APPENDIX 20A: Loan Portfolio Risk and Management

In this chapter, we have evaluated the risk of a loan on a stand-alone basis. However, as is shown below, the credit risk of a portfolio of loans will be generally less than the sum of the risks of loans when viewed on a stand-alone basis. Let the expected return on a portfolio of loans be equal to R_P, where:

$$R_P = \sum_{i=1}^{N} X_i \bar{R}_i$$

The variance of returns or risk on a portfolio (σ_p^2) of loans can be calculated as:

$$\sigma_p^2 = \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij}$$

where

R_P = Expected or mean return on the loan portfolio
$\bar{R}_i$ = Mean return on the *i*th loan in the portfolio
X_i = Proportion of the loan portfolio invested in the *i*th loan
σ_i^2 = Variance of returns on the *i*th loan
σ_{ij} = Covariance of returns between the *i*th and *j*th loans (this reflects the correlation or covariance between the default risks of borrowers *i* and *j*)

The fundamental lesson of modern portfolio theory (MPT) is that by taking advantage of its size, an FI can diversify considerable amounts of credit risk as long as the returns on different loans are imperfectly correlated.[28]

Consider the equation for variance of returns, σ_p^2. If many loans have negative correlations or covariances of returns (σ_{ij} are negative)—that is, when one borrower's loan does badly, another's does well—the sum of the individual credit risks of loans viewed independently overestimates the risk of the whole portfolio. This is what we meant in Chapter 19 when we stated that by pooling funds, FIs can reduce risk by taking advantage of the law of large numbers in their investment decisions.[29]

Example 20-7 Calculation of Return and Risk on a Two-Asset Portfolio

Suppose that an FI holds two loans with the following characteristics:[30]

Loan *i*	X_i	$\bar{R}_i$	σ_i	σ_i^2	
1	.40	10%	9.80%	96.0%	σ_{12} = −80%
2	.60	12	8.57	73.5	

[28]One objection to using MPT for loans is that the returns on individual loans are not normally or symmetrically distributed. In particular, most loans have limited upside returns and long-tail downside risks (see the discussion in Chapter 19). Nevertheless, default correlations in general are likely to be low. For example, the joint probability of two major companies such as General Motors and Ford defaulting on their loans at exactly the same time is quite small, even though they are both in the same industry.

Since loans are not publicly traded, assumptions have to be made about the returns, risks, and correlations among loans. For a full description of different approaches, see A. Saunders and L. Allen, *Credit Risk Measurement: New Approaches to Value at Risk and Other Paradigms,* 2nd ed. (New York: John Wiley & Sons, 1999).

[29]CreditMetrics, released by J.P. Morgan and its co-sponsors (Bank of America, BZW, Deutsche Morgan Greenfell, Swiss Bank Corporation, Union Bank of Switzerland, and KMV Corporation) in 1997, is the first publicly available model that applies portfolio theory and value-at-risk methodology to evaluate credit risk across a broad range of instruments and portfolios of these instruments, including traditional loans, commitments, and letters of credit; fixed-income instruments; commercial contracts (such as trade credits and receivables); and derivative instruments (such as swaps, forwards, and futures). In general, CreditMetrics and other vendors have found that default correlations are low (in the zero to .3 range), suggesting considerable credit portfolio risk diversification possibilities.

[30]Note that variance (σ^2) is measured in percent squared; standard deviation (σ) is measured in percent. Also, since the correlation coefficient, p_{12}, equals $\sigma_{12}/(\sigma_1 \times \sigma_2)$, then $\sigma_{12} = -80/(9.8)(8.57) = -.95$.

The return on the loan portfolio is:

$$\bar{R}_p = .4(10\%) + .6(12\%) = 11.2\%$$

while the risk of the portfolio is:

$$\sigma_p^{\ 2} = (.4)^2(96.0\%) + (.6)^2(73.5\%) + 2(.4)(.6)(-80\%) = 3.42\%$$

thus:

$$\sigma_p = \sqrt{3.42\%} = 1.85\%$$

Notice that the risk (or standard deviation of returns) of the portfolio, σ_p (1.85 percent), is less than the risk of either individual loan (9.8 percent and 8.57 percent, respectively). The negative covariance of the returns of the two loans (−80 percent) results in an overall reduction of risk when they are put together in an FI's portfolio.

Consider the advantages of diversification in Figure 20–3. Note that *A* is an undiversified portfolio with heavy investment concentration in just a few loans. By fully exploiting diversification potential with loans whose returns are negatively correlated or that have a low positive correlation with those in the existing portfolio, the FI manager can lower the credit risk on the portfolio from σ_{pA} to σ_{pB} while earning the same expected return. That is, portfolio *B* is the "efficient" (lowest risk) portfolio associated with portfolio return level $\bar{R}_{i\,p}$. By varying the required portfolio return level $\bar{R}_{i\,p}$ up and down, the manager can identify an entire frontier of efficient portfolio mixes of loans. Each portfolio mix is efficient in the sense that it offers the lowest risk level to the FI manager at each possible level of portfolio returns. As Figure 20–3 indicates, however, of all possible efficient portfolios that can be generated, portfolio *B* produces the lowest possible risk level for the FI manager—that is, it maximizes the gains from diversifying across all available loans so that the manager cannot reduce the risk of the portfolio below σ_{pB}. For this reason, σ_{pB} is usually considered the **minimum risk portfolio.**

minimum risk portfolio

A portfolio for which a combination of assets reduces the variance of portfolio returns to the lowest feasible level.

Even though *B* is clearly the minimum *risk* portfolio, it does not generate the highest returns. Consequently, portfolio *B* may be chosen only by the most risk-averse FI managers, whose sole objective is to minimize portfolio risk regardless of the portfolio's return. Most portfolio managers have some desired return–risk trade-off in mind; they are willing to accept more risk if they are compensated with higher expected returns.[31] One such possibility would be portfolio *C* in Figure 20–3. This is an efficient portfolio because the FI manager has mixed loans to produce a portfolio risk level that is a minimum for that higher expected return level. This portfolio dominates all other portfolios that can produce the same expected return level.[32]

[31] The point that is chosen depends on the manager's risk aversion and the degree of separation of ownership from control. If the FI is managed by agents who perform the task of maximizing the value of the firm, they act as risk-neutral agents. They would know that stockholders who are well diversified could, through homemade diversification, hold the shares of many firms to eliminate borrower-specific risk. Thus, managers would seek to maximize expected return subject to any regulatory constraints on risk-taking behavior (i.e., they would likely pick a point in the region *C* in Figure 20–A1). However, if managers are risk averse because of their human capital invested in the FI and make lending decisions based on their own risk preferences rather than those of the stockholders, they are likely to choose a relatively low-risk portfolio, something closer to the minimum risk portfolio.

[32] For a detailed discussion of portfolio risk calculation, see R. A. Brealey, S. C. Myers, and A. J. Marcus, *Fundamentals of Corporate Finance* (New York: McGraw-Hill, 1999), pp. 225–29.

Figure 20–3 **FI Portfolio Diversification**