecasted electricity

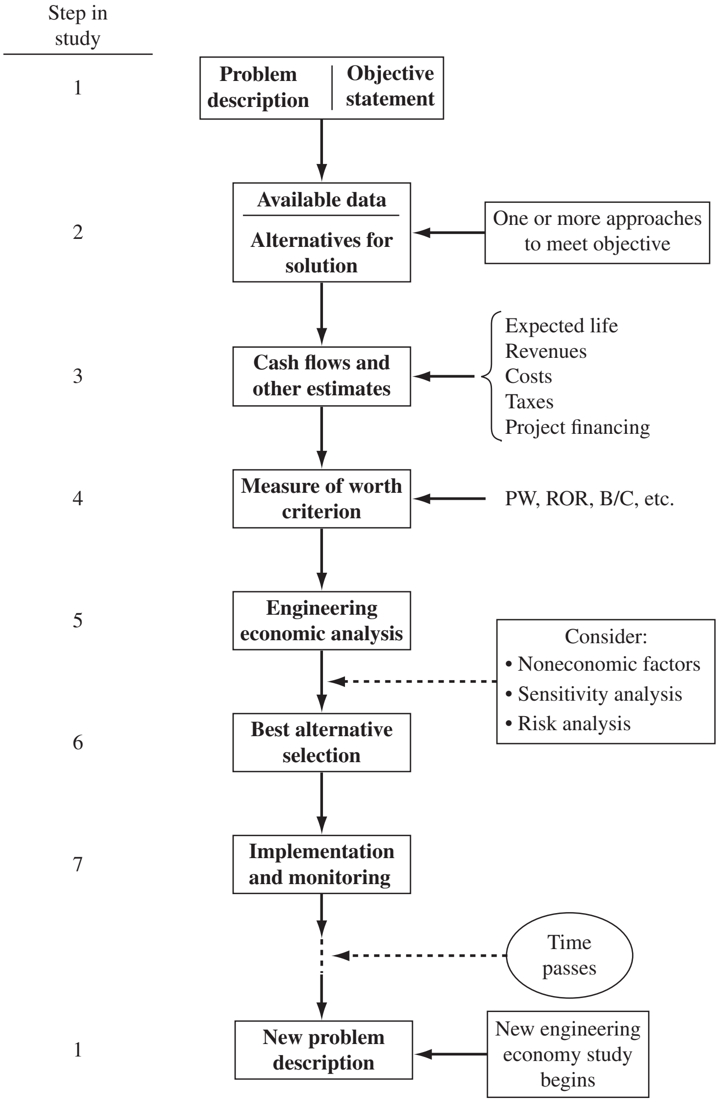


Figure 1–1
Steps in an engineering economy study.

needed for 2015 and beyond, plus to not exceed all the projected emission allowances in these future years.

Alternatives These are stand-alone descriptions of viable solutions to problems that can meet the objectives. Words, pictures, graphs, equipment and service descriptions, simulations, etc. define each alternative. The best estimates for parameters are also part of the alternative. Some parameters include equipment first cost, expected life, salvage value (estimated trade-in, resale, or market value), and annual operating cost (AOC), which can also be termed *maintenance and operating* (M&O) *cost,* and subcontract cost for specific services. If changes in income (revenue) may occur, this parameter must be estimated.

Detailing all viable alternatives at this stage is crucial. For example, if two alternatives are described and analyzed, one will likely be selected and implementation initiated. If a third, more attractive method that was available is later recognized, a wrong decision was made.

Cash Flows All cash flows are estimated for each alternative. Since these are future expenditures and revenues, the results of step 3 usually prove to be inaccurate when an alternative is actually in place and operating. When cash flow estimates for specific parameters are expected to vary significantly from a *point estimate* made now, risk and sensitivity analyses (step 5) are needed to improve the chances of selecting the best alternative. Sizable variation is usually expected in estimates of revenues, AOC, salvage values, and subcontractor costs. Estimation of costs is discussed in Chapter 15, and the elements of variation (risk) and sensitivity analysis are included throughout the text.

Engineering Economy Analysis The techniques and computations that you will learn and use throughout this text utilize the cash flow estimates, time value of money, and a selected measure of worth. The result of the analysis will be one or more numerical values; this can be in one of several terms, such as money, an interest rate, number of years, or a probability. In the end, a selected measure of worth mentioned in the previous section will be used to select the best alternative.

Before an economic analysis technique is applied to the cash flows, some decisions about what to include in the analysis must be made. Two important possibilities are taxes and inflation. Federal, state or provincial, county, and city taxes will impact the costs of every alternative. An after-tax analysis includes some additional estimates and methods compared to a before-tax analysis. If taxes and inflation are expected to impact all alternatives equally, they may be disregarded in the analysis. However, if the size of these projected costs is important, taxes and inflation should be considered. Also, if the impact of inflation over time is important to the decision, an additional set of computations must be added to the analysis; Chapter 14 covers the details.

Selection of the Best Alternative The measure of worth is a primary basis for selecting the best economic alternative. For example, if alternative A has a rate of return (ROR) of 15.2% per year and alternative B will result in an ROR of 16.9% per year, B is better economically. However, there can always be **noneconomic** or **intangible factors** that must be considered and that may alter the decision. There are many possible noneconomic factors; some typical ones are

- Market pressures, such as need for an increased international presence
- Availability of certain resources, e.g., skilled labor force, water, power, tax incentives
- Government laws that dictate safety, environmental, legal, or other aspects
- Corporate management's or the board of director's interest in a particular alternative
- Goodwill offered by an alternative toward a group: employees, union, county, etc.

As indicated in Figure 1–1, once all the economic, noneconomic, and risk factors have been evaluated, a final decision of the "best" alternative is made.

At times, only one viable alternative is identified. In this case, the **do-nothing (DN) alternative** may be chosen provided the measure of worth and other factors result in the alternative being a poor choice. The do-nothing alternative maintains the status quo.

Whether we are aware of it or not, we use criteria every day to choose between alternatives. For example, when you drive to campus, you decide to take the "best" route. But how did you define *best?* Was the best route the safest, shortest, fastest, cheapest, most scenic, or what? Obviously, depending upon which criterion or combination of criteria is used to identify the best, a different route might be selected each time. In economic analysis, **financial units (dollars or other currency)** are generally used as the tangible basis for evaluation. Thus, when there are several ways of accomplishing a stated objective, the alternative with the lowest overall cost or highest overall net income is selected.

1.3 Professional Ethics and Economic Decisions

Many of the fundamentals of engineering ethics are intertwined with the roles of money and economics-based decisions in the making of professionally ethical judgments. Some of these integral connections are discussed here, plus sections in later chapters discuss additional aspects of ethics and economics. For example, Chapter 9, Benefit/Cost Analysis and Public Sector Economics, includes material on the ethics of public project contracts and public policy. Although it is very limited in scope and space, it is anticipated that this coverage of the important role of economics in engineering ethics will prompt further interest on the part of students and instructors of engineering economy.

The terms **morals** and **ethics** are commonly used interchangeably, yet they have slightly different interpretations. Morals usually relate to the underlying tenets that form the character and conduct of a person in judging right and wrong. Ethical practices can be evaluated by using a code of morals or **code of ethics** that forms the standards to guide decisions and actions of individuals and organizations in a profession, for example, electrical, chemical, mechanical, industrial, or civil engineering. There are several different levels and types of morals and ethics.

Universal or common morals These are fundamental moral beliefs held by virtually all people. Most people agree that to steal, murder, lie, or physically harm someone is wrong.

It is possible for **actions** and **intentions** to come into conflict concerning a common moral. Consider the World Trade Center buildings in New York City. After their collapse on September 11, 2001, it was apparent that the design was not sufficient to withstand the heat generated by the firestorm caused by the impact of an aircraft. The structural engineers who worked on the design surely did not have the intent to harm or kill occupants in the buildings. However, their design actions did not foresee this outcome as a measurable possibility. Did they violate the common moral belief of not doing harm to others or murdering?

Individual or personal morals These are the moral beliefs that a person has and maintains over time. These usually parallel the common morals in that stealing, lying, murdering, etc. are immoral acts.

It is quite possible that an individual strongly supports the common morals and has excellent personal morals, but these may conflict from time to time when decisions must be made. Consider the engineering student who genuinely believes that cheating is wrong. If he or she does not know how to work some test problems, but must make a certain minimum grade on the final exam to graduate, the decision to cheat or not on the final exam is an exercise in following or violating a personal moral.

Professional or engineering ethics Professionals in a specific discipline are guided in their decision making and performance of work activities by a formal standard or code. The code states the commonly accepted standards of honesty and integrity that each individual is expected to demonstrate in her or his practice. There are codes of ethics for medical doctors, attorneys, and, of course, engineers.

Although each engineering profession has its own code of ethics, the **Code of Ethics for Engineers** published by the National Society of Professional Engineers (NSPE) is very commonly used and quoted. This code, reprinted in its entirety in Appendix C, includes numerous sections that have direct or indirect economic and financial impact upon the designs, actions, and decisions that engineers make in their professional dealings. Here are three examples from the Code:

"Engineers, in the fulfillment of their duties, shall hold paramount the *safety, health, and welfare of the public*." (section I.1)

"Engineers shall *not accept financial or other considerations*, including free engineering designs, from material or equipment suppliers for specifying their product." (section III.5.a)

"Engineers using designs supplied by a client recognize that the *designs remain the property of the client* and may not be duplicated by the engineer for others without express permission." (section III.9.b)

As with common and personal morals, conflicts can easily rise in the mind of an engineer between his or her own ethics and that of the employing corporation. Consider a manufacturing engineer who has recently come to firmly disagree morally with war and its negative effects on human beings. Suppose the engineer has worked for years in a military defense contractor's facility and does the detailed cost estimations and economic evaluations of producing fighter jets for the Air Force. The Code of Ethics for Engineers is silent on the ethics of producing and using war materiel. Although the employer and the engineer are not violating any ethics code, the engineer, as an individual, is stressed in this position. Like many people during a declining national economy, retention of this job is of paramount importance to the family and the engineer. Conflicts such as this can place individuals in real dilemmas with no or mostly unsatisfactory alternatives.

At first thought, it may not be apparent how activities related to engineering economics may present an ethical challenge to an individual, a company, or a public servant in government service. Many money-related situations, such as those that follow, can have ethical dimensions.

In the design stage:

- Safety factors are compromised to ensure that a price bid comes in as low as possible.
- Family or personal connections with individuals in a company offer unfair or insider information that allows costs to be cut in strategic areas of a project.
- A potential vendor offers specifications for company-specific equipment, and the design engineer does not have sufficient time to determine if this equipment will meet the needs of the project being designed and costed.

While the system is operating:

- Delayed or below-standard maintenance can be performed to save money when cost overruns exist in other segments of a project.
- Opportunities to purchase cheaper repair parts can save money for a subcontractor working on a fixed-price contract.
- Safety margins are compromised because of cost, personal inconvenience to workers, tight time schedules, etc.

A good example of the last item—safety is compromised while operating the system—is the situation that arose in 1984 in Bhopal, India (Martin and Schinzinger 2005, pp. 245–8). A Union Carbide plant manufacturing the highly toxic pesticide chemical methyl isocyanate (MIC) experienced a large gas leak from high-pressure tanks. Some 500,000 persons were exposed to inhalation of this deadly gas that burns moist parts of the body. There were 2500 to 3000 deaths within days, and over the following 10-year period, some 12,000 death claims and 870,000 personal injury claims were recorded. Although Union Carbide owned the facility, the Indian government had only Indian workers in the plant. Safety practices clearly eroded due to cost-cutting measures, insufficient repair parts, and reduction in personnel to save salary money. However, one of the surprising practices that caused unnecessary harm to workers was the fact that masks, gloves, and other protective gear were not worn by workers in close proximity to the tanks containing MIC. Why? Unlike in plants in the United States and other countries, there was no air conditioning in the Indian plant, resulting in high ambient temperatures in the facility.

Many ethical questions arise when corporations operate in international settings where the corporate rules, worker incentives, cultural practices, and costs in the home country differ from those in the host country. Often these ethical dilemmas are fundamentally based in the economics that provide cheaper labor, reduced raw material costs, less government oversight, and a host of other cost-reducing factors. When an engineering economy study is performed, it is important for the engineer performing the study to consider all ethically related matters to ensure that the cost and revenue estimates reflect what is likely to happen once the project or system is operating.

It is important to understand that the translation from universal morals to personal morals and professional ethics does vary from one culture and country to another. As an example, consider the common belief (universal moral) that the awarding of contracts and financial arrangements for services to be performed (for government or business) should be accomplished in a fair and transparent fashion. In some societies and cultures, corruption in the process of contract making is common and often "overlooked" by the local authorities, who may also be involved in the affairs. Are these immoral or unethical practices? Most would say, "Yes, this should not be allowed. Find and punish the individuals involved." Yet, such practices do continue, thus indicating the differences in interpretation of common morals as they are translated into the ethics of individuals and professionals.

EXAMPLE 1.2

Jamie is an engineer employed by Burris, a United States–based company that develops subway and surface transportation systems for medium-sized municipalities in the United States and Canada. He has been a registered professional engineer (PE) for the last 15 years. Last year, Carol, an engineer friend from university days who works as an individual consultant, asked Jamie to help her with some cost estimates on a metro train job. Carol offered to pay for his time and talent, but Jamie saw no reason to take money for helping with data commonly used by him in performing his job at Burris. The estimates took one weekend to complete, and once Jamie delivered them to Carol, he did not hear from her again; nor did he learn the identity of the company for which Carol was preparing the estimates.

Yesterday, Jamie was called into his supervisor's office and told that Burris had not received the contract award in Sharpstown, where a metro system is to be installed. The project estimates were prepared by Jamie and others at Burris over the past several months. This job was greatly needed by Burris, as the country and most municipalities were in a real economic slump, so much so that Burris was considering furloughing several engineers if the Sharpstown bid was not accepted. Jamie was told he was to be laid off immediately, not because the bid was rejected, but because he had been secretly working without management approval for a prime consultant of Burris' main competitor. Jamie was astounded and angry. He knew he had done nothing to warrant firing, but the evidence was clearly there. The numbers used by the competitor to win the Sharpstown award were the same numbers that Jamie had prepared for Burris on this bid, and they closely matched the values that he gave Carol when he helped her.

Jamie was told he was fortunate, because Burris' president had decided to not legally charge Jamie with unethical behavior and to not request that his PE license be rescinded. As a result, Jamie was escorted out of his office and the building within one hour and told to not ask anyone at Burris for a reference letter if he attempted to get another engineering job.

Discuss the ethical dimensions of this situation for Jamie, Carol, and Burris' management. Refer to the NSPE Code of Ethics for Engineers (Appendix C) for specific points of concern.

Solution

There are several obvious errors and omissions present in the actions of Jamie, Carol, and Burris' management in this situation. Some of these mistakes, oversights, and possible code violations are summarized here.

Jamie

- Did not learn identity of company Carol was working for and whether the company was to be a bidder on the Sharpstown project
- Helped a friend with confidential data, probably innocently, without the knowledge or approval of his employer
- Assisted a competitor, probably unknowingly, without the knowledge or approval of his employer
- Likely violated, at least, Code of Ethics for Engineers section II.1.c, which reads, "Engineers shall not reveal facts, data, or information without the prior consent of the client or employer except as authorized or required by law or this Code."

Carol

- Did not share the intended use of Jamie’s work
- Did not seek information from Jamie concerning his employer’s intention to bid on the same project as her client
- Misled Jamie in that she did not seek approval from Jamie to use and quote his information and assistance
- Did not inform her client that portions of her work originated from a source employed by a possible bid competitor
- Likely violated, at least, Code of Ethics for Engineers section III.9.a, which reads, “Engineers shall, whenever possible, name the person or persons who may be individually responsible for designs, inventions, writings, or other accomplishments.”

Burris’ management

- Acted too fast in dismissing Jamie; they should have listened to Jamie and conducted an investigation
- Did not put him on administrative leave during a review
- Possibly did not take Jamie’s previous good work record into account

These are not all ethical considerations; some are just plain good business practices for Jamie, Carol, and Burris.

1.4 Interest Rate and Rate of Return

Interest is the manifestation of the time value of money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest. There are always two perspectives to an amount of interest—interest paid and interest earned. These are illustrated in Figure 1–2. Interest is **paid** when a person or organization borrowed money (obtained a loan) and repays a larger amount over time. Interest is **earned** when a person or organization saved, invested, or lent money and obtains a return of a larger amount over time. The numerical values and formulas used are the same for both perspectives, but the interpretations are different.

Interest paid on borrowed funds (a loan) is determined using the original amount, also called the *principal,*

$$\text{Interest} = \text{amount owed now} - \text{principal} \qquad [1.1]$$

When interest paid over a *specific time unit* is expressed as a percentage of the principal, the result is called the **interest rate.**

$$\text{Interest rate (\%)} = \frac{\text{interest accrued per time unit}}{\text{principal}} \times 100\% \qquad [1.2]$$

The time unit of the rate is called the **interest period.** By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as 1% per month. Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, 8.5%, a 1-year interest period is assumed.

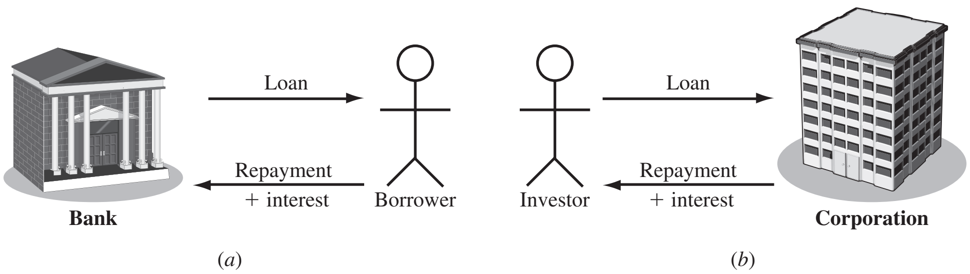


Figure 1–2

(*a*) Interest paid over time to lender. (*b*) Interest earned over time by investor.

EXAMPLE 1.3

An employee at LaserKinetics.com borrows \$10,000 on May 1 and must repay a total of \$10,700 exactly 1 year later. Determine the interest amount and the interest rate paid.

Solution

The perspective here is that of the borrower since \$10,700 repays a loan. Apply Equation [1.1] to determine the interest paid.

$$\text{Interest paid} = \$10{,}700 - 10{,}000 = \$700$$

Equation [1.2] determines the interest rate paid for 1 year.

$$\text{Percent interest rate} = \frac{\$700}{\$10{,}000} \times 100\% = 7\% \text{ per year}$$

EXAMPLE 1.4

Stereophonics, Inc., plans to borrow \$20,000 from a bank for 1 year at 9% interest for new recording equipment. (*a*) Compute the interest and the total amount due after 1 year. (*b*) Construct a column graph that shows the original loan amount and total amount due after 1 year used to compute the loan interest rate of 9% per year.

Solution

(*a*) Compute the total interest accrued by solving Equation [1.2] for interest accrued.

$$\text{Interest} = \$20{,}000(0.09) = \$1800$$

The total amount due is the sum of principal and interest.

$$\text{Total due} = \$20{,}000 + 1800 = \$21{,}800$$

(*b*) Figure 1–3 shows the values used in Equation [1.2]: \$1800 interest, \$20,000 original loan principal, 1-year interest period.

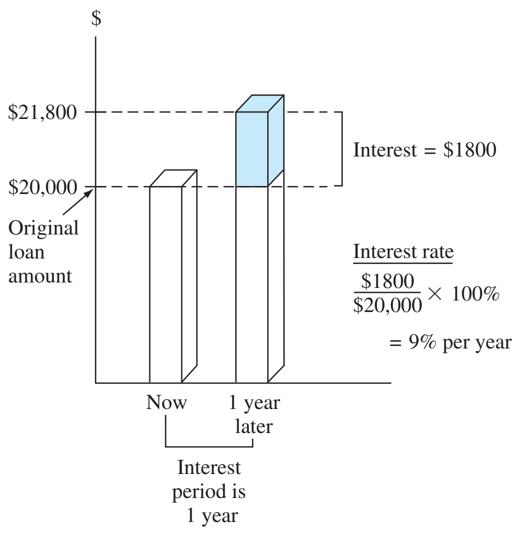


Figure 1–3
Values used to compute an interest rate of 9% per year. Example 1.4.

Comment

Note that in part (*a*), the total amount due may also be computed as

$$\text{Total due} = \text{principal}(1 + \text{interest rate}) = \$20{,}000(1.09) = \$21{,}800$$

Later we will use this method to determine future amounts for times longer than one interest period.

From the perspective of a saver, a lender, or an investor, **interest earned** (Figure 1–2*b*) is the final amount minus the initial amount, or principal.

$$\text{Interest earned} = \text{total amount now} - \text{principal} \qquad [1.3]$$

Interest earned over a specific period of time is expressed as a percentage of the original amount and is called **rate of return (ROR).**

$$\textbf{Rate of return (\%)} = \frac{\textbf{interest accrued per time unit}}{\textbf{principal}} \times 100\% \qquad [1.4]$$

The time unit for rate of return is called the **interest period,** just as for the borrower's perspective. Again, the most common period is 1 year.

The term **return on investment (ROI)** is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineering-oriented programs.

The numerical values in Equations [1.2] and [1.4] are the same, but the term *interest rate paid* is more appropriate for the borrower's perspective, while the *rate of return earned* is better for the investor's perspective.

EXAMPLE 1.5

(*a*) Calculate the amount deposited 1 year ago to have \$1000 now at an interest rate of 5% per year.
(*b*) Calculate the amount of interest earned during this time period.

Solution

(*a*) The total amount accrued (\$1000) is the sum of the original deposit and the earned interest. If X is the original deposit,

$$\text{Total accrued} = \text{deposit} + \text{deposit(interest rate)}$$

$$\$1000 = X + X(0.05) = X(1 + 0.05) = 1.05X$$

The original deposit is

$$X = \frac{1000}{1.05} = \$952.38$$

(*b*) Apply Equation [1.3] to determine the interest earned.

$$\text{Interest} = \$1000 - 952.38 = \$47.62$$

In Examples 1.3 to 1.5 the interest period was 1 year, and the interest amount was calculated at the end of one period. When more than one interest period is involved, e.g., the amount of interest after 3 years, it is necessary to state whether the interest is accrued on a *simple* or *compound* basis from one period to the next. This topic is covered later in this chapter.

Since **inflation** can significantly increase an interest rate, some comments about the fundamentals of inflation are warranted at this early stage. By definition, inflation represents a decrease in the value of a given currency. That is, \$10 now will not purchase the same amount of gasoline for your car (or most other things) as \$10 did 10 years ago. The changing value of the currency affects market interest rates.

Inflation

> In simple terms, interest rates reflect two things: a so-called real rate of return *plus* the expected inflation rate. The real rate of return allows the investor to purchase more than he or she could have purchased before the investment, while inflation raises the real rate to the market rate that we use on a daily basis.

The safest investments (such as government bonds) typically have a 3% to 4% real rate of return built into their overall interest rates. Thus, a market interest rate of, say, 8% per year on a bond means that investors expect the inflation rate to be in the range of 4% to 5% per year. Clearly, inflation causes interest rates to rise.

From the borrower's perspective, the rate of inflation is another interest rate *tacked on to the real interest rate*. And from the vantage point of the saver or investor in a fixed-interest account, inflation *reduces the real rate of return* on the investment. Inflation means that cost and revenue cash flow estimates increase over time. This increase is due to the changing value of money that is forced upon a country's currency by inflation, thus making a unit of currency (such as the dollar) worth less relative to its value at a previous time. We see the effect of inflation in that money purchases less now than it did at a previous time. Inflation contributes to

- A reduction in purchasing power of the currency
- An increase in the CPI (consumer price index)
- An increase in the cost of equipment and its maintenance
- An increase in the cost of salaried professionals and hourly employees
- A reduction in the real rate of return on personal savings and certain corporate investments

In other words, inflation can materially contribute to changes in corporate and personal economic analysis.

Commonly, engineering economy studies assume that inflation affects all estimated values equally. Accordingly, an interest rate or rate of return, such as 8% per year, is applied throughout the analysis without accounting for an additional inflation rate. However, if inflation were explicitly taken into account, and it was reducing the value of money at, say, an average of 4% per year, then it would be necessary to perform the economic analysis using an inflated interest rate. (The rate is 12.32% per year using the relations derived in Chapter 14.)

1.5 Terminology and Symbols

The equations and procedures of engineering economy utilize the following terms and symbols. Sample units are indicated.

P = value or amount of money at a time designated as the present or time 0. Also P is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); monetary units, such as dollars

F = value or amount of money at some future time. Also F is called future worth (FW) and future value (FV); dollars

A = series of consecutive, equal, end-of-period amounts of money. Also A is called the annual worth (AW) and equivalent uniform annual worth (EUAW); dollars per year, euros per month

n = number of interest periods; years, months, days

i = interest rate per time period; percent per year, percent per month

t = time, stated in periods; years, months, days

The symbols P and F represent one-time occurrences: A occurs with the same value in each interest period for a specified number of periods. It should be clear that a present value P represents a single sum of money at some time prior to a future value F or prior to the first occurrence of an equivalent series amount A.

It is important to note that the symbol A always represents a uniform amount (i.e., the same amount each period) that extends through *consecutive* interest periods. Both conditions must exist before the series can be represented by A.

The interest rate i is expressed in percent per interest period, for example, 12% per year. Unless stated otherwise, assume that the rate applies throughout the entire n years or interest periods. The decimal equivalent for i is always used in formulas and equations in engineering economy computations.

All engineering economy problems involve the element of time expressed as n and interest rate i. In general, every problem will involve at least four of the symbols P, F, A, n, and i, with at least three of them estimated or known.

Additional symbols used in engineering economy are defined in Appendix E.

EXAMPLE 1.6

Today, Julie borrowed $5000 to purchase furniture for her new house. She can repay the loan in either of the two ways described below. Determine the engineering economy symbols and their value for each option.

(*a*) Five equal annual installments with interest based on 5% per year.
(*b*) One payment 3 years from now with interest based on 7% per year.

Solution

(*a*) The repayment schedule requires an equivalent annual amount A, which is unknown.

$$P = \$5000 \qquad i = 5\% \text{ per year} \qquad n = 5 \text{ years} \qquad A = ?$$

(*b*) Repayment requires a single future amount F, which is unknown.

$$P = \$5000 \qquad i = 7\% \text{ per year} \qquad n = 3 \text{ years} \qquad F = ?$$

EXAMPLE 1.7

You plan to make a lump-sum deposit of \$5000 now into an investment account that pays 6% per year, and you plan to withdraw an equal end-of-year amount of \$1000 for 5 years, starting next year. At the end of the sixth year, you plan to close your account by withdrawing the remaining money. Define the engineering economy symbols involved.

Solution

All five symbols are present, but the future value in year 6 is the unknown.

$$\begin{aligned} P &= \$5000 \\ A &= \$1000 \text{ per year for 5 years} \\ F &= \text{? at end of year 6} \\ i &= 6\% \text{ per year} \\ n &= 5 \text{ years for the } A \text{ series and 6 for the } F \text{ value} \end{aligned}$$

EXAMPLE 1.8

Last year Jane's grandmother offered to put enough money into a savings account to generate \$5000 in interest this year to help pay Jane's expenses at college. (*a*) Identify the symbols, and (*b*) calculate the amount that had to be deposited exactly 1 year ago to earn \$5000 in interest now, if the rate of return is 6% per year.

Solution

(*a*) Symbols P (last year is -1) and F (this year) are needed.

$$\begin{aligned} P &= ? \\ i &= 6\% \text{ per year} \\ n &= 1 \text{ year} \\ F &= P + \text{interest} = ? + \$5000 \end{aligned}$$

(*b*) Let F = total amount now and P = original amount. We know that $F - P = \$5000$ is accrued interest. Now we can determine P. Refer to Equations [1.1] through [1.4].

$$F = P + Pi$$

The \$5000 interest can be expressed as

$$\begin{aligned} \text{Interest} &= F - P = (P + Pi) - P \\ &= Pi \\ \$5000 &= P(0.06) \\ P &= \frac{\$5000}{0.06} = \$83{,}333.33 \end{aligned}$$

1.6 Cash Flows: Estimation and Diagramming ● ● ●

As mentioned in earlier sections, cash flows are the amounts of money estimated for future projects or observed for project events that have taken place. All cash flows occur during specific time periods, such as 1 month, every 6 months, or 1 year. Annual is the most common time period. For example, a payment of \$10,000 once every year in December for 5 years is a series of 5 outgoing cash flows. And an estimated receipt of \$500 every month for 2 years is a series of 24 incoming cash flows. Engineering economy bases its computations on the timing, size, and direction of cash flows.

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A **plus sign** indicates a cash inflow.

Cash flow

Cash outflows are costs, disbursements, expenses, and taxes caused by projects and business activity. A **negative or minus sign** indicates a cash outflow. When a project involves only costs, the minus sign may be omitted for some techniques, such as benefit/cost analysis.

Of all the steps in Figure 1–1 that outline the engineering economy study, estimating cash flows (step 3) is the most difficult, primarily because it is an attempt to predict the future. Some examples of cash flow estimates are shown here. As you scan these, consider how the cash inflow or outflow may be estimated most accurately.

Cash Inflow Estimates

Income: +\$150,000 per year from sales of solar-powered watches
Savings: +\$24,500 tax savings from capital loss on equipment salvage
Receipt: +\$750,000 received on large business loan plus accrued interest
Savings: +\$150,000 per year saved by installing more efficient air conditioning
Revenue: +\$50,000 to +\$75,000 per month in sales for extended battery life iPhones

Cash Outflow Estimates

Operating costs: −\$230,000 per year annual operating costs for software services
First cost: −\$800,000 next year to purchase replacement earthmoving equipment
Expense: −\$20,000 per year for loan interest payment to bank
Initial cost: −\$1 to −\$1.2 million in capital expenditures for a water recycling unit

All of these are **point estimates,** that is, *single-value estimates* for cash flow elements of an alternative, except for the last revenue and cost estimates listed above. They provide a **range estimate,** because the persons estimating the revenue and cost do not have enough knowledge or experience with the systems to be more accurate. For the initial chapters, we will utilize point estimates. The use of risk and sensitivity analysis for range estimates is covered in the later chapters of this book.

Once all cash inflows and outflows are estimated (or determined for a completed project), the **net cash flow** for each time period is calculated.

$$\text{Net cash flow} = \text{cash inflows} - \text{cash outflows} \qquad [1.5]$$
$$\text{NCF} = R - D \qquad [1.6]$$

where NCF is net cash flow, R is receipts, and D is disbursements.

At the beginning of this section, the *timing, size, and direction of cash flows* were mentioned as important. Because cash flows may take place at any time during an interest period, as a matter of convention, all cash flows are assumed to occur at the end of an interest period.

The end-of-period convention means that all cash inflows and all cash outflows are assumed to take place at the **end of the interest period** in which they actually occur. When several inflows and outflows occur within the same period, the *net* cash flow is assumed to occur at the *end* of the period.

End-of-period convention

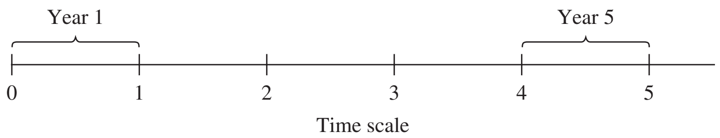


Figure 1–4
A typical cash flow time scale for 5 years.

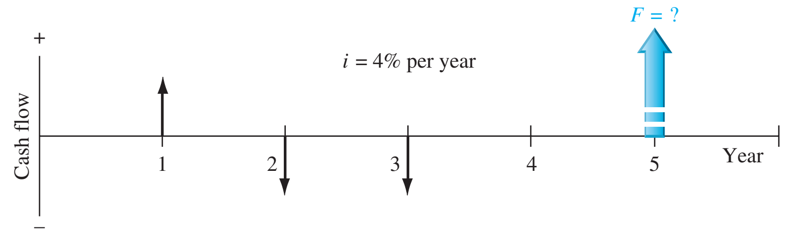


Figure 1–5
Example of positive and negative cash flows.

In assuming end-of-period cash flows, it is important to understand that future (F) and uniform annual (A) amounts are located at the end of the interest period, which is not necessarily December 31. If in Example 1.7 the lump-sum deposit took place on July 1, 2011, the withdrawals will take place on July 1 of each succeeding year for 6 years. Remember, end of the period means end of interest period, not end of calendar year.

The **cash flow diagram** is a very important tool in an economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on the y axis with a time scale on the x axis. The diagram includes what is known, what is estimated, and what is needed. That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

Cash flow diagram time $t = 0$ is the present, and $t = 1$ is the end of time period 1. We assume that the periods are in years for now. The time scale of Figure 1–4 is set up for 5 years. Since the end-of-year convention places cash flows at the ends of years, the "1" marks the end of year 1.

While it is not necessary to use an exact scale on the cash flow diagram, you will probably avoid errors if you make a neat diagram to approximate scale for both time and relative cash flow magnitudes.

The direction of the arrows on the diagram is important to differentiate income from outgo. A vertical arrow pointing up indicates a positive cash flow. Conversely, a down-pointing arrow indicates a negative cash flow. **We will use a bold, colored arrow to indicate what is unknown and to be determined.** For example, if a future value F is to be determined in year 5, a wide, colored arrow with $F = ?$ is shown in year 5. The interest rate is also indicated on the diagram. Figure 1–5 illustrates a cash inflow at the end of year 1, equal cash outflows at the end of years 2 and 3, an interest rate of 4% per year, and the unknown future value F after 5 years. The arrow for the unknown value is generally drawn in the opposite direction from the other cash flows; however, the engineering economy computations will determine the actual sign on the F value.

Before the diagramming of cash flows, a perspective or vantage point must be determined so that + or – signs can be assigned and the economic analysis performed correctly. Assume you borrow \$8500 from a bank today to purchase an \$8000 used car for cash next week, and you plan to spend the remaining \$500 on a new paint job for the car two weeks from now. There are several perspectives possible when developing the cash flow diagram—those of the borrower (that's you), the banker, the car dealer, or the paint shop owner. The cash flow signs and amounts for these perspectives are as follows.

Perspective	Activity	Cash flow with Sign, $	Time, week
You	Borrow	+8500	0
	Buy car	–8000	1
	Paint job	–500	2
Banker	Lender	–8500	0
Car dealer	Car sale	+8000	1
Painter	Paint job	+500	2

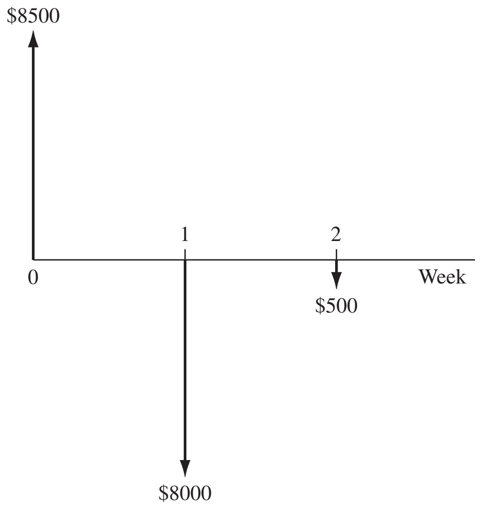


Figure 1–6
Cash flows from perspective of borrower for loan and purchases.

One, and only one, of the perspectives is selected to develop the diagram. For your perspective, all three cash flows are involved and the diagram appears as shown in Figure 1–6 with a time scale of weeks. Applying the end-of-period convention, you have a receipt of +$8500 now (time 0) and cash outflows of −$8000 at the end of week 1, followed by −$500 at the end of week 2.

EXAMPLE 1.9

Each year Exxon-Mobil expends large amounts of funds for mechanical safety features throughout its worldwide operations. Carla Ramos, a lead engineer for Mexico and Central American operations, plans expenditures of $1 million *now* and each of the next 4 years just for the improvement of field-based pressure-release valves. Construct the cash flow diagram to find the equivalent value of these expenditures at the end of year 4, using a cost of capital estimate for safety-related funds of 12% per year.

Solution

Figure 1–7 indicates the uniform and negative cash flow series (expenditures) for five periods, and the unknown F value (positive cash flow equivalent) at exactly the same time as the fifth expenditure. Since the expenditures start immediately, the first $1 million is shown at time 0, not time 1. Therefore, the last negative cash flow occurs at the end of the fourth year, when F also occurs. To make this diagram have a full 5 years on the time scale, the addition of the year −1 completes the diagram. This addition demonstrates that year 0 is the end-of-period point for the year −1.

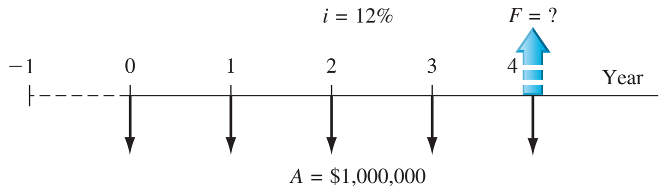


Figure 1–7
Cash flow diagram, Example 1.9.

EXAMPLE 1.10

An electrical engineer wants to deposit an amount P now such that she can withdraw an equal annual amount of $A_1 = \$2000$ per year for the first 5 years, starting 1 year after the deposit, and a different annual withdrawal of $A_2 = \$3000$ per year for the following 3 years. How would the cash flow diagram appear if $i = 8.5\%$ per year?

Solution

The cash flows are shown in Figure 1–8. The negative cash outflow P occurs now. The withdrawals (positive cash inflow) for the A_1 series occur at the end of years 1 through 5, and A_2 occurs in years 6 through 8.

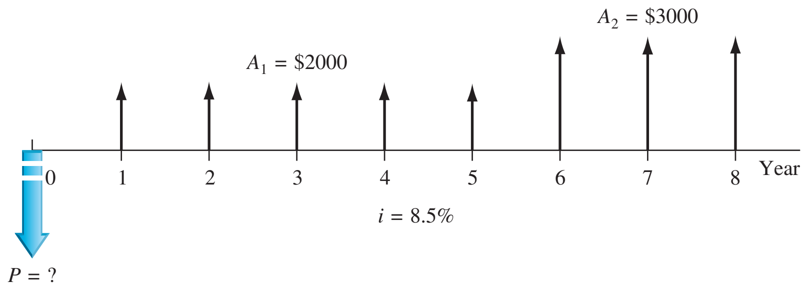


Figure 1–8
Cash flow diagram with two different A series, Example 1.10.

EXAMPLE 1.11

A rental company spent \$2500 on a new air compressor 7 years ago. The annual rental income from the compressor has been \$750. The \$100 spent on maintenance the first year has increased each year by \$25. The company plans to sell the compressor at the end of next year for \$150. Construct the cash flow diagram from the company's perspective and indicate where the present worth now is located.

Solution

Let now be time $t = 0$. The incomes and costs for years -7 through 1 (next year) are tabulated below with net cash flow computed using Equation [1.5]. The net cash flows (one negative, eight positive) are diagrammed in Figure 1–9. Present worth P is located at year 0.

End of Year	Income	Cost	Net Cash Flow
–7	\$ 0	\$2500	\$–2500
–6	750	100	650
–5	750	125	625
–4	750	150	600
–3	750	175	575
–2	750	200	550
–1	750	225	525
0	750	250	500
1	750 + 150	275	625

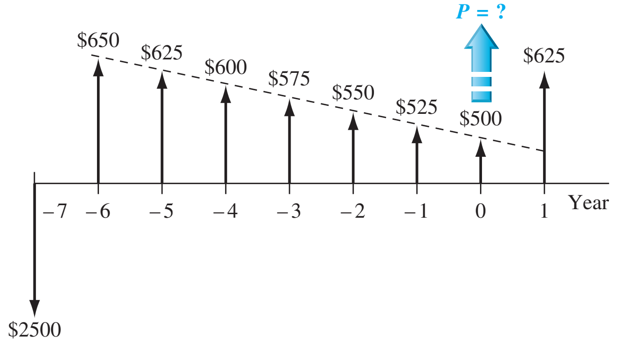


Figure 1–9
Cash flow diagram, Example 1.11.

1.7 Economic Equivalence ● ● ●

Economic equivalence is a fundamental concept upon which engineering economy computations are based. Before we delve into the economic aspects, think of the many types of equivalency we may utilize daily by transferring from one scale to another. Some example transfers between scales are as follows:

Length:
12 inches = 1 foot 3 feet = 1 yard 39.370 inches = 1 meter
100 centimeters = 1 meter 1000 meters = 1 kilometer 1 kilometer = 0.621 mile

Pressure:
1 atmosphere = 1 newton/meter^2 = 10^3 pascal = 1 kilopascal

Often equivalency involves two or more scales. Consider the equivalency of a *speed* of 110 kilometers per hour (kph) into miles per minute using conversions between distance and time scales with three-decimal accuracy.

Speed:
1 mile = 1.609 kilometers 1 hour = 60 minutes
110 kph = 68.365 miles per hour (mph) 68.365 mph = 1.139 miles per minute

Four scales—time in minutes, time in hours, length in miles, and length in kilometers—are combined to develop these equivalent statements on speed. Note that throughout these statements, the fundamental relations of 1 mile = 1.609 kilometers and 1 hour = 60 minutes are applied. If a fundamental relation changes, the entire equivalency is in error.

Now we consider economic equivalency.

Economic equivalence is a combination of **interest rate** and **time value of money** to determine the different amounts of money at different points in time that are equal in economic value.

Economic equivalence

As an illustration, if the interest rate is 6% per year, \$100 today (present time) is equivalent to \$106 one year from today.

$$\text{Amount accrued} = 100 + 100(0.06) = 100(1 + 0.06) = \$106$$

If someone offered you a gift of \$100 today or \$106 one year from today, it would make no difference which offer you accepted from an economic perspective. In either case you have \$106 one year from today. However, the two sums of money are equivalent to each other *only* when the interest rate is 6% per year. At a higher or lower interest rate, \$100 today is not equivalent to \$106 one year from today.

In addition to future equivalence, we can apply the same logic to determine equivalence for previous years. A total of \$100 now is equivalent to \$100/1.06 = \$94.34 one year ago at an interest rate of 6% per year. From these illustrations, we can state the following: \$94.34 last year, \$100 now, and \$106 one year from now are equivalent at an interest rate of 6% per year. The fact that these sums are equivalent can be verified by computing the two interest rates for 1-year interest periods.

$$\frac{\$6}{\$100} \times 100\% = 6\% \text{ per year}$$

and

$$\frac{\$5.66}{\$94.34} \times 100\% = 6\% \text{ per year}$$

The cash flow diagram in Figure 1–10 indicates the amount of interest needed each year to make these three different amounts equivalent at 6% per year.

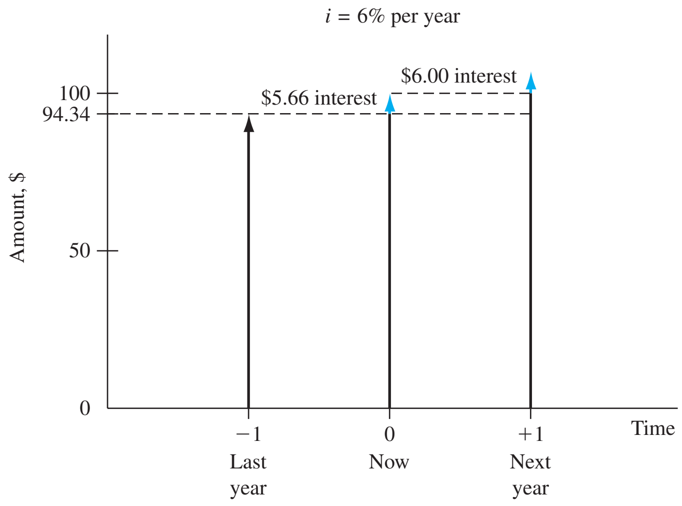


Figure 1–10
Equivalence of money at 6% per year interest.

EXAMPLE 1.12

Manufacturers make backup batteries for computer systems available to Batteries+ dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Batteries+ outlet. Make the calculations necessary to show which of the following statements are true and which are false about battery costs.

(*a*) The amount of $98 now is equivalent to a cost of $105.60 one year from now.
(*b*) A truck battery cost of $200 one year ago is equivalent to $205 now.
(*c*) A $38 cost now is equivalent to $39.90 one year from now.
(*d*) A $3000 cost now is equivalent to $2887.14 one year earlier.
(*e*) The carrying charge accumulated in 1 year on an investment of $20,000 worth of batteries is $1000.

Solution

(*a*) Total amount accrued = 98(1.05) = $102.90 ≠ $105.60; therefore, it is false. Another way to solve this is as follows: Required original cost is 105.60/1.05 = $100.57 ≠ $98.
(*b*) Equivalent cost 1 year ago is 205.00/1.05 = $195.24 ≠ $200; therefore, it is false.
(*c*) The cost 1 year from now is $38(1.05) = $39.90; true.
(*d*) Cost now is 2887.14(1.05) = $3031.50 ≠ $3000; false.
(*e*) The charge is 5% per year interest, or $20,000(0.05) = $1000; true.

Comparison of alternative cash flow series requires the use of equivalence to determine when the series are economically equal or if one is economically preferable to another. The keys to the analysis are the interest rate and the timing of the cash flows. Example 1.13 demonstrates how easy it is to be misled by the size and timing of cash flows.

EXAMPLE 1.13

Howard owns a small electronics repair shop. He wants to borrow $10,000 now and repay it over the next 1 or 2 years. He believes that new diagnostic test equipment will allow him to work on a wider variety of electronic items and increase his annual revenue. Howard received 2-year repayment options from banks A and B.

Year	Amount to pay, \$ per year Bank A	Bank B
1	−5,378.05	−5,000.00
2	−5,378.05	−5,775.00
Total paid	−10,756.10	−10,775.00

After reviewing these plans, Howard decided that he wants to repay the \$10,000 after only 1 year based on the expected increased revenue. During a family conversation, Howard's brother-in-law offered to lend him the \$10,000 now and take \$10,600 after exactly 1 year. Now Howard has three options and wonders which one to take. Which one is economically the best?

Solution

The repayment plans for both banks are economically equivalent at the interest rate of 5% per year. (This is determined by using computations that you will learn in Chapter 2.) Therefore, Howard can choose either plan even though the bank B plan requires a slightly larger sum of money over the 2 years.

The brother-in-law repayment plan requires a total of \$600 in interest 1 year later plus the principal of \$10,000, which makes the interest rate 6% per year. Given the two 5% per year options from the banks, this 6% plan should not be chosen as it is not economically better than the other two. Even though the sum of money repaid is smaller, the timing of the cash flows and the interest rate make it less desirable. The point here is that cash flows themselves, or their sums, cannot be relied upon as the primary basis for an economic decision. The interest rate, timing, and economic equivalence must be considered.

1.8 Simple and Compound Interest ● ● ●

The terms *interest, interest period,* and *interest rate* (introduced in Section 1.4) are useful in calculating equivalent sums of money for one interest period in the past and one period in the future. However, for more than one interest period, the terms *simple interest* and *compound interest* become important.

Simple interest is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as

$$\text{Simple interest} = (\text{principal})(\text{number of periods})(\text{interest rate}) \qquad [1.7]$$
$$I = Pni$$

where I is the amount of interest earned or paid and the interest rate i is expressed in decimal form.

EXAMPLE 1.14

GreenTree Financing lent an engineering company \$100,000 to retrofit an environmentally unfriendly building. The loan is for 3 years at 10% per year simple interest. How much money will the firm repay at the end of 3 years?

Solution

The interest for each of the 3 years is

$$\text{Interest per year} = \$100{,}000(0.10) = \$10{,}000$$

Total interest for 3 years from Equation [1.7] is

$$\text{Total interest} = \$100{,}000(3)(0.10) = \$30{,}000$$

The amount due after 3 years is

$$\text{Total due} = \$100{,}000 + 30{,}000 = \$130{,}000$$

The interest accrued in the first year and in the second year does not earn interest. The interest due each year is $10,000 calculated only on the $100,000 loan principal.

In most financial and economic analyses, we use **compound interest** calculations.

For *compound interest,* the interest accrued for each interest period is calculated on the **principal plus the total amount of interest accumulated in all previous periods.** Thus, compound interest means interest on top of interest.

Compound interest reflects the effect of the time value of money on the interest also. Now the interest for one period is calculated as

$$\textbf{Compound interest} = \textbf{(principal + all accrued interest)(interest rate)} \qquad [1.8]$$

In mathematical terms, the interest I_t for time period t may be calculated using the relation.

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_J\right)(i) \qquad [1.9]$$

EXAMPLE 1.15

Assume an engineering company borrows $100,000 at 10% per year compound interest and will pay the principal and all the interest after 3 years. Compute the annual interest and total amount due after 3 years. Graph the interest and total owed for each year, and compare with the previous example that involved simple interest.

Solution

To include compounding of interest, the annual interest and total owed each year are calculated by Equation [1.8].

Interest, year 1:	100,000(0.10) = $10,000
Total due, year 1:	100,000 + 10,000 = $110,000
Interest, year 2:	110,000(0.10) = $11,000
Total due, year 2:	110,000 + 11,000 = $121,000
Interest, year 3:	121,000(0.10) = $12,100
Total due, year 3:	121,000 + 12,100 = $133,100

The repayment plan requires no payment until year 3 when all interest and the principal, a total of $133,100, are due. Figure 1–11 uses a cash flow diagram format to compare end-of-year (*a*) simple and (*b*) compound interest and total amounts owed. The differences due to compounding are clear. An extra $133,100 – 130,000 = $3100 in interest is due for the compounded interest loan.

Note that while simple interest due each year is constant, the compounded interest due grows geometrically. Due to this geometric growth of compound interest, the difference between simple and compound interest accumulation increases rapidly as the time frame increases. For example, if the loan is for 10 years, not 3, the extra paid for compounding interest may be calculated to be $59,374.

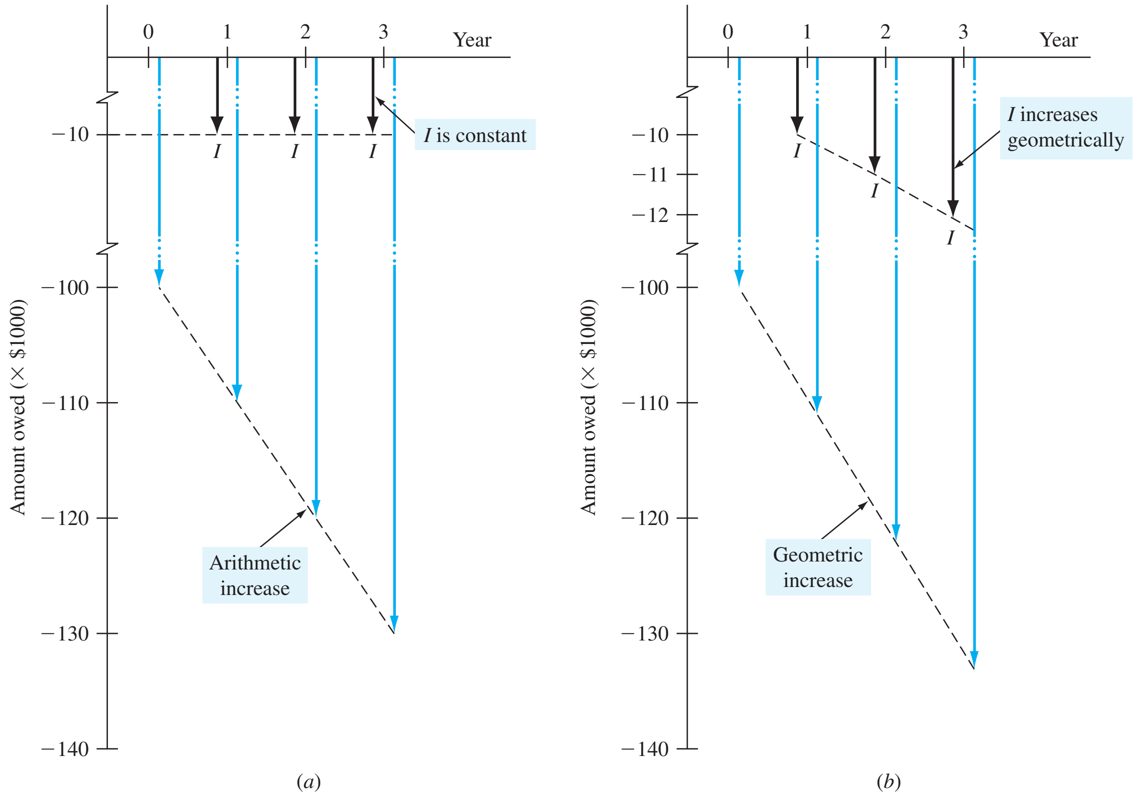


Figure 1–11
Interest I owed and total amount owed for (*a*) simple interest (Example 1.14) and (*b*) compound interest (Example 1.15).

A more efficient way to calculate the total amount due after a number of years in Example 1.15 is to utilize the fact that compound interest increases geometrically. This allows us to skip the year-by-year computation of interest. In this case, the **total amount due at the end of each year** is

Year 1: $\$100{,}000(1.10)^1 = \$110{,}000$
Year 2: $\$100{,}000(1.10)^2 = \$121{,}000$
Year 3: $\$100{,}000(1.10)^3 = \$133{,}100$

This allows future totals owed to be calculated directly without intermediate steps. The general form of the equation is

$$\text{Total due after } n \text{ years} = \text{principal}(1 + \text{interest rate})^{n \text{ years}} \qquad [1.10]$$
$$= P(1 + i)^n$$

where i is expressed in decimal form. Equation [1.10] was applied above to obtain the \$133,100 due after 3 years. This fundamental relation will be used many times in the upcoming chapters.

We can combine the concepts of interest rate, compound interest, and equivalence to demonstrate that different loan repayment plans may be equivalent, but differ substantially in amounts paid from one year to another and in the total repayment amount. This also shows that there are many ways to take into account the time value of money.

EXAMPLE 1.16

Table 1–1 details four different loan repayment plans described below. Each plan repays a $5000 loan in 5 years at 8% per year compound interest.

- **Plan 1: Pay all at end.** No interest or principal is paid until the end of year 5. Interest accumulates each year on the total of principal and all accrued interest.
- **Plan 2: Pay interest annually, principal repaid at end.** The accrued interest is paid each year, and the entire principal is repaid at the end of year 5.
- **Plan 3: Pay interest and portion of principal annually.** The accrued interest and one-fifth of the principal (or $1000) are repaid each year. The outstanding loan balance decreases each year, so the interest (column 2) for each year decreases.
- **Plan 4: Pay equal amount of interest and principal.** Equal payments are made each year with a portion going toward principal repayment and the remainder covering the accrued interest. Since the loan balance decreases at a rate slower than that in plan 3 due to the equal end-of-year payments, the interest decreases, but at a slower rate.

TABLE 1–1 Different Repayment Schedules Over 5 Years for $5000 at 8% Per Year Compound Interest

(1) End of Year	(2) Interest Owed for Year	(3) Total Owed at End of Year	(4) End-of-Year Payment	(5) Total Owed After Payment
Plan 1: Pay All at End				
0				$5000.00
1	$400.00	$5400.00	—	5400.00
2	432.00	5832.00	—	5832.00
3	466.56	6298.56	—	6298.56
4	503.88	6802.44	—	6802.44
5	544.20	7346.64	$−7346.64	
Total			$−7346.64	
Plan 2: Pay Interest Annually; Principal Repaid at End				
0				$5000.00
1	$400.00	$5400.00	$−400.00	5000.00
2	400.00	5400.00	−400.00	5000.00
3	400.00	5400.00	−400.00	5000.00
4	400.00	5400.00	−400.00	5000.00
5	400.00	5400.00	−5400.00	
Total			$−7000.00	
Plan 3: Pay Interest and Portion of Principal Annually				
0				$5000.00
1	$400.00	$5400.00	$−1400.00	4000.00
2	320.00	4320.00	−1320.00	3000.00
3	240.00	3240.00	−1240.00	2000.00
4	160.00	2160.00	−1160.00	1000.00
5	80.00	1080.00	−1080.00	
Total			$−6200.00	
Plan 4: Pay Equal Annual Amount of Interest and Principal				
0				$5000.00
1	$400.00	$5400.00	$–1252.28	4147.72
2	331.82	4479.54	–1252.28	3227.25
3	258.18	3485.43	–1252.28	2233.15
4	178.65	2411.80	–1252.28	1159.52
5	92.76	1252.28	–1252.28	
Total			$–6261.40	

(*a*) Make a statement about the *equivalence* of each plan at 8% compound interest.
(*b*) Develop an 8% per year *simple* interest repayment plan for this loan using the same approach as plan 2. Comment on the total amounts repaid for the two plans.

Solution

(*a*) The amounts of the annual payments are different for each repayment schedule, and the total amounts repaid for most plans are different, even though each repayment plan requires exactly 5 years. The difference in the total amounts repaid can be explained by the time value of money and by the partial repayment of principal prior to year 5.

A loan of $5000 at time 0 made at 8% per year compound interest is equivalent to each of the following:

Plan 1 $7346.64 at the end of year 5

Plan 2 $400 per year for 4 years and $5400 at the end of year 5

Plan 3 Decreasing payments of interest and partial principal in years 1 ($1400) through 5 ($1080)

Plan 4 $1252.28 per year for 5 years

An engineering economy study typically uses plan 4; interest is compounded, and a constant amount is paid each period. This amount covers accrued interest and a partial amount of principal repayment.

(*b*) The repayment schedule for 8% per year simple interest is detailed in Table 1–2. Since the annual accrued interest of $400 is paid each year and the principal of $5000 is repaid in year 5, the schedule is exactly the same as that for 8% per year compound interest, and the total amount repaid is the same at $7000. In this unusual case, simple and compound interest result in the same total repayment amount. Any deviation from this schedule will cause the two plans and amounts to differ.

TABLE 1–2 A 5-Year Repayment Schedule of $5000 at 8% per Year Simple Interest

End of Year	Interest Owed for Year	Total Owed at End of Year	End-of-Year Payment	Total Owed After Payment
0				$5000
1	$400	$5400	$−400	5000
2	400	5400	−400	5000
3	400	5400	−400	5000
4	400	5400	−400	5000
5	400	5400	−5400	0
Total			$−7000	

1.9 Minimum Attractive Rate of Return

For any investment to be profitable, the investor (corporate or individual) expects to receive more money than the amount of capital invested. In other words, a fair *rate of return,* or *return on investment,* must be realizable. The definition of ROR in Equation [1.4] is used in this discussion, that is, amount earned divided by the principal.

Engineering alternatives are evaluated upon the prognosis that a reasonable ROR can be expected. Therefore, some reasonable rate must be established for the selection criteria (step 4) of the engineering economy study (Figure 1–1).

Minimum Attractive Rate of Return (MARR)

The Minimum Attractive Rate of Return (MARR) is a reasonable rate of return established for the evaluation and selection of alternatives. A project is not economically viable unless it is **expected to return at least the MARR.** MARR is also referred to as the *hurdle rate, cutoff rate, benchmark rate,* and *minimum acceptable rate of return.*

Figure 1–12 indicates the relations between different rate of return values. In the United States, the current U.S. Treasury Bill return is sometimes used as the benchmark safe rate. The MARR will always be higher than this, or a similar, safe rate. The MARR is not a rate that is calculated as a ROR. The MARR is established by (financial) managers and is used as a criterion against which an alternative's ROR is measured, when making the accept/reject investment decision.

To develop a foundation-level understanding of how a MARR value is established and used to make investment decisions, we return to the term **capital** introduced in Section 1.1. Although the MARR is used as a criterion to decide on investing in a project, the size of MARR is fundamentally connected to how much it costs to obtain the needed capital funds. It always costs money in the form of interest to raise capital. The interest, expressed as a percentage rate per year, is called the **cost of capital.** As an example on a personal level, if you want to purchase a new widescreen HDTV, but do not have sufficient money (capital), you could obtain a bank loan for, say, a cost of capital of 9% per year and pay for the TV in cash now. Alternatively, you might choose to use your credit card and pay off the balance on a monthly basis. This approach will probably cost you at least 15% per year. Or, you could use funds from your savings account that earns 5% per year and pay cash. This approach means that you also forgo future returns from these funds. The 9%, 15%, and 5% rates are your cost of capital estimates to raise the capital for the system by different methods of capital financing. In analogous ways, corporations estimate the **cost of capital** from different sources to raise funds for engineering projects and other types of projects.

Cost of capital

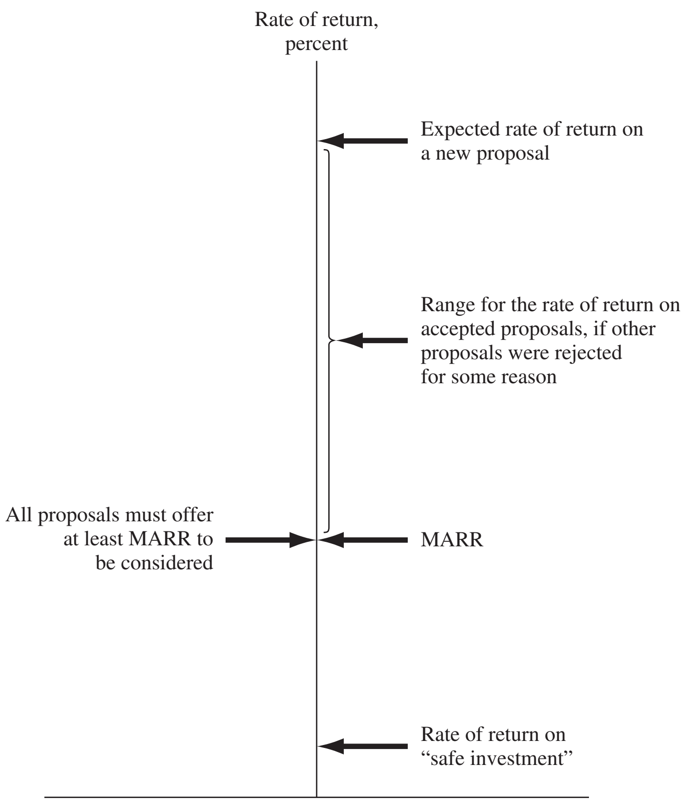


Figure 1–12
Size of MAAR relative to other rate of return values.

In general, capital is developed in two ways—equity financing and debt financing. A combination of these two is very common for most projects. Chapter 10 covers these in greater detail, but a snapshot description follows.

Equity financing The corporation uses its own funds from cash on hand, stock sales, or retained earnings. Individuals can use their own cash, savings, or investments. In the example above, using money from the 5% savings account is equity financing.

Debt financing The corporation borrows from outside sources and repays the principal and interest according to some schedule, much like the plans in Table 1–1. Sources of debt capital may be bonds, loans, mortgages, venture capital pools, and many others. Individuals, too, can utilize debt sources, such as the credit card (15% rate) and bank options (9% rate) described above.

Combinations of debt-equity financing mean that a **weighted average cost of capital (WACC)** results. If the HDTV is purchased with 40% credit card money at 15% per year and 60% savings account funds earning 5% per year, the weighted average cost of capital is 0.4(15) + 0.6(5) = 9% per year.

For a corporation, the *established MARR* used as a criterion to accept or reject an investment alternative will usually be *equal to or higher than the WACC* that the corporation must bear to obtain the necessary capital funds. So the inequality

$$\text{ROR} \geq \text{MARR} > \text{WACC} \qquad [1.11]$$

must be correct for an accepted project. Exceptions may be government-regulated requirements (safety, security, environmental, legal, etc.), economically lucrative ventures expected to lead to other opportunities, etc.

Often there are many alternatives that are expected to yield a ROR that exceeds the MARR as indicated in Figure 1–12, but there may not be sufficient capital available for all, or the project's risk may be estimated as too high to take the investment chance. Therefore, new projects that are undertaken usually have an expected return at least as great as the return on another alternative that is not funded. The expected rate of return on the unfunded project is called the **opportunity cost.**

The opportunity cost is the rate of return of a forgone opportunity caused by the inability to pursue a project. Numerically, it is the **largest rate of return of all the projects not accepted (forgone) due to the lack of capital funds or other resources.** When no specific MARR is established, the de facto MARR is the opportunity cost, i.e., the ROR of the first project not undertaken due to unavailability of capital funds.

Opportunity cost

As an illustration of opportunity cost, refer to Figure 1–12 and assume a MARR of 12% per year. Further, assume that a proposal, call it A, with an expected ROR = 13% is not funded due to a lack of capital. Meanwhile, proposal B has a ROR = 14.5% and is funded from available capital. Since proposal A is not undertaken due to the lack of capital, its estimated ROR of 13% is the *opportunity cost;* that is, the opportunity to make an additional 13% return is forgone.

1.10 Introduction to Spreadsheet Use

The functions on a computer spreadsheet can greatly reduce the amount of hand work for equivalency computations involving *compound interest* and the terms P, F, A, i, and n. The use of a calculator to solve most simple problems is preferred by many students and professors as described in Appendix D. However, as cash flow series become more complex, the spreadsheet offers a good alternative. Microsoft Excel is used throughout this book because it is readily available and easy to use. Appendix A is a primer on using spreadsheets and Excel. The functions used in engineering economy are described there in detail, with explanations of all the parameters. Appendix A also includes a section on spreadsheet layout that is useful when the economic analysis is presented to someone else—a coworker, a boss, or a professor.

A total of seven Excel functions can perform most of the fundamental engineering economy calculations. The functions are great supplemental tools, but they do not replace the understanding of engineering economy relations, assumptions, and techniques. Using the symbols P, F, A, i, and n defined in the previous section, the functions most used in engineering economic analysis are formulated as follows.

To find the present value P: **= PV(*i*%,*n*,*A*,*F*)**

To find the future value F: **= FV(*i*%,*n*,*A*,*P*)**

To find the equal, periodic value A: **= PMT(*i*%,*n*,*P*,*F*)**

To find the number of periods n: **= NPER(*i*%,*A*,*P*,*F*)**

To find the compound interest rate i: **= RATE(*n*,*A*,*P*,*F*)**

To find the compound interest rate i: **= IRR(first_cell:last_cell)**

To find the present value P of any series: **= NPV(i%, second_cell:last_cell) + first_cell**

If some of the parameters don't apply to a particular problem, they can be omitted and zero is assumed. For readability, spaces can be inserted between parameters within parentheses. If the parameter omitted is an interior one, the comma must be entered. The last two functions require that a series of numbers be entered into contiguous spreadsheet cells, but the first five can be used with no supporting data. In all cases, the function must be preceded by an equals sign (=) in the cell where the answer is to be displayed.

To understand how the spreadsheet functions work, look back at Example 1.6*a*, where the equivalent annual amount A is unknown, as indicated by $A = ?$. (In Chapter 2, we learn how engineering economy factors calculate A, given P, i, and n.) To find A using a spreadsheet function, simply enter the PMT function = PMT(5%,5,5000). Figure 1–13 is a screen image of a spreadsheet with the PMT function entered into cell B4. The answer ($1154.87) is displayed. The answer may appear in red and in parentheses, or with a minus sign on your screen to indicate a negative amount from the perspective of a reduction in the account balance. The right side of Figure 1–13 presents the solution to Example 1.6*b*. The future value F is determined by using the FV function. The FV function appears in the formula bar; and many examples throughout this text will include cell tags, as shown here, to indicate the format of important entries.

The following example demonstrates the use of a spreadsheet to develop relations (not built-in functions) to calculate interest and cash flows. Once set up, the spreadsheet can be used to perform sensitivity analysis for estimates that are subject to change. We will illustrate the use of spreadsheets throughout the chapters. (*Note:* The spreadsheet examples may be omitted, if spreadsheets are not used in the course. A solution by hand is included in virtually all examples.)

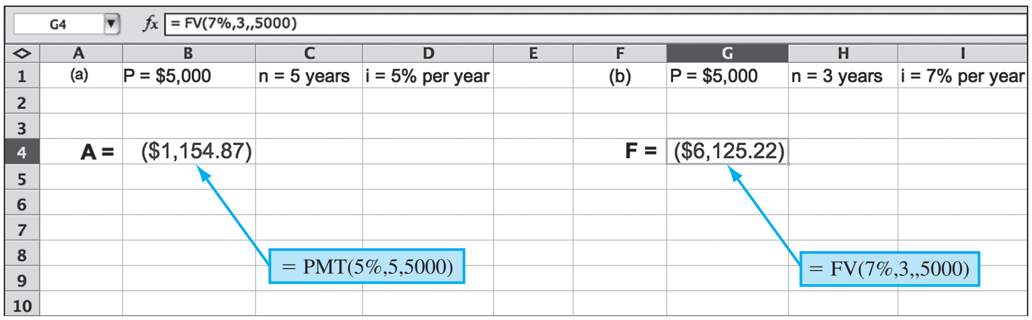


Figure 1–13

Use of spreadsheet functions PMT and FV, Example 1.6.

EXAMPLE 1.17

A Japan-based architectural firm has asked a United States–based software engineering group to infuse GPS sensing capability via satellite into monitoring software for high-rise structures in order to detect greater than expected horizontal movements. This software could be very beneficial as an advance warning of serious tremors in earthquake-prone areas in Japan and the United States. The inclusion of accurate GPS data is estimated to increase annual revenue over that for the current software system by $200,000 for each of the next 2 years, and by $300,000 for each of years 3 and 4. The planning horizon is only 4 years due to the rapid advances made internationally in building-monitoring software. Develop spreadsheets to answer the questions below.

(*a*) Determine the total interest and total revenue after 4 years, using a compound rate of return of 8% per year.
(*b*) Repeat part (*a*) if estimated revenue increases from $300,000 to $600,000 in years 3 and 4.
(*c*) Repeat part (*a*) if inflation is estimated to be 4% per year. This will decrease the *real rate of return* from 8% to 3.85% per year (Chapter 14 shows why).

Solution by Spreadsheet

Refer to Figure 1–14*a* to *d* for the solutions. All the spreadsheets contain the same information, but some cell values are altered as required by the question. (Actually, all the questions can be answered on one spreadsheet by changing the numbers. Separate spreadsheets are shown here for explanation purposes only.)

The Excel functions are constructed with reference to the cells, not the values themselves, so that sensitivity analysis can be performed without function changes. This approach treats the value in a cell as a *global variable* for the spreadsheet. For example, the 8% rate in cell B2 will be referenced in all functions as B2, not 8%. Thus, a change in the rate requires only one alteration in the cell B2 entry, not in every relation where 8% is used. See Appendix A for additional information about using cell referencing and building spreadsheet relations.

(*a*) Figure 1–14*a* shows the results, and Figure 1–14*b* presents all spreadsheet relations for estimated interest and revenue (yearly in columns C and E, cumulative in columns D and F). As an illustration, for year 3 the interest I_3 and revenue plus interest R_3 are

$$\begin{aligned} I_3 &= \text{(cumulative revenue through year 2)(rate of return)} \\ &= \$416{,}000(0.08) \\ &= \$33{,}280 \end{aligned}$$

$$\begin{aligned} R_3 &= \text{revenue in year 3} + I_3 \\ &= \$300{,}000 + 33{,}280 \\ &= \$333{,}280 \end{aligned}$$

The detailed relations shown in Figure 1–14*b* calculate these values in cells C8 and E8.

Cell C8 relation for I_3: = F7*B2
Cell E8 relation for CF_3: = B8 + C8

The equivalent amount after 4 years is $1,109,022, which is comprised of $1,000,000 in total revenue and $109,022 in interest compounded at 8% per year. The shaded cells in Figure 1–14*a* and *b* indicate that the sum of the annual values and the last entry in the cumulative columns must be equal.

(*b*) To determine the effect of increasing estimated revenue for years 3 and 4 to $600,000, use the same spreadsheet and change the entries in cells B8 and B9 as shown in Figure 1–14*c*. Total interest increases 22%, or $24,000, from $109,222 to $133,222.

(*c*) Figure 1–14*d* shows the effect of changing the original *i* value from 8% to an inflation-adjusted rate of 3.85% in cell B2 on the first spreadsheet. [Remember to return to the $300,000 revenue estimates for years 3 and 4 after working part (*b*).] Inflation has now reduced total interest by 53% from $109,222 to $51,247, as shown in cell C10.

	A	B	C	D	E	F
1	Part (a) - Find totals in year 4					
2	i =	8.0%				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200,000	0	0	200,000	200,000
7	2	200,000	16,000	16,000	216,000	416,000
8	3	300,000	33,280	49,280	333,280	749,280
9	4	300,000	59,942	109,222	359,942	1,109,222
10			109,222		1,109,222	

(*a*) Total interest and revenue for base case, year 4

	A	B	C	D	E	F
1	Part (a) - Find totals in year 4					
2	i =	0.08				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200000	0	=C6	=B6 + C6	=E6
7	2	200000	=F6*B1	=C7 + D6	=B7 + C7	=E7 + F6
8	3	300000	=F7*B1	=C8 + D7	=B8 + C8	=E8 + F7
9	4	300000	=F8*B1	=C9 + D8	=B9 + C9	=E9 + F8
10			=SUM(C6:C9)		=SUM(E6:E9)	

(*b*) Spreadsheet relations for base case

	A	B	C	D	E	F
1	Part (b) - Find totals in year 4 with increased revenues					
2	i =	8.0%				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200,000	0	0	200,000	200,000
7	2	200,000	16,000	16,000	216,000	416,000
8	3	600,000	33,280	49,280	633,280	1,049,280
9	4	600,000	83,942	133,222	683,942	1,733,222
10			133,222		1,733,222	
11						
12			Revenue changed			
13						

(*c*) Totals with increased revenue in years 3 and 4

	A	B	C	D	E	F
1	Part (c) - Find totals in year 4 considering 4% inflation					
2	i =	3.85%				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200,000	0	0	200,000	200,000
7	2	200,000	7,700	7,700	207,700	407,700
8	3	300,000	15,696	23,396	315,696	723,396
9	4	300,000	27,851	51,247	327,851	1,051,247
10			51,247		1,051,247	

Rate of return changed

(*d*) Totals with inflation of 4% per year considered

Figure 1–14
Spreadsheet solutions with sensitivity analysis, Example 1.17*a* to *c*.

Comment

Later we will learn how to utilize the NPV and FV Excel financial functions to obtain the same answers determined in Figure 1–14, where we developed each basic relation.

When you are working with an Excel spreadsheet, it is possible to display all of the entries and functions on the screen as shown in Figure 1–14*b* by simultaneously touching the <Ctrl> and <`> keys, which may be in the upper left of the keyboard on the key with <~>.

CHAPTER SUMMARY

Engineering economy is the application of economic factors and criteria to evaluate alternatives, considering the time value of money. The engineering economy study involves computing a specific economic measure of worth for estimated cash flows over a specific period of time.

The concept of *equivalence* helps in understanding how different sums of money at different times are equal in economic terms. The differences between simple interest (based on principal only) and compound interest (based on principal and interest upon interest) have been described in formulas, tables, and graphs. This power of compounding is very noticeable, especially over extended periods of time, and for larger sums of money.

The MARR is a reasonable rate of return established as a hurdle rate to determine if an alternative is economically viable. The MARR is always higher than the return from a safe investment and the cost to acquire needed capital.

Also, we learned a lot about cash flows:

End-of-year convention for cash flow location

Net cash flow computation

Different perspectives in determining the cash flow sign

Construction of a cash flow diagram

Difficulties with estimating future cash flows accurately

PROBLEMS

Basic Concepts

1.1 List the four essential elements involved in decision making in engineering economic analysis.

1.2 What is meant by (*a*) limited capital funds and (*b*) sensitivity analysis?

1.3 List three measures of worth that are used in engineering economic analysis.

1.4 Identify the following factors as either economic (tangible) or noneconomic (intangible): first cost, leadership, taxes, salvage value, morale, dependability, inflation, profit, acceptance, ethics, interest rate.

Ethics

1.5 Stefanie is a design engineer with an international railroad locomotive manufacturing company in Illinois. Management wants to return some of the engineering design work to the United States rather than export all of it to India, where the primary design work has been accomplished for the last decade. This transfer will employ more people locally and could improve the economic conditions for families in and around Illinois.

Stefanie and her design team were selected as a test case to determine the quality and speed of the design work they could demonstrate on a more fuel-efficient diesel locomotive. Neither she nor any of her team members have done such a significant design job, because their jobs had previously entailed only the interface with the subcontracted engineers in India. One of her team members had a great design idea on a key element that will improve fuel efficiency by approximately 15%. She told Stefanie it came from one of the Indian-generated documents, but that it would probably be okay for the team to use it and remain silent as to its origin, since it was quite clear the U.S. management was about to cancel the foreign contract. Although reluctant at first, Stefanie did go forward with a design that included the efficiency improvement, and no mention of the origin of the idea was made at the time of the oral presentation or documentation delivery. As a result, the Indian contract was canceled and full design responsibility was transferred to Stefanie's group.

Consult the NSPE Code of Ethics for Engineers (Appendix C) and identify sections that are points of concern about Stefanie's decisions and actions.

1.6 Consider the common moral precept that stealing is wrong. Hector is with a group of friends in a local supermarket. One of Hector's buddies takes a high-energy drink from a six-pack on the shelf, opens it, drinks it, and returns the empty can to the package, with no intention of paying for it. He then invites the others to do the same, saying, "It's only one drink. Others do it all the time." All the others, except Hector, have now consumed a drink of their choice. Personally, Hector believes this is a form of stealing. State three actions that Hector can take, and evaluate them from the personal moral perspective.

1.7 While going to work this morning off site from his office, an engineer accidently ran a stop sign and was in a car accident that resulted in the death of a 5-year-old child. He has a strong belief in the universal moral that it is wrong to do serious harm to another person. Explain the conflict that can arise for him between the universal moral and his personal moral about doing serious harm, given the accident was deemed his fault.

1.8 Claude is a fourth-year engineering university student who has just been informed by his instructor that he made a very low grade on his Spanish language final test for the year. Although he had a passing score prior to the final, his final grade was so low that he has now flunked the entire year and will likely have to extend his graduation another semester or two.

Throughout the year, Claude, who hated the course and his instructor, has copied homework, cheated on tests, and never seriously studied for anything in the course. He did realize during the semester that he was doing something that even he considered wrong morally and ethically. He knew he had done badly on the final. The classroom was reconfigured for the final exam in a way that he could not get any answers from classmates, and cell phones were collected prior to the exam, thus removing texting possibilities to friends outside the classroom who might help him on the final exam. Claude is now face to face with the instructor in her office. The question to Claude is, "What have you been doing throughout this year to make passing scores repeatedly, but demonstrate such a poor command of Spanish on the final exam?"

From an ethical viewpoint, what options does Claude have in his answer to this question? Also, discuss some of the possible effects that this experience may have upon Claude's future actions and moral dilemmas.

Interest Rate and Rate of Return

1.9 RKI Instruments borrowed \$3,500,000 from a private equity firm for expansion of its manufacturing facility for making carbon monoxide monitors/controllers. The company repaid the loan after 1 year with a single payment of \$3,885,000. What was the interest rate on the loan?

1.10 Emerson Processing borrowed \$900,000 for installing energy-efficient lighting and safety equipment in its La Grange manufacturing facility. The terms of the loan were such that the company could pay *interest only* at the end of each year for up to 5 years, after which the company would have to pay the entire amount due. If the interest rate on the loan was 12% per year and the company paid only the interest for 4 years, determine the following:

(*a*) The amount of each of the four interest payments
(*b*) The amount of the final payment at the end of year 5

1.11 Which of the following 1-year investments has the highest rate of return?

(*a*) \$12,500 that yields \$1125 in interest,
(*b*) \$56,000 that yields \$6160 in interest, or
(*c*) \$95,000 that yields \$7600 in interest.

1.12 A new engineering graduate who started a consulting business borrowed money for 1 year to furnish the office. The amount of the loan was \$23,800, and it had an interest rate of 10% per year. However, because the new graduate had not built up a credit history, the bank made him buy loan-default insurance that cost 5% of the loan amount. In addition, the bank charged a loan setup fee of \$300. What was the effective interest rate the engineer paid for the loan?

1.13 When the inflation rate is expected to be 8% per year, what is the market interest rate likely to be?

Terms and Symbols

1.14 The symbol *P* represents an amount of money at a time designated as present. The following symbols also represent a present amount of money and require similar calculations. Explain what each symbol stands for: PW, PV, NPV, DCF, and CC.

1.15 Identify the four engineering economy symbols and their values from the following problem statement. Use a question mark with the symbol whose value is to be determined.

Thompson Mechanical Products is planning to set aside \$150,000 now for possibly replacing its large synchronous refiner motors whenever it becomes necessary. If the replacement is not needed for 7 years, how much will the company have in its investment set-aside account, provided it achieves a rate of return of 11% per year?

1.16 Identify the four engineering economy symbols and their values from the following problem statement.

Use a question mark with the symbol whose value is to be determined.

Atlas Long-Haul Transportation is considering installing Valutemp temperature loggers in all of its refrigerated trucks for monitoring temperatures during transit. If the systems will reduce insurance claims by $100,000 two years from now, how much should the company be willing to spend now, if it uses an interest rate of 12% per year?

1.17 Identify the four engineering economy symbols and their values from the following problem statement. Use a question mark with the symbol whose value is to be determined.

A green algae, *Chlamydomonas reinhardtii,* can produce hydrogen when temporarily deprived of sulfur for up to 2 days at a time. A small company needs to purchase equipment costing $3.4 million to commercialize the process. If the company wants to earn a rate of return of 10% per year and recover its investment in 8 years, what must be the net value of the hydrogen produced each year?

1.18 Identify the four engineering economy symbols and their values from the following problem statement. Use a question mark with the symbol whose value is to be determined.

Vision Technologies, Inc., is a small company that uses ultra-wideband technology to develop devices that can detect objects (including people) inside of buildings, behind walls, or below ground. The company expects to spend $100,000 per year for labor and $125,000 per year for supplies before a product can be marketed. At an interest rate of 15% per year, what is the total equivalent future amount of the company's expenses at the end of 3 years?

Cash Flows

1.19 What is meant by end-of-period convention?

1.20 Identify the following as cash inflows or outflows to commercial air carriers: fuel cost, pension plan contributions, fares, maintenance, freight revenue, cargo revenue, extra-bag charges, water and sodas, advertising, landing fees, seat preference fees.

1.21 Many credit unions use semiannual interest periods to pay interest on customer savings accounts. For a credit union that uses June 30 and December 31 as its semiannual interest periods, determine the end-of-period amounts that will be recorded for the deposits shown in the table.

Month	Deposit, $
Jan	50
Feb	70
Mar	—
Apr	120
May	20
June	—
July	150
Aug	90
Sept	—
Oct	—
Nov	40
Dec	110

1.22 For a company that uses a year as its interest period, determine the *net cash flow* that will be recorded at the *end of the year* from the cash flows shown.

Month	Receipts, $1000	Disbursements, $1000
Jan	500	300
Feb	800	500
Mar	200	400
Apr	120	400
May	600	500
June	900	600
July	800	300
Aug	700	300
Sept	900	500
Oct	500	400
Nov	400	400
Dec	1800	700

1.23 Construct a cash flow diagram for the following cash flows: $25,000 outflow at time 0, $9000 per year inflow in years 1 through 5 at an interest rate of 10% per year, and an unknown future amount in year 5.

1.24 Construct a cash flow diagram to find the present worth in year 0 at an interest rate of 15% per year for the following situation.

Year	Cash Flow, $
0	−19,000
1–4	+8,100

1.25 Construct a cash flow diagram that represents the amount of money that will be accumulated in 15 years from an investment of $40,000 now at an interest rate of 8% per year.

Equivalence

1.26 At an interest rate of 15% per year, an investment of $100,000 one year ago is equivalent to how much now?

1.27 During a recession, the price of goods and services goes down because of low demand. A company that makes Ethernet adapters is planning to expand its production facility at a cost of $1,000,000 one year from now. However, a contractor who needs work has offered to do the job for $790,000 if the company will do the expansion now instead of 1 year from now. If the interest rate is 15% per year, how much of a discount is the company getting?

1.28 As a principal in the consulting firm where you have worked for 20 years, you have accumulated 5000 shares of company stock. One year ago, each share of stock was worth $40. The company has offered to buy back your shares for $225,000. At what interest rate would the firm's offer be equivalent to the worth of the stock last year?

1.29 A design/build engineering company that usually gives year-end bonuses in the amount of $8000 to each of its engineers is having cash flow problems. The company said that although it could not give bonuses this year, it would give each engineer two bonuses next year, the regular one of $8000 plus an amount equivalent to the $8000 that each engineer should have gotten this year. If the interest rate is 8% per year, what will be the total amount of bonus money the engineers should get next year?

1.30 University tuition and fees can be paid by using one of two plans.

Early-bird: Pay total amount due *1 year in advance* and get a 10% discount.

On-time: Pay total amount due when classes start.

The cost of tuition and fees is $10,000 per year.

(*a*) How much is paid in the early-bird plan?
(*b*) What is the equivalent amount of the savings compared to the on-time payment at the time that the on-time payment is made?

Simple and Compound Interest

1.31 If a company sets aside $1,000,000 now into a contingency fund, how much will the company have in 2 years, if it does not use any of the money and the account grows at a rate of 10% per year?

1.32 Iselt Welding has extra funds to invest for future capital expansion. If the selected investment pays simple interest, what interest rate would be required for the amount to grow from $60,000 to $90,000 in 5 years?

1.33 To finance a new product line, a company that makes high-temperature ball bearings borrowed $1.8 million at 10% per year interest. If the company repaid the loan in a lump sum amount after 2 years, what was (*a*) the amount of the payment and (*b*) the amount of interest?

1.34 Because market interest rates were near all-time lows at 4% per year, a hand tool company decided to call (i.e., pay off) the high-interest bonds that it issued 3 years ago. If the interest rate on the bonds was 9% per year, how much does the company have to pay the bond holders? The face value (principal) of the bonds is $6,000,000.

1.35 A solid waste disposal company borrowed money at 10% per year interest to purchase new haulers and other equipment needed at the company-owned landfill site. If the company got the loan 2 years ago and paid it off with a single payment of $4,600,000, what was the principal amount *P* of the loan?

1.36 If interest is compounded at 20% per year, how long will it take for $50,000 to accumulate to $86,400?

1.37 To make CDs look more attractive than they really are, some banks advertise that their rates are higher than their competitors' rates; however, the fine print says that the rate is a simple interest rate. If a person deposits $10,000 at 10% per year simple interest, what compound interest rate would yield the same amount of money in 3 years?

MARR and Opportunity Cost

1.38 Give three other names for minimum attractive rate of return.

1.39 Identify the following as either equity or debt financing: bonds, stock sales, retained earnings, venture capital, short-term loan, capital advance from friend, cash on hand, credit card, home equity loan.

1.40 What is the weighted average cost of capital for a corporation that finances an expansion project using 30% retained earnings and 70% venture capital? Assume the interest rates are 8% for the equity financing and 13% for the debt financing.

1.41 Managers from different departments in Zenith Trading, a large multinational corporation, have offered six projects for consideration by the corporate office. A staff member for the chief financial officer used key words to identify the projects and then listed them in order of projected rate of return as shown below. If the company wants to grow rapidly through high leverage and uses only 10% equity financing that has a cost of equity capital of 9% and 90% debt financing with a cost of debt capital of 16%, which projects should the company undertake?

Project ID	Projected ROR, % per year
Inventory	30
Technology	28.4
Warehouse	19
Products	13.1
Energy	9.6
Shipping	8.2

Spreadsheet Functions

1.42 State the purpose for each of the following built-in spreadsheet functions.
(*a*) PV(i%,n,A,F)
(*b*) FV(i%,n,A,P)
(*c*) RATE(n,A,P,F)
(*d*) IRR(first_cell:last_cell)
(*e*) PMT(i%,n,P,F)
(*f*) NPER(i%,A,P,F)

1.43 What are the values of the engineering economy symbols P, F, A, i, and n in the following functions? Use a question mark for the symbol that is to be determined.
(a) NPER(8%,−1500,8000,2000)
(*b*) FV(7%,102000,−9000)
(*c*) RATE(10,1000,−12000,2000)
(*d*) PMT(11%,20,,14000)
(*e*) PV(8%,15,−1000,800)

1.44 Write the engineering economy symbol that corresponds to each of the following spreadsheet functions.
(*a*) PMT (*b*) FV (*c*) NPER (*d*) PV (*e*) IRR

1.45 In a built-in spreadsheet function, if a certain parameter is not present, (*a*) under what circumstances can it be left blank and (*b*) when must a comma be entered in its place?

1.46 Sheryl and Marcelly both invest $1000 at 10% per year for 4 years. Sheryl receives simple interest and Marcelly gets compound interest. Use a spreadsheet and cell reference formats to develop relations that show a total of $64 more interest for Marcelly at the end of the 4 years. Assume no withdrawals or further deposits are made during the 4 years.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

1.47 The concept that different sums of money at different points in time can be said to be equal to each other is known as:
(*a*) Evaluation criterion
(*b*) Equivalence
(*c*) Cash flow
(*d*) Intangible factors

1.48 The evaluation criterion that is usually used in an economic analysis is:
(*a*) Time to completion
(*b*) Technical feasibility
(*c*) Sustainability
(*d*) Financial units (dollars or other currency)

1.49 All of the following are examples of cash outflows, *except*:
(*a*) Asset salvage value
(*b*) Income taxes
(*c*) Operating cost of asset
(*d*) First cost of asset

1.50 In most engineering economy studies, the best alternative is the one that:
(*a*) Will last the longest time
(*b*) Is most politically correct
(*c*) Is easiest to implement
(*d*) Has the lowest cost

1.51 The following annual maintenance and operation (M&O) costs for a piece of equipment were collected over a 5-year period: $12,300, $8900, $9200, $11,000, and $12,100. The average is $10,700. In conducting a sensitivity analysis, the most reasonable range of costs to use (i.e., percent from the average) is:
(*a*) ±5% (*b*) ±11% (*c*) ±17% (*d*) ±25%

1.52 At an interest rate of 10% per year, the equivalent amount of $10,000 *one year ago* is closest to:
(*a*) $8264 (*b*) $9091 (*c*) $11,000 (*d*) $12,000

1.53 Assume that you and your best friend each have $1000 to invest. You invest your money in a fund that pays 10% per year *compound* interest. Your friend invests her money at a bank that pays 10% per year *simple* interest. At the end of 1 year, the difference in the total amount for each of you is:
(*a*) You have $10 more than she does
(*b*) You have $100 more than she does
(*c*) You both have the same amount of money
(*d*) She has $10 more than you do

1.54 The time it would take for a given sum of money to double at 4% per year *simple* interest is closest to:
(*a*) 30 years (*b*) 25 years
(*c*) 20 years (*d*) 10 years

1.55 All of the following are examples of equity financing, *except*:
(*a*) Mortgage
(*b*) Money from savings
(*c*) Cash on hand
(*d*) Retained earnings

1.56 To finance a new project costing $30 million, a company borrowed $21 million at 16% per year interest and used retained earnings valued at 12% per year for the remainder of the investment. The company's weighted average cost of capital for the project was closest to:
(*a*) 12.5% (*b*) 13.6% (*c*) 14.8% (*d*) 15.6%

CASE STUDY

RENEWABLE ENERGY SOURCES FOR ELECTRICITY GENERATION

Background

Pedernales Electric Cooperative (PEC) is the largest member-owned electric co-op in the United States with over 232,000 meters in 12 Central Texas counties. PEC has a capacity of approximately 1300 MW (megawatts) of power, of which 277 MW, or about 21%, is from renewable sources. The latest addition is 60 MW of power from a wind farm in south Texas close to the city of Corpus Christi. A constant question is how much of PEC's generation capacity should be from renewable sources, especially given the environmental issues with coal-generated electricity and the rising costs of hydrocarbon fuels.

Wind and nuclear sources are the current consideration for the PEC leadership as Texas is increasing its generation by nuclear power and the state is the national leader in wind farm–produced electricity.

Consider yourself a member of the board of directors of PEC. You are an engineer who has been newly elected by the PEC membership to serve a 3-year term as a director-at-large. As such, you do not represent a specific district within the entire service area; all other directors do represent a specific district. You have many questions about the operations of PEC, plus you are interested in the economic and societal benefits of pursuing more renewable source generation capacity.

Information

Here are some data that you have obtained. The information is sketchy, as this point, and the numbers are very approximate. Electricity generation cost estimates are national in scope, not PEC-specific, and are provided in cents per kilowatt-hour (¢/kWh).

	Generation Cost, ¢/kWh	
Fuel Source	**Likely Range**	**Reasonable Average**
Coal	4 to 9	7.4
Natural gas	4 to 10.5	8.6
Wind	4.8 to 9.1	8.2
Solar	4.5 to 15.5	8.8

National average cost of electricity to residential customers: 11¢/kWh

PEC average cost to residential customers: 10.27 ¢/kWh (from primary sources) and 10.92 ¢/kWh (renewable sources)

Expected life of a generation facility: 20 to 40 years (it is likely closer to 20 than 40)

Time to construct a facility: 2 to 5 years

Capital cost to build a generation facility: $900 to $1500 per kW

You have also learned that the PEC staff uses the well-recognized *levelized energy cost* (LEC) method to determine the price of electricity that must be charged to customers to break even. The formula takes into account the capital cost of the generation facilities, the cost of capital of borrowed money, annual maintenance and operation (M&O) costs, and the expected life of the facility. The LEC formula, expressed in dollars per kWh for ($t = 1, 2, \ldots, n$), is

$$\text{LEC} = \frac{\displaystyle\sum_{t=1}^{t=n} \frac{P_t + A_t + C_t}{(1+i)^t}}{\displaystyle\sum_{t=1}^{t=n} \frac{E_t}{(1+i)^t}}$$

where P_t = capital investments made in year t
A_t = annual maintenance and operating (M&O) costs for year t
C_t = fuel costs for year t
E_t = amount of electricity generated in year t
n = expected life of facility
i = discount rate (cost of capital)

Case Study Exercises

1. If you wanted to know more about the new arrangement with the wind farm in south Texas for the additional 60 MW per year, what types of questions would you ask of a staff member in your first meeting with him or her?
2. Much of the current generation capacity of PEC facilities utilizes coal and natural gas as the primary fuel source. What about the ethical aspects of the government's allowance for these plants to continue polluting the atmosphere with the emissions that may cause health problems for citizens and further the effects of global warming? What types of regulations, if any, should be developed for PEC (and other generators) to follow in the future?

3. You developed an interest in the LEC relation and the publicized cost of electricity of 10.27¢/kWh for this year. You wonder if the addition of 60 MW of wind-sourced electricity will make any difference in the LEC value for this next year. You did learn the following:

   This is year $t = 11$ for LEC computation purposes

   $n = 25$ years

   $i = 5\%$ per year

   $E_{11} = 5.052$ billion kWh

   LEC last year was 10.22 ¢/kWh (last year's breakeven cost to customers)

From these sketchy data, can you determine the value of unknowns in the LEC relation for this year? Is it possible to determine if the wind farm addition of 60 MW makes any difference in the electricity rate charged to customers? If not, what additional information is necessary to determine the LEC with the wind source included?

CASE STUDY

REFRIGERATOR SHELLS

Background

Large refrigerator manufacturers such as Whirlpool, General Electric, Frigidaire, and others may subcontract the molding of their plastic liners and door panels. One prime national subcontractor is Innovations Plastics. Because of improvements in mechanical properties, the molded plastic can sustain increased vertical and horizontal loading, thus significantly reducing the need for attached metal anchors for some shelving. However, improved molding equipment is needed to enter this market now. The company president wants a recommendation on whether Innovations should offer the new technology to the major manufacturers and an estimate of the necessary capital investment to enter this market.

You work as an engineer for Innovations. At this stage, you are not expected to perform a complete engineering economic analysis, for not enough information is available. You are asked to formulate reasonable alternatives, determine what data and estimates are needed for each one, and ascertain what criteria (economic and noneconomic) should be utilized to make the final decision.

Information

Some information useful at this time is as follows:

- The technology and equipment are expected to last about 10 years before new methods are developed.
- Inflation and income taxes will not be considered in the analysis.
- The expected returns on capital investment used for the last three new technology projects were compound rates of 15%, 5%, and 18%. The 5% rate was the criterion for enhancing an employee-safety system on an existing chemical-mixing process.
- Equity capital financing beyond \$5 million is not possible. The amount of debt financing and its cost are unknown.
- Annual operating costs have been averaging 8% of first cost for major equipment.
- Increased annual training costs and salary requirements for handling the new plastics and operating new equipment can range from \$800,000 to \$1.2 million.

There are two manufacturers working on the new-generation equipment. You label these options as alternatives A and B.

Case Study Exercises

1. Use the first four steps of the decision-making process to generally describe the alternatives and identify what economic-related estimates you will need to complete an engineering economy analysis for the president.
2. Identify any noneconomic factors and criteria to be considered in making the alternative selection.
3. During your inquiries about alternative B from its manufacturer, you learn that this company has already produced a prototype molding machine and has sold it to a company in Germany for \$3 million (U.S. dollars). Upon inquiry, you further discover that the German company already has unused capacity on the equipment for manufacturing plastic shells. The company is willing to sell time on the equipment to Innovations immediately to produce its own shells for U.S. delivery. This could allow immediate market entry into the United States. Consider this as alternative C, and develop the estimates necessary to evaluate C at the same time as alternatives A and B.