Chapter 1

1. (*a*) Applying formula (1.1)

$$A(t) = t^2 + 2t + 3 \quad \text{and} \quad A(0) = 3$$

so that

$$a(t) = \frac{A(t)}{k} = \frac{A(t)}{A(0)} = \frac{1}{3}\left(t^2 + 2t + 3\right).$$

(*b*) The three properties are listed on p. 2.

(1) $a(0) = \frac{1}{3}(3) = 1.$

(2) $a'(t) = \frac{1}{3}(2t + 2) > 0 \quad \text{for} \quad t \geq 0,$

so that $a(t)$ is an increasing function.

(3) $a(t)$ is a polynomial and thus is continuous.

(*c*) Applying formula (1.2)

$$\begin{aligned} I_n &= A(n) - A(n-1) = \left[n^2 + 2n + 3\right] - \left[(n-1)^2 + 2(n-1) + 3\right] \\ &= n^2 + 2n + 3 - n^2 + 2n - 1 - 2n + 2 - 3 \\ &= 2n + 1. \end{aligned}$$

2. (*a*) Appling formula (1.2)

$$\begin{aligned} I_1 + I_2 + \ldots + I_n &= \left[A(1) - A(0)\right] + \left[A(2) - A(1)\right] + \cdots + \left[A(n) - A(n-1)\right] \\ &= A(n) - A(0). \end{aligned}$$

(*b*) The LHS is the increment in the fund over the *n* periods, which is entirely attributable to the interest earned. The RHS is the sum of the interest earned during each of the *n* periods.

3. Using ratio and proportion

$$\frac{5000}{11{,}130}\left(12{,}153.96 - 11{,}575.20\right) = \$260.$$

4. We have $a(t) = at^2 + b,$ so that

$$\begin{aligned} a(0) &= \quad\; b = 1 \\ a(3) &= 9a + b = 1.72. \end{aligned}$$

Solving two equations in two unknowns $a = .08$ and $b = 1.$ Thus,

$$a(5) = 5^2(.08) + 1 = 3$$

$$a(10) = 10^2(.08) + 1 = 9.$$

and the answer is $100\dfrac{a(10)}{a(5)} = 100\dfrac{9}{3} = 300.$

5. (*a*) From formula (1.4*b*) and $A(t) = 100 + 5t$

$$i_5 = \frac{A(5) - A(4)}{A(4)} = \frac{125 - 120}{120} = \frac{5}{120} = \frac{1}{24}.$$

(*b*) $i_{10} = \dfrac{A(10) - A(9)}{A(9)} = \dfrac{150 - 145}{145} = \dfrac{5}{145} = \dfrac{1}{29}.$

6. (*a*) $A(t) = 100(1.1)^t$ and

$$i_5 = \frac{A(5) - A(4)}{A(4)} = \frac{100\left[(1.1)^5 - (1.1)^4\right]}{100(1.1)^4} = 1.1 - 1 = .1.$$

(*b*) $i_{10} = \dfrac{A(10) - A(9)}{A(9)} = \dfrac{100\left[(1.1)^{10} - (1.1)^9\right]}{100(1.1)^9} = 1.1 - 1 = .1.$

7. From formula (1.4*b*)

$$i_n = \frac{A(n) - A(n-1)}{A(n-1)}$$

so that

$$A(n) - A(n-1) = i_n A(n-1)$$

and

$$A(n) = (1 + i_n)A(n-1).$$

8. We have $i_5 = .05$, $i_6 = .06$, $i_7 = .07$, and using the result derived in Exercise 7

$$\begin{aligned} A(7) &= A(4)(1+i_5)(1+i_6)(1+i_7) \\ &= 1000(1.05)(1.06)(1.07) = \$1190.91. \end{aligned}$$

9. (*a*) Applying formula (1.5)

$$615 = 500(1 + 2.5i) = 500 + 1250i$$

so that

$$1250i = 115 \quad \text{and} \quad i = 115/1250 = .092, \quad \text{or } 9.2\%.$$

(*b*) Similarly,

$$630 = 500(1+.078t) = 500 + 39t$$

so that

$$39t = 130 \quad \text{and} \quad t = 130/39 = 10/3 = 3\tfrac{1}{3} \text{ years.}$$

10. We have

$$1110 = 1000(1+it) = 1000 + 1000it$$
$$1000it = 110 \quad \text{and} \quad it = .11$$

so that

$$500\left[1+\left(\frac{3}{4}\right)(i)(2t)\right] = 500[1+1.5it]$$
$$= 500[1+(1.5)(.11)] = \$582.50.$$

11. Applying formula (1.6)

$$i_n = \frac{i}{1+i(n-1)} \quad \text{and} \quad .025 = \frac{.04}{1+.04(n-1)}$$

so that

$$.025 + .001(n-1) = .04, \quad .001n = .016, \quad \text{and} \quad n = 16.$$

12. We have

$$i_1 = .01 \quad i_2 = .02 \quad i_3 = .03 \quad i_4 = .04 \quad i_5 = .05$$

and adapting formula (1.5)

$$1000\left[1+(i_1+i_2+i_3+i_4+i_5)\right] = 1000(1.15) = \$1150.$$

13. Applying formula (1.8)

$$600(1+i)^2 = 600 + 264 = 864$$

which gives

$$(1+i)^2 = 864/600 = 1.44, \quad 1+i = 1.2, \quad \text{and} \quad i = .2$$

so that

$$2000(1+i)^3 = 2000(1.2)^3 = \$3456.$$

14. We have

$$\frac{(1+i)^n}{(1+j)^n} = (1+r)^n \quad \text{and} \quad 1+r = \frac{1+i}{1+j}$$

so that

$$r = \frac{1+i}{1+j} - 1 = \frac{(1+i)-(1+j)}{1+j} = \frac{i-j}{1+j}.$$

This type of analysis will be important in Sections 4.7 and 9.4.

15. From the information given:

$$\begin{aligned} (1+i)^a &= 2 & \quad (1+i)^a &= 2 \\ 2(1+i)^b &= 3 & \quad (1+i)^b &= 3/2 \\ 3(1+i)^c &= 15 & \quad (1+i)^c &= 5 \\ 6(1+i)^n &= 10 & \quad (1+i)^n &= 5/3. \end{aligned}$$

By inspection $\frac{5}{3} = 5 \cdot \frac{2}{3} \cdot \frac{1}{2}$. Since exponents are addictive with multiplication, we have $n = c - a - b.$

16. For one unit invested the amount of interest earned in each quarter is:

Quarter:	1	2	3	4
Simple:	.03	.03	.03	.03
Compound:	$1.03-1$	$(1.03)^2 - 1.03$	$(1.03)^3 - (1.03)^2$	$(1.03)^4 - (1.03)^3$

Thus, we have

$$\frac{D(4)}{D(3)} = \frac{\left[(1.03)^4 - (1.03)^3\right] - .03}{\left[(1.03)^3 - (1.03)^2\right] - .03} = 1.523.$$

17. Applying formula (1.12)

$$\begin{aligned} A:&\quad 10,000\left[(1.06)^{-18} + (1.06)^{-19}\right] = 6808.57 \\ B:&\quad 10,000\left[(1.06)^{-20} + (1.06)^{-21}\right] = \underline{6059.60} \\ &\quad \text{Difference} = \$748.97. \end{aligned}$$

18. We have

$$v^n + v^{2n} = 1$$

and multiplying by $(1+i)^{2n}$

$$(1+i)^n + 1 = (1+i)^{2n}$$

or

$$(1+i)^{2n} - (1+i)^n - 1 = 0 \quad \text{which is a quadratic.}$$

Solving the quadratic

$$(1+i)^n = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2} \quad \text{rejecting the negative root.}$$

Finally,

$$(1+i)^{2n} = \left(\frac{1+\sqrt{5}}{2}\right)^2 = \frac{1+2\sqrt{5}+5}{4} = \frac{3+\sqrt{5}}{2}.$$

19. From the given information $500(1+i)^{30} = 4000$ or $(1+i)^{30} = 8.$
The sum requested is

$$10,000\left(v^{20}+v^{40}+v^{60}\right) = 10,000\left(8^{-2/3}+8^{-4/3}+8^{-2}\right)$$
$$= 10,000\left(\frac{1}{4}+\frac{1}{16}+\frac{1}{64}\right) = \$3281.25.$$

20. (*a*) Applying formula (1.13) with $a(t) = 1+it = 1+.1t$, we have

$$d_5 = \frac{I_5}{A_5} = \frac{a(5)-a(4)}{a(5)} = \frac{1.5-1.4}{1.5} = \frac{.1}{1.5} = \frac{1}{15}.$$

(*b*) A similar approach using formula (1.18) gives

$$a^{-1}(t) = 1-dt = 1-.1t$$

and

$$d_5 = \frac{I_5}{A_5} = \frac{a(5)-a(4)}{a(5)} = \frac{(1-.5)^{-1}-(1-.4)^{-1}}{(1-.5)^{-1}}$$
$$= \frac{1/.5-1/.6}{1-1/.5} = \frac{2-5/3}{2} = \frac{6-5}{2\cdot 3} = \frac{1}{6}.$$

21. From formula (1.16) we know that $v = 1-d$, so we have

$$200+300(1-d) = 600(1-d)^2$$
$$6d^2-12d+6-2-3+3d = 0$$
$$6d^2-9d+1 = 0 \quad \text{which is a quadratic.}$$

Solving the quadratic

$$d = \frac{9 \pm \sqrt{(-9)^2-(4)(6)(1)}}{2\cdot 6} = \frac{9-\sqrt{57}}{12}$$

rejecting the root > 1, so that

$$d = .1208, \quad \text{or } 12.08\%.$$

22. Amount of interest: $iA = 336.$
Amount of discount: $dA = 300.$

Applying formula (1.14)

$$i = \frac{d}{1-d} \quad \text{and} \quad \frac{336}{A} = \frac{300/A}{1-300/A} = \frac{300}{A-300}$$

so that

$$336(A-300) = 300A$$
$$36A = 100{,}800 \quad \text{and} \quad A = \$2800.$$

23. Note that this Exercise is based on material covered in Section 1.8. The quarterly discount rate is .08/4 = .02, while 25 months is $8\frac{1}{3}$ quarters.

(*a*) The exact answer is

$$5000v^{25/3} = 5000(1-.02)^{25/3} = \$4225.27.$$

(*b*) The approximate answer is based on formula (1.20)

$$5000v^8\left(1-\tfrac{1}{3}d\right) = 5000(1-.02)^8\left[1-\left(\tfrac{1}{3}\right)(.02)\right] = \$4225.46.$$

The two answers are quite close in value.

24. We will algebraically change both the RHS and LHS using several of the basic identities contained in this Section.

$$\text{RHS} = \frac{(i-d)^2}{1-v} = \frac{(id)^2}{d} = i^2 d \quad \text{and}$$
$$\text{LHS} = \frac{d^3}{(1-d)^2} = \frac{i^3v^3}{v^2} = i^3 v = i^2 d.$$

25. Simple interest: $a(t) = 1 + it$ from formula (1.5).
Simple discount: $a^{-1}(t) = 1 - dt$ from formula (1.18).

Thus,

$$1 + it = \frac{1}{1-dt}$$

and

$$1 - dt + it - idt^2 = 1$$
$$it - dt = idt^2$$
$$i - d = idt.$$

26. (*a*) From formula (1.23*a*)

$$\left(1-\frac{d^{(4)}}{4}\right)^{-4}=\left(1+\frac{i^{(3)}}{3}\right)^{3}$$

so that

$$d^{(4)}=4\left[1-\left(1+\frac{i^{(3)}}{3}\right)^{-\frac{3}{4}}\right].$$

(*b*)

$$\left(1+\frac{i^{(6)}}{6}\right)^{6}=\left(1-\frac{d^{(2)}}{2}\right)^{-2}$$

so that

$$i^{(6)}=6\left[\left(1-\frac{d^{(2)}}{2}\right)^{-\frac{1}{3}}-1\right].$$

27. (*a*) From formula (1.24)

$$i^{(m)}-d^{(m)}=\frac{i^{(m)}d^{(m)}}{m}$$

so that

$$i^{(m)}=d^{(m)}\left(1+\frac{i^{(m)}}{m}\right)=d^{(m)}(1+i)^{1/m}.$$

(*b*) $i^{(m)}$ measures interest at the ends of *m*ths of a year, while $d^{(m)}$ is a comparable measure at the beginnings of *m*ths of a year. Accumulating $d^{(m)}$ from the beginning to the end of the *m*thly periods gives $i^{(m)}$.

28. (*a*) We have $j=\frac{i^{(4)}}{4}=\frac{.06}{4}=.015$ and $n=2\cdot 4=8$ quarters, so that the accumulated value is

$$100(1.015)^{8}=\$112.65.$$

(*b*) Here we have an unusual and uncommon situation in which the conversion frequency is less frequent than annual. We have $j=4(.06)=.24$ per 4-year period and $n=2(1/4)=1/2$ such periods, so that the accumulated value is

$$100(1-.24)^{-.5}=100(.76)^{-.5}=\$114.71.$$

29. From formula (1.24)

$$i^{(m)}-d^{(m)}=\frac{i^{(m)}d^{(m)}}{m}$$

so that

$$m = \frac{i^{(m)}d^{(m)}}{i^{(m)} - d^{(m)}} = \frac{(.1844144)(.1802608)}{.1844144 - .1802608} = 8.$$

30. We know that

$$1 + \frac{i^{(4)}}{4} = (1+i)^{\frac{1}{4}} \quad \text{and} \quad 1 + \frac{i^{(5)}}{5} = (1+i)^{\frac{1}{5}}$$

so that

$$\text{RHS} = (1+i)^{\frac{1}{4}-\frac{1}{5}} = (1+i)^{\frac{1}{20}}$$

$$\text{LHS} = (1+i)^{\frac{1}{n}} \quad \text{and} \quad n = 20.$$

31. We first need to express v in terms of $i^{(4)}$ and $d^{(4)}$ as follows:

$$v = 1 - d = \left(1 - \frac{d^{(4)}}{4}\right)^4 \quad \text{so that} \quad d^{(4)} = 4\left(1 - v^{.25}\right)$$

and

$$v = (1+i)^{-1} = \left(1 - \frac{i^{(4)}}{4}\right)^{-4} \quad \text{so that} \quad i^{(4)} = 4\left(v^{-.25} - 1\right).$$

Now

$$r = \frac{i^{(4)}}{d^{(4)}} = \frac{4\left(v^{-.25} - 1\right)}{4\left(1 - v^{.25}\right)} = v^{-.25} \quad \text{so that} \quad v^{.25} = r^{-1} \quad \text{and} \quad v = r^{-4}.$$

32. We know that $d < i$ from formula (1.14) and that $d^{(m)} < i^{(m)}$ from formula (1.24). We also know that $i^{(m)} = i$ and $d^{(m)} = d$ if $m = 1$. Finally, in the limit $i^{(m)} \to \delta$ and $d^{(m)} \to \delta$ as $m \to \infty$. Thus, putting it all together, we have

$$d < d^{(m)} < \delta < i^{(m)} < i.$$

33. (*a*) Using formula (1.26), we have

$$A(t) = Ka^t b^{t^2} d^{c^t}$$

$$\ln A(t) = \ln K + t \ln a + t^2 \ln b + c^t \ln d$$

and

$$\delta_t = \frac{d}{dt} \ln A(t) = \ln a + 2t \ln b + c^t \ln c \ln d.$$

(*b*) Formula (1.26) is much more convenient since it involves differentiating a sum, while formula (1.25) involves differentiating a product.

34. Fund A: $a^A(t)=1+.10t$ and $\delta_t^A = \dfrac{\frac{d}{dt}a^A(t)}{a^A(t)} = \dfrac{.10}{1+.10t}.$

Fund B: $a^B(t)=(1-.05t)^{-1}$ and $\delta_t^B = \dfrac{\frac{d}{dt}a^B(t)}{a^B(t)} = \dfrac{.05}{1+.05t}.$

Equating the two and solving for t, we have

$$\frac{.10}{1+.10t} = \frac{.05}{1-.05t} \quad \text{and} \quad .10-.005t = .05+.005t$$

so that $.01t = .05$ and $t = 5$.

35. The accumulation function is a second degree polynomial, i.e. $a(t) = at^2+bt+c$.

$a(0)$	=	$c =$	1	from Section 1.2
$a(.5)$	=	$.25a+.5b+c =$	1.025	5% convertible semiannually
$a(1)$	=	$a+\ b\ +c =$	1.07	7% effective for the year

Solving three equations in three unknowns, we have

$$a=.04 \quad b=.03 \quad c=1.$$

36. Let the excess be denoted by E_t. We then have

$$E_t = (1+it)-(1+i)^t$$

which we want to maximize. Using the standard approach from calculus

$$\frac{d}{dt}E_t = i-(1+i)^t\ln(1+i) = i-\delta(1+i)^t = 0$$

$$(1+i)^t = \frac{i}{\delta} \quad \text{and} \quad t\ln(1+i) = t\delta = \ln i - \ln\delta$$

so that

$$t = \frac{\ln i - \ln\delta}{\delta}.$$

37. We need to modify formula (1.39) to reflect rates of discount rather than rates of interest. Then from the definition of equivalency, we have

$$a(3) = (1+i)^3 = (1-d_1)^{-1}(1-d_2)^{-1}(1-d_3)^{-1}$$

$$= (.92)^{-1}(.93)^{-1}(.94)^{-1} = .804261^{-1}$$

and

$$i = (.804264)^{-1/3} - 1 = .0753, \quad \text{or} \quad 7.53\%.$$

38. (*a*) From formula (1.39)

$$a(n) = (1+i_1)(1+i_2)\ldots(1+i_n) \quad \text{where} \quad 1+i_k = (1+r)^k(1+i)$$

so that

$$a(n) = [(1+r)(1+i)]\left[(1+r)^2(1+i)\right]\ldots\left[(1+r)^n(1+i)\right]$$

and using the formula for the sum of the first *n* positive integers in the exponent, we have

$$a(n) = (1+r)^{n(n+1)/2}(1+i)^n.$$

(*b*) From part (*a*)

$$(1+j)^n = (1+r)^{n(n+1)/2}(1+i)^n \quad \text{so that} \quad j = (1+r)^{(n+1)/2} - 1.$$

39. Adapting formula (1.42) for $t = 10$, we have

$$a(10) = e^{5(.06)}e^{5\delta} = 2, \quad \text{so that} \quad e^{5\delta} = 2e^{-.3}$$

and

$$\delta = \frac{1}{5}\ln\left(2e^{-.3}\right) = .0786, \quad \text{or} \quad 7.86\%.$$

40. Fund X: $a^X(20) = e^{\int_0^{20}(.01t+.1)dt} = e^4$ performing the integration in the exponent.

Fund Y: $a^Y(20) = (1+i)^{20} = e^4$ equating the fund balances at time $t = 20$.

The answer is

$$a^Y(1.5) = (1+i)^{1.5} = \left[(1+i)^{20}\right]^{.075} = \left(e^4\right)^{.075} = e^{.3}.$$

41. Compound discount:

$$a(3) = (1-d_1)^{-1}(1-d_2)^{-1}(1-d_3)^{-1} = (.93)^{-1}(.92)^{-1}(.91)^{-1} = 1.284363$$

using the approach taken in Exercise 37.

Simple interest: $a(3) = 1 + 3i$.

Equating the two and solving for *i*, we have

$$1 + 3i = 1.284363 \quad \text{and} \quad i = .0948, \quad \text{or} \quad 9.48\%.$$

42. Similar to Exercise 35 we need to solve three equations in three unknowns. We have

$$A(t) = At^2 + Bt + C$$

and using the values of $A(t)$ provided

$$\begin{aligned} A(0) &= & C &= 100 \\ A(1) &= & A + B + C &= 110 \\ A(2) &= & 4A + 2B + C &= 136 \end{aligned}$$

which has the solution $A = 8 \quad B = 2 \quad C = 100$.

(*a*) $i_2 = \dfrac{A(2) - A(1)}{A(1)} = \dfrac{136 - 110}{110} = \dfrac{26}{110} = .236$, or 23.6%.

(*b*) $\dfrac{A(1.5) - A(.5)}{A(1.5)} = \dfrac{121 - 103}{121} = \dfrac{18}{121} = .149$, or 14.9%.

(*c*) $\delta_t = \dfrac{A'(t)}{A(t)} = \dfrac{16t + 2}{8t^2 + 2t + 100}$ so that $\delta_{1.2} = \dfrac{21.2}{113.92} = .186$, or 18.6%.

(*d*) $\dfrac{A(.75)}{A(1.25)} = \dfrac{106}{115} = .922.$

43. The equation for the force of interest which increases linearly from 5% at time $t = 0$ to 8% at time $t = 6$ is given by

$$\delta_t = .05 + .005t \quad \text{for} \quad 0 \le t \le 6.$$

Now applying formula (1.27) the present value is

$$1{,}000{,}000a^{-1}(6) = 1{,}000{,}000e^{-\int_0^6 (.05+.005t)dt} = 1{,}000{,}000e^{-.39} = \$677{,}057.$$

44. The interest earned amounts are given by

$$A: \quad X\left[\left(1+\frac{i}{2}\right)^{16} - \left(1+\frac{i}{2}\right)^{15}\right] = X\left(1+\frac{i}{2}\right)^{15}\left(\frac{i}{2}\right)$$

$$B: \quad 2X \cdot \frac{i}{2}.$$

Equating two expressions and solving for *i*

$$X\left(1+\frac{i}{2}\right)^{15}\left(\frac{i}{2}\right) = 2X \cdot \frac{i}{2} \qquad \left(1+\frac{i}{2}\right)^{15} = 2 \qquad i = 2\left(2^{1/15} - 1\right) = .0946, \text{ or } 9.46\%.$$

45. Following a similar approach to that taken in Exercise 44, but using rates of discount rather than rates of interest, we have

$$A: \quad X = 100\left[(1-d)^{-11} - (1-d)^{-10}\right] = 100(1-d)^{-10}\left[(1-d)^{-1} - 1\right]$$

$$B: \quad X = 50\left[(1-d)^{-17} - (1-d)^{-16}\right] = 50(1-d)^{-16}\left[(1-d)^{-1} - 1\right].$$

Equating the two expressions and solving for *d*

$$100(1-d)^{-10} = 50(1-d)^{-16} \quad (1-d)^{-6} = 2 \quad (1-d)^{-1} = 2^{1/6}.$$

Finally, we need to solve for *X*. Using *A* we have

$$X = 100 \cdot 2^{10/6}\left(2^{1/6} - 1\right) = 38.88.$$

46. For an investment of one unit at $t=2$ the value at $t=n$ is

$$a(n)=e^{\int_2^n \delta_t dt}=e^{2\int_2^n (t-1)^{-1}dt}=e^{2\ln(t-1)]_2^n}=\frac{(n-1)^2}{(2-1)^2}=(n-1)^2.$$

Now applying formula (1.13)

$$d_n=\frac{a(n+1)-a(n)}{a(n+1)}=\frac{n^2-(n-1)^2}{n^2}$$

and

$$1-dn=\left(\frac{n-1}{n}\right)^2.$$

Finally, the equivalent $d_n^{(2)}$ is

$$d_n^{(2)}=2\left[1-(1-d_n)^{1/2}\right]=2\left[1-\frac{n-1}{n}\right]=\frac{2}{n}.$$

47. We are given $i=.20=\frac{1}{5}$, so that

$$d=\frac{i}{1+i}=\frac{1/5}{1+1/5}=\frac{1}{6}.$$

We then have

$$\text{PV}_A=(1.20)^{-1}\left[1+\frac{1}{2}\cdot\frac{1}{5}\right]^{-1}$$

$$\text{PV}_B=(1.20)^{-1}\left[1-\frac{1}{2}\cdot\frac{1}{6}\right]$$

and the required ratio is

$$\frac{\text{PV}_A}{\text{PV}_B}=\frac{\left(1+1/10\right)^{-1}}{1-1/12}=\frac{10}{11}\cdot\frac{12}{11}=\frac{120}{121}.$$

48. (*a*) $i=e^\delta-1=\delta+\frac{\delta^2}{2!}+\frac{\delta^3}{3!}+\frac{\delta^4}{4!}+\cdots$

using the standard power series expansion for e^δ.

(*b*) $\delta=\ln(1+i)=i-\frac{i^2}{2}+\frac{i^3}{3}-\frac{i^4}{4}+\cdots$

using a Taylor series expansion.

(*c*) $d = \dfrac{i}{1+i} = i(1+i)^{-1} = i(1 - i^2 + i^2 - i^3 + \cdots) = i - i^2 + i^3 - i^4 + \cdots$

using the sum of an infinite geometric progression.

(*d*) $\delta = -\ln(1-d) = -\left(-d - \dfrac{d^2}{2} - \dfrac{d^3}{3} - \dfrac{d^4}{4} - \cdots\right)$

adapting the series expansion in part (*b*).

49. (*a*) $\dfrac{dd}{di} = \dfrac{d}{di}\left(\dfrac{i}{1+i}\right) = \dfrac{(1+i)-i}{(1+i)^2} = (1+i)^{-2}$.

(*b*) $\dfrac{d\delta}{di} = \dfrac{d}{di}\ln(1+i) = \dfrac{1}{1+i} = (1+i)^{-1}$.

(*c*) $\dfrac{d\delta}{di} = \dfrac{d}{dv}(-\ln v) = -\dfrac{1}{v} = -v^{-1}$.

(*d*) $\dfrac{dd}{d\delta} = \dfrac{d}{d\delta}(1 - e^{-\delta}) = -e^{-\delta}(-1) = e^{-\delta}$.

50. (*a*) (1) $a(t) = e^{\int_0^t (a+br)dr} = e^{at+bt^2/2}$.

(2) $1 + i_n = \dfrac{a(n)}{a(n-1)} = \dfrac{e^{an+.5bn^2}}{e^{a(n-1)+.5b(n-1)^2}} = e^{an+.5bn^2-an+a-.5bn^2+bn-.5b} = e^{(a-b/2)+bn}$.

(*b*) (1) $a(t) = e^{\int_0^t ab^r dr} = e^{a(b^t-1)/\ln b}$.

(2) $1 + i_n = \dfrac{a(n)}{a(n-1)} = e^{\frac{a}{\ln b}\left[(b^n-1)-(b^{n-1}-1)\right]} = e^{a(b-1)b^{n-1}/\ln b}$.