Chapter 3

1. The equation of value using a comparison date at time $t = 20$ is
$$50,000 = 1000s_{\overline{20}|} + Xs_{\overline{10}|} \text{ at } 7\%.$$
Thus,
$$X = \frac{50,000 - 1000s_{\overline{20}|}}{s_{\overline{10}|}} = \frac{50,000 - 40,995.49}{13.81645} = \$651.72.$$

2. The down payment (*D*) plus the amount of the loan (*L*) must equal the total price paid for the automobile. The monthly rate of interest is $j = .18/12 = .015$ and the amount of the loan (*L*) is the present value of the payments, i.e.
$$L = 250a_{\overline{48}|.015} = 250(34.04255) = 8510.64.$$
Thus, the down payment needed will be
$$D = 10,000 - 8510.64 = \$1489.36.$$

3. The monthly interest rate on the first loan (L_1) is $j_1 = .06/12 = .005$ and
$$L_1 = 500a_{\overline{48}|.005} = (500)(42.58032) = 21,290.16.$$
The monthly interest rate on the second loan (L_2) is $j_2 = .075/12 = .00625$ and
$$L_2 = 25,000 - L_1 = 25,000 - 21,290.16 = 3709.84.$$
The payment on the second loan (*R*) can be determined from
$$3709.84 = Ra_{\overline{12}|.00625}$$
giving
$$R = \frac{3709.84}{11.52639} = \$321.86.$$

4. A's loan: $20,000 = Ra_{\overline{8}|.085}$
$$R = \frac{20,000}{5.639183} = 3546.61$$
so that the total interest would be
$$(8)(3546.61) - 20,000 = 8372.88.$$

B's loan: The annual interest is
$$(.085)(20,000) = 1700.00$$
so that the total interest would be
$$(8)(1700.00) = 13,600.00.$$
Thus, the difference is
$$13,600.00 - 8372.88 = \$5227.12.$$

5. Using formula (3.2), the present value is

$$na_{\overline{n}|} = \frac{n\left[1-(1+i)^{-n}\right]}{i} \quad \text{where} \quad i = \frac{1}{n}.$$

This expression then becomes

$$\frac{n\left[1-\left(\frac{n+1}{n}\right)^{-n}\right]}{\frac{1}{n}} = n^2\left[1-\left(\frac{n}{n+1}\right)^{n}\right].$$

6. We are given $a_{\overline{n}|} = \frac{1-v^n}{i} = x,$ so that $v^n = 1-ix.$ Also, we are given $a_{\overline{2n}|} = \frac{1-v^{2n}}{i} = y,$ so that $v^{2n} = 1-iy.$ But $v^{2n} = \left(v^n\right)^2$ so that $1-iy = (1-ix)^2.$ This equation is the quadratic $x^2i^2 - (2x-y)i = 0$ so that $i = \frac{2x-y}{x^2}.$ Then applying formula (1.15a), we have $d = \frac{i}{1+i} = \frac{2x-y}{x^2+2x-y}.$

7. We know that $d = 1-v,$ and directly applying formula (3.8), we have

$$\ddot{a}_{\overline{8}|} = \frac{1-v^8}{d} = \frac{1-(1-d)^8}{d} = \frac{1-(.9)^8}{.1} = 5.695.$$

8. The semiannual interest rate is $j = .06/2 = .03.$ The present value of the payments is

$$100\left(\ddot{a}_{\overline{21}|} + \ddot{a}_{\overline{9}|}\right) = 100(15.87747 + 8.01969) = \$2389.72.$$

9. We will use a comparison date at the point where the interest rate changes. The equation of value at age 65 is

$$3000\ddot{s}_{\overline{25}|.08} = R\ddot{a}_{\overline{15}|.07}$$

so that

$$R = \frac{3000\ddot{s}_{\overline{25}|.08}}{\ddot{a}_{\overline{15}|.07}} = \frac{236,863.25}{9.74547} = \$24,305$$

to the nearest dollar.

10. (*a*) Using formulas (3.1) and (3.7)

$$\begin{aligned}\ddot{a}_{\overline{n}|} &= \left(1+v+v^2+\cdots+v^{n-1}\right)+v^n-v^n \\ &= \left(v+v^2+\cdots+v^n\right)+1-v^n = a_{\overline{n}|}+1-v^n.\end{aligned}$$

(*b*) Using formulas (3.3) and (3.9)

$$\begin{aligned}\ddot{s}_{\overline{n}|} &= \left[(1+i)^n + (1+i)^{n-1} + \cdots + (1+i)\right] + 1 - 1 \\ &= \left[(1+i)^{n-1} + \cdots + (1+i) + 1\right] + (1+i)^n - 1 \\ &= s_{\overline{n}|} - 1 + (1+i)^n .\end{aligned}$$

(*c*) Each formula can be explained from the above derivations by putting the annuity-immediate payments on a time diagram and adjusting the beginning and end of the series of payments to turn each into an annuity-due.

11. We know that

$$\ddot{a}_{\overline{p}|} = x = \frac{1-v^p}{d} \quad \text{and} \quad s_{\overline{q}|} = y = \frac{(1+i)^q - 1}{i}.$$

Thus, $v^p = 1 - dx = 1 - ivx$ and $(1+i)^q = 1 + iy$, so that $v^q = (1+iy)^{-1}$.

Finally,

$$\begin{aligned}a_{\overline{p+q}|} &= \frac{1-v^{p+q}}{i} = \frac{1}{i}\left(1 - \frac{1-ivx}{1+iy}\right) \\ &= \frac{(1+iy)-(1-ivx)}{i(1+iy)} = \frac{vx+y}{1+iy}.\end{aligned}$$

12. We will call September 7, $z-1$ $t=0$
so that March 7, $z+8$ is $t=34$
and June 7, $z+12$ is $t=51$
where time t is measured in quarters. Payments are made at $t=3$ through $t=49$, inclusive. The quarterly rate of interest is $j=.06/4=.015$.

(*a*) $\text{PV} = 100\left(a_{\overline{49}|} - a_{\overline{2}|}\right) = 100(34.5247 - 1.9559) = \$3256.88.$

(*b*) $\text{CV} = 100\left(s_{\overline{32}|} + a_{\overline{15}|}\right) = 100(40.6883 + 13.3432) = \$5403.15.$

(*c*) $\text{AV} = 100\left(s_{\overline{49}|} - s_{\overline{2}|}\right) = 100(71.6087 - 2.0150) = \$6959.37.$

13. One approach is to sum the geometric progression

$$a_{\overline{15}|}\left(1+v^{15}+v^{30}\right) = a_{\overline{15}|}\frac{1-v^{45}}{1-v^{15}} = a_{\overline{15}|}\frac{a_{\overline{45}|}}{a_{\overline{15}|}} = a_{\overline{45}|}.$$

The formula also can be derived by observing that

$$a_{\overline{15}|}\left(1+v^{15}+v^{30}\right) = a_{\overline{15}|} + {}_{15|}a_{\overline{15}|} + {}_{30|}a_{\overline{15}|} = a_{\overline{45}|}$$

by splitting the 45 payments into 3 sets of 15 payments each.

14. We multiply numerator and denominator by $(1+i)^4$ to change the comparison date from time $t=0$ to $t=4$ and obtain

$$\frac{a_{\overline{7|}}}{a_{\overline{11|}}}=\frac{a_{\overline{7|}}(1+i)^4}{a_{\overline{11|}}(1+i)^4}=\frac{a_{\overline{3|}}+s_{\overline{4|}}}{a_{\overline{7|}}+s_{\overline{4|}}}.$$

Therefore $x=4,\ y=7,$ and $z=4.$

15. The present value of annuities *X* and *Y* are:

$$\text{PV}_X=a_{\overline{30|}}+v^{10}a_{\overline{10|}} \quad \text{and}$$

$$\text{PV}_Y=K\left(a_{\overline{10|}}+v^{20}a_{\overline{10|}}\right).$$

We are given that $\text{PV}_X=\text{PV}_Y$ and $v^{10}=.5.$ Multiplying through by *i*, we have

$$1-v^{30}+v^{10}\left(1-v^{10}\right)=K\left(1-v^{10}\right)\left(1+v^{20}\right)$$

so that

$$K=\frac{1+v^{10}-v^{20}-v^{30}}{1-v^{10}+v^{20}-v^{30}}=\frac{1+.5-.25-.125}{1-.5+.25-.125}=\frac{1.125}{.625}=1.8.$$

16. We are given ${}_{5|}a_{\overline{10|}}=3\cdot{}_{10|}a_{\overline{5|}}$ or $v^5a_{\overline{10|}}=3v^{10}a_{\overline{5|}}$ and $v^5\dfrac{1-v^{10}}{i}=3v^{10}\dfrac{1-v^5}{i}.$

Therefore, we have

$$v^5-v^{15}=3v^{10}-3v^{15} \text{ or } 2v^{15}-3v^{10}+v^5=0 \text{ or } 2-3(1+i)^5+(1+i)^{10}=0$$

which is a quadratic in $(1+i)^5$. Solving the quadratic

$$(1+i)^5=\frac{3\pm\sqrt{(-3)^2-(4)(2)(1)}}{2}=\frac{3\pm1}{2}=2$$

rejecting the root $i=0.$

17. The semiannual interest rate is $j=.09/2=.045.$ The present value of the annuity on October 1 of the prior year is $2000a_{\overline{10|}}.$ Thus, the present value on January 1 is

$$2000a_{\overline{10|}}(1.045)^{.5}$$

$$=(2000)(7.91272)(1.02225)=\$16{,}178$$

to the nearest dollar.

18. The equation of value at time $t=0$ is

$$1000\ddot{a}_{\overline{20|}}=R\cdot v^{30}\cdot\ddot{a}_{\overline{\infty|}}$$

or

$$1000\frac{1-v^{20}}{d}=R\cdot v^{30}\frac{1}{d}$$

so that

$$R = 1000\frac{1-v^{20}}{v^{30}} = 1000\left(1-v^{20}\right)\left(1+i\right)^{30}$$
$$= 1000\left[\left(1+i\right)^{30} - \left(1+i\right)^{10}\right].$$

19. We are given $i = \frac{1}{9}$ so that $d = \frac{i}{1+i} = \frac{1}{10}$. The equation of value at time $t = 0$ is

$$6561 = 1000v^n \ddot{a}_{\overline{\infty}|} \quad \text{or} \quad 6.561 = \frac{(1-d)^n}{d} = \frac{(1-.1)^n}{.1}.$$

Therefore, $(.9)^n = (.1)(6.561) = .6561$ and $n = 4$.

20. The equation of value at age 60 is

$$50,000a_{\overline{\infty}|} = Rv^5 a_{\overline{20}|}$$

or

$$\frac{50,000}{i} = Rv^5 \frac{1-v^{20}}{i}$$

so that

$$R = \frac{50,000}{v^5 - v^{25}} \quad \text{at} \quad i = .05$$
$$= \frac{50,000}{.7835262 - .2953028} = \$102,412$$

to the nearest dollar.

21. Per dollar of annuity payment, we have $\text{PV}_A = \text{PV}_D$ which gives

$$\frac{1}{3}a_{\overline{n}|} = v^n \cdot a_{\overline{\infty}|} \quad \text{or} \quad a_{\overline{n}|} = 3v^n a_{\overline{\infty}|}$$

and $1 - v^n = 3v^n$, so that

$$4v^n = 1 \quad \text{or} \quad v^n = .25 \quad \text{and} \quad \left(1+i\right)^n = 4.$$

22. Per dollar of annuity payment, we have

$$\text{PV}_A = a_{\overline{n}|},\ \ \text{PV}_B = v^n a_{\overline{n}|},\ \ \text{PV}_C = v^{2n} a_{\overline{n}|} \quad \text{and} \quad \text{PV}_D = v^{3n} a_{\overline{\infty}|}.$$

We are given

$$\frac{\text{PV}_C}{\text{PV}_A} = v^{2n} = .49 \quad \text{or} \quad v^n = .7.$$

Finally,

$$\frac{\text{PV}_B}{\text{PV}_D} = \frac{v^n a_{\overline{n}|}}{v^{3n} a_{\overline{\infty}|}} = \frac{v^n\left(1-v^n\right)}{v^{3n}}$$

$$= \frac{1-v^n}{v^{2n}} = \frac{1-.7}{(.7)^2} = \frac{.30}{.49} = \frac{30}{49}.$$

23. (a) $\quad a_{\overline{5.25}|} = a_{\overline{5}|} + v^{5.25}\left[\frac{(1+i)^{.25}-1}{i}\right] \quad \text{at} \quad i = .05$

$$= 4.32946 + (.77402)\left[\frac{(1.05)^{.25}-1}{.05}\right] = 4.5195.$$

(b) $\quad a_{\overline{5.25}|} = a_{\overline{5}|} + .25v^{5.25}$

$$= 4.32946 + (.25)(.77402) = 4.5230.$$

(c) $\quad a_{\overline{5.25}|} = a_{\overline{5}|} + .25v^{6}$

$$= 4.23946 + (.25)(.74621) = 4.5160.$$

24. At time $t = 0$ we have the equation of value

$$1000 = 100\left(a_{\overline{n}|} - a_{\overline{4}|}\right) \quad \text{or}$$

$$a_{\overline{n}|} = 10 + a_{\overline{4}|} = 13.5875 \quad \text{at} \quad i = .045.$$

Now using a financial calculator, we find that $n = 21$ full payments plus a balloon payment. We now use time $t = 21$ as the comparison date to obtain

$$1000(1.045)^{21} = 100s_{\overline{17}|} + K$$

or

$$K = 1000(1.045)^{21} - 100s_{\overline{17}|}$$

$$= 2520.2412 - 100(24.74171) = 46.07$$

Thus, the balloon payment is

$$100 + 46.07 = \$146.07 \quad \text{at time} \quad t = 21.$$

25. We are given $\text{PV}_1 = \text{PV}_2$ where

$$\text{PV}_1 = 4a_{\overline{36}|} \quad \text{and} \quad \text{PV}_2 = 5a_{\overline{18}|}.$$

We are also given that $(1+i)^n = 2.$ Thus, we have

$$4 \cdot \frac{1-v^{36}}{i} = 5\frac{1-v^{18}}{i} \quad \text{or}$$

$$4\left(1-v^{36}\right) = 4\left(1-v^{18}\right)\left(1+v^{18}\right) = 5\left(1-v^{18}\right).$$

Thus, we have

$$4\left(1+v^{18}\right)=5 \quad \text{or} \quad v^{18}=.25.$$

Finally, we have $(1+i)^{18}=4,$ so that $(1+i)^{9}=2$ which gives $n=9.$

26. At time $t=20,$ the fund balance would be

$$500\ddot{s}_{\overline{20}|}=24,711.46 \quad \text{at} \quad i=.08.$$

Let n be the number of years full withdrawals of 1000 can be made, so that the equation of value is

$$1000s_{\overline{n}|}=24,711.46 \quad \text{or} \quad s_{\overline{n}|}=24.71146\,.$$

Using a financial calculator we find that only $n=14$ full withdrawals are possible.

27. (*a*) The monthly rate of interest is $j=.12/12=.01.$ The equation of value at time $t=0$ is

$$6000v^k=100a_{\overline{60}|}=4495.5038$$

$$v^k=.749251 \quad \text{so that} \quad k=\frac{-\ln(.749251)}{\ln(1.01)}=29.$$

(*b*) Applying formula (2.2) we have

$$\bar{t}=\frac{1000(1+2+\cdots+60)}{100(60)}=\frac{(60)(61)}{2(60)}=\frac{61}{2}=30.5.$$

28.(*a*) Set: $\text{N}=48 \quad \text{PV}=12,000 \quad \text{PMT}=-300$ and CPT I to obtain $j=.7701\%.$ The answer is $12j=9.24\%.$

(*b*) We have $300a_{\overline{48}|}=12,000$ or $a_{\overline{48}|}=40.$ Applying formula (3.21) with $n=48$ and $g=40,$ we have

$$j\approx\frac{2(n-g)}{g(n+1)}=\frac{2(48-40)}{40(48+1)}=.8163\%.$$

The answer is $12j=9.80\%.$

29. We have

$$a_{\overline{2}|}=v+v^2 \quad \text{or} \quad 1.75=(1+i)^{-1}+(1+i)^{-2}.$$

Multiplying through $(1+i)^2$ gives

$$1.75(1+i)^2=(1+i)+1$$

$$1.75\left(1+2i+i^2\right)=2+i$$

and $1.75i^2+2.5-.25$ or $7i^2+10-1=0$ which is a quadratic. Solving for *i*

$$i = \frac{-10 \pm \sqrt{(10)^2 - (4)(7)(-1)}}{(2)(7)} = \frac{-10 \pm \sqrt{128}}{14}$$

$$= \frac{4\sqrt{2} - 5}{7} \text{ rejecting the negative root.}$$

30. We have the following equation of value

$$10,000 = 1538 a_{\overline{10}|} = 1072 a_{\overline{20}|}.$$

Thus $1538(1 - v^{10}) = 1072(1 - v^{20}) = 1072(1 - v^{10})(1 + v^{10})$, so that $1 + v^{10} = \frac{1538}{1072}$ or $v^{10} = .43470.$

Solving for i, we obtain

$$(1+i)^{-10} = .43470 \quad \text{and} \quad i = (.43470)^{-.1} - 1 = .0869, \quad \text{or} \quad 8.69\%.$$

31. We are given that the following present values are equal

$$a_{\overline{\infty}|7.25\%} = a_{\overline{50}|j} = a_{\overline{n}|j-1}.$$

Using the financial calculator

$$a_{\overline{50}|j} = \frac{1}{.0725} = 13.7931$$

and solving we obtain $j = 7.00\%$. Since $j - 1 = 6\%$, we use the financial calculator again

$$a_{\overline{n}|6\%} = 13.7931 \quad \text{to obtain} \quad n = 30.2.$$

32. (*a*) We have $j_1 = .08/2 = .04$ and $j_2 = .07/2 = .035$. The present value is

$$a_{\overline{6}|.04} + a_{\overline{4}|.035}(1.04)^{-6} = 5.2421 + (3.6731)(.79031)$$
$$= 8.145.$$

(*b*) The present value is

$$a_{\overline{6}|.04} + a_{\overline{4}|.035}(1.035)^{-6} = 5.2421 + (3.6731)(.81350)$$
$$= 8.230.$$

(*c*) Answer (*b*) is greater than answer (*a*) since the last four payments are discounted over the first three years at a lower interest rate.

33. (*a*) Using formula (3.24)

$$a_{\overline{5}|} = v + v^2 + v^3 + v^4 + v^5$$
$$= \frac{1}{1.06} + \frac{1}{(1.062)^2} + \frac{1}{(1.064)^3} + \frac{1}{(1.066)^4} + \frac{1}{(1.068)^5}$$
$$= 4.1543.$$

(*b*) Using formula (3.23)

$$a_{\overline{5}|} = \frac{1}{1.06} + \frac{1}{(1.06)(1.062)} + \frac{1}{(1.06)(1.062)(1.064)} + \frac{1}{(1.06)(1.062)(1.064)(1.066)}$$
$$+ \frac{1}{(1.06)(1.062)(1.064)(1.066)(1.068)} = 4.1831.$$

34. Payments are R at time $t = .5$ and $2R$ at time $t = 1.5,\ 2.5, \ldots,\ 9.5.$ The present value of these payments is equal to P. Thus, we have

$$P = R\left[1 + 2a_{\overline{4}|\,i} + 2a_{\overline{a}|\,j}(1+i)^{-4}\right](1+i)^{-1/2}$$

and

$$R = \frac{P(1+i)^{1/2}}{1 + 2a_{\overline{4}|\,i} + 2(1+i)^{-4}\,a_{\overline{5}|\,j}}.$$

35. The payments occur at $t = 0,\ 1,\ 2, \ldots,\ 19$ and we need the current value at time $t = 2$ using the variable effective rate of interest given. The current value is

$$\left(1+\frac{1}{9}\right)\left(1+\frac{1}{10}\right) + \left(1+\frac{1}{10}\right) + 1 + \left(1+\frac{1}{11}\right)^{-1} + \left(1+\frac{1}{11}\right)^{-1}\left(1+\frac{1}{12}\right)^{-1}$$
$$+ \cdots + \left(1+\frac{1}{11}\right)^{-1}\left(1+\frac{1}{12}\right)^{-1}\cdots\left(1+\frac{1}{27}\right)^{-1}$$
$$= \left(\frac{10}{9}\right)\left(\frac{11}{10}\right) + \frac{11}{10} + 1 + \frac{11}{12} + \left(\frac{11}{12}\right)\left(\frac{12}{13}\right) + \cdots + \left(\frac{11}{12}\cdot\frac{12}{13}\cdots\frac{27}{28}\right)$$
$$= \frac{11}{9} + \frac{11}{10} + \frac{11}{11} + \frac{11}{12} + \frac{11}{13} + \cdots + \frac{11}{28} = \sum_{t=9}^{28}\frac{11}{t}.$$

36. We know that $a^{-1}(t) = 1 - dt$ using simple discount. Therefore, we have

$$a_{\overline{n}|} = \sum_{t=1}^{n} a^{-1}(t) = \sum_{t=1}^{n}(1 - dt) = n - \tfrac{1}{2}n(n+1)d$$

by summing the first n positive integers.

37. We have $a(t) = \dfrac{1}{\log_2(t+2) - \log_2(t+1)} = \dfrac{1}{\log_2 \dfrac{t+2}{t+1}}$, so that $a^{-1}(t) = \log_2 \dfrac{t+2}{t+1}$.

Now

$$\ddot{a}_{\overline{n}|} = \sum_{t=0}^{n-1} a^{-1}(t) = \sum_{t=0}^{n-1} \log_2 \frac{t+2}{t+1}$$
$$= \log_2 \frac{2}{1} + \log_2 \frac{3}{2} + \cdots + \log_2 \frac{n+1}{n}$$
$$= \log_2 \left(\frac{2}{1} \cdot \frac{3}{2} \cdot \ \cdots \ \cdot \frac{n+1}{n} \right) = \log_2 (n+1).$$

38. The accumulated value of 1 paid at time *t* accumulated to time 10 is

$$e^{\int_t^{10} \delta_r dr} = e^{\int_t^{10} \frac{1}{20-r} dr} = e^{\ln(20-r) - \ln 10} = \frac{20-r}{10}.$$

Then

$$s_{\overline{10}|} = \sum_{r=1}^{10} \frac{20-r}{10} = \frac{19}{10} + \frac{18}{10} + \cdots + \frac{10}{10} = 14.5.$$

39. A: $\text{PV}_A = \frac{1}{1.01} + \frac{1}{1.02} + \frac{1}{1.03} + \frac{1}{1.04} + \frac{1}{1.05} = 4.8553$

B: $\text{AV}_B = 1.04 + 1.03 + 1.02 + 1.01 + 1.00 = 5.1000$

and taking the present value

$$\text{PV}_B = \frac{5.1000}{1.05} = 4.8571.$$

The answers differ by 4.8571 - 4.8553=.0018.

40. The present value of the payments in (*ii*) is

$$30a_{\overline{10}|} + 60v^{10}a_{\overline{10}|} + 90v^{20}a_{\overline{10}|} = a_{\overline{10}|}\left(30 + 60v^{10} + 90v^{20}\right).$$

The present value of the payments in (*i*) is

$$55a_{\overline{20}|} = 55a_{\overline{10}|}\left(1 + v^{10}\right).$$

Equating the two values we have the quadratic $90v^{20} + 5v^{10} - 25 = 0$. Solving the quadratic

$$v^{10} = \frac{-5 \pm \sqrt{(5)^2 - (4)(90)(-25)}}{(2)(90)} = \frac{90}{180} = .5$$

rejecting the negative root. Now $v^{10} = .5$ or $(1+i)^{10} = 2$ and $i = .0718$. Finally, $X = 55a_{\overline{20}|.0718} = 574.60.$

41. We have the equation of value at time $t = 3n$

$$98s_{\overline{3n}|} + 98s_{\overline{2n}|} = 8000$$

or

$$\frac{(1+i)^{3n}-1}{i}+\frac{(1+i)^{2n}-1}{i}=\frac{8000}{98}=81.6327.$$

We are given that $(1+i)^n=2.$ Therefore, $\frac{2^3-1}{i}+\frac{2^2-1}{i}=\frac{10}{i}=81.6327$ and $i=.1225,$ or 12.25%.

42. At time $t=0$ we have the equation of value

$$10,000=4ka_{\overline{20|}}-ka_{\overline{15|}}-ka_{\overline{10|}}-ka_{\overline{5|}}$$

so that

$$k=\frac{10,000}{4a_{\overline{20|}}-a_{\overline{15|}}-a_{\overline{10|}}-a_{\overline{5|}}}.$$

43. The present values given are:

(i) $2a_{\overline{2n|}}+a_{\overline{n|}}=36$ or $2(1-v^{2n})+(1-v^n)=36i,$ and

(ii) $2v^na_{\overline{n|}}=6$ or $2v^n(1-v^n)=6i.$

Thus, $2(1-v^{2n})+(1-v^n)=(6)(2)v^n(1-v^n)$ which simplifies to the quadratic

$$10v^{2n}-13v^n+3=0.$$

Solving,

$$v^n=\frac{13\pm\sqrt{(-13)^2-(4)(10)(3)}}{(2)(10)}=\frac{6}{20}=.3$$

rejecting the root $v^n=1.$ Substituting back into (ii)

$(2)(.3)\frac{1-.3}{i}=6,$ so that $i=\frac{(2)(.3)(.7)}{6}=.07,$ or 7%.

44. An equation of value at time $t=10$ is

$$10,000(1.04)^{10}-K(1.05)(1.04)^6-K(1.05)(1.04)^5$$
$$-K(1.04)^4-K(1.04)^3=10,000.$$

Thus, we have

$$K=\frac{10,000\left[(1.04)^{10}-1\right]}{(1.05)(1.04)^6+(1.05)(1.04)^5+(1.04)^4+(1.04)^3}$$
$$=\$980 \quad \text{to the nearest dollar.}$$

45. $\sum_{n=15}^{40}s_{\overline{n|}}=\frac{1}{i}\sum_{n=15}^{40}\left[(1+i)^n-1\right]=\frac{s_{\overline{41|}}-s_{\overline{15|}}-26}{i}$

using formula (3.3) twice and recognizing that there are 26 terms in the summation.