# Chapter 5

1. The quarterly interest rate is $j = .06/4 = .015$. The end of the second year is the end of the eighth quarter. There are a total of 20 installment payments, so

$$R = \frac{1000}{a_{\overline{20}|.015}}$$

and using the prospective method

$$B_8^p = Ra_{\overline{12}|.015} = \frac{1000a_{\overline{12}|.015}}{a_{\overline{20}|.015}} = \frac{1000(10.90751)}{17.16864} = \$635.32.$$

2. Use the retrospective method to bypass having to determine the final irregular payment. We then have

$$\begin{aligned} B_5^r &= 10,000(1.12)^5 - 2000s_{\overline{5}|.12} \\ &= (10,000)(1.76234) - (2000)(6.35283) \\ &= \$4918 \quad \text{to the nearest dollar.} \end{aligned}$$

3. The quarterly interest rate is $j = .10/2 = .025$. Applying the retrospective method we have $B_4^r = L(1+j)^4 - Rs_{\overline{4}|j}$ and solving for $L$

$$\begin{aligned} L = \frac{B_4^r + Rs_{\overline{4}|j}}{(1+j)^4} &= \frac{12,000 + 1500(4.15252)}{1.10381} \\ &= \$16,514 \quad \text{to the nearest dollar.} \end{aligned}$$

4. The installment payment is $R = \dfrac{20,000}{a_{\overline{12}|}}$ and the fourth loan balance prospectively is

$$B_4^p = \frac{20,000}{a_{\overline{12}|}} a_{\overline{8}|} = \frac{20,000(1-v^8)}{1-v^{12}} = \frac{20,000(1-2^{-2})}{1-2^{-3}} = \$17,143 \text{ to the nearest dollar.}$$

5. We have

$$R = \frac{20,000}{a_{\overline{20}|}} \quad \text{and} \quad B_5^P = Ra_{\overline{15}|}.$$

The revised loan balance at time $t = 7$ is $B_7' = B_5^p(1+i)^2$, since no payments are made for two years. The revised installment payment thus becomes

$$R' = \frac{B_7}{a_{\overline{13}|}} = 20,000\frac{a_{\overline{15}|}(1+i)^2}{a_{\overline{20}|}a_{\overline{13}|}}.$$

6. The installment payment is $R = \dfrac{L}{a_{\overline{n}|}} = \dfrac{1}{a_{\overline{25}|}}$. Using the original payment schedule

$$B_5^p = Ra_{\overline{20}|} = \frac{a_{\overline{20}|}}{a_{\overline{25}|}}$$

and using the revised payment schedule $B_5^p = Ra_{\overline{15}|} + Ka_{\overline{5}|}$. Equating the two and solving for $K$ we have

$$K = \frac{1}{a_{\overline{5}|}}\left(\frac{a_{\overline{20}|}}{a_{\overline{25}|}} - \frac{a_{\overline{15}|}}{a_{\overline{25}|}}\right) = \frac{a_{\overline{20}|} - a_{\overline{15}|}}{a_{\overline{25}|}a_{\overline{5}|}}.$$

7. We have

$$R = \frac{150,000}{a_{\overline{15}|.065}} = \frac{150,000}{9.4026689} = 15,952.92$$

and

$$B_5^p = Ra_{\overline{10}|.065}(15,952.92)(7.1888302) = 114,682.83.$$

The revised fifth loan balance becomes

$$B_5' = 114,682.83 + 80,000 = 194,682.83$$

and the revised term of the loan is $n' = 15 - 5 + 7 = 17$. Thus, the revised installment payment is

$$R' = \frac{194,682.83}{a_{\overline{17}|.075}} = \frac{194,682.83}{9.4339598} = \$20,636 \quad \text{to the nearest dollar.}$$

8. The quarterly interest rate is $j = .12/4 = .03$. Directly from formula (5.5), we have

$$P_6 = 1000v_{.03}^{20-6+1} = 1000(1.03)^{-15} = \$641.86.$$

9. The installment payment is

$$R = \frac{10,000}{a_{\overline{20}|}}$$

and applying formula (5.4) we have

$$I_{11} = \frac{10,000}{a_{\overline{20}|}}\left(1 - v^{20-11+1}\right) = \frac{10,000(.1)\left(1 - v^{10}\right)}{1 - v^{20}}$$

$$= \frac{1000\left(1 - v^{10}\right)}{\left(1 - v^{10}\right)\left(1 + v^{10}\right)} = \frac{1000}{1 + v^{10}}.$$

10. The quarterly interest rate is $j = .10/4 = .025$. The total number of payments is $n = 5 \times 4 = 20$. Using the fact that the principal repaid column in Table 5.1 is a geometric progression, we have the answer

$$100\left[(1+i)^{13} + (1+i)^{14} + (1+i)^{15} + (1+i)^{16} + (1+i)^{17}\right]$$
$$= 100\left(s_{\overline{18}|} - s_{\overline{13}|}\right) = 100(22.38635 - 15.14044) = \$724.59.$$

11. (*a*) We have $B_4^p = a_{\overline{6}|\, i} + v_i^6 a_{\overline{10}|\, j}$ so that $I_5 = i \cdot B_4 = i\left(a_{\overline{6}|\, i} + v_i^6 a_{\overline{10}|\, j}\right)$.

(*b*) After 10 years, the loan becomes a standard loan at one interest rate. Thus applying formula (5.5)

$$P_{15} = v_j^{20-15+1} = v_j^6.$$

12. After the seventh payment we have $B_7^p = a_{\overline{13}|}$. If the principal $P_8 = v^{20-8+1} = v^{13}$ in the next line of the amortization schedule is also paid at time $t = 7$; then, in essence, the next line in the amortization schedule drops out and we save $1 - v^{13}$ in interest over the life of the loan. The loan is exactly prepaid one year early at time $t = 19$.

13. (*a*) The amount of principal repaid in the first 5 payments is

$$B_0 - B_5 = L - B_5^p = L - \left(\frac{L}{a_{\overline{10}|}}\right)a_{\overline{5}|} = L\left(1 - \frac{a_{\overline{5}|}}{a_{\overline{10}|}}\right) = L\left(1 - \frac{1-v^5}{1-v^{10}}\right) = L\left(1 - \frac{1 - \frac{2}{3}}{1 - \frac{4}{9}}\right) = .4L.$$

(*b*) The answer is

$$B_5(1+i)^5 = (L - .4L)\frac{3}{2} = .9L.$$

14. We are given

$$I_8 = R\left(1 - v^{28}\right) = 135 \quad \text{and} \quad I_{22} = R\left(1 - v^{14}\right) = 108.$$

Taking the ratio

$$\frac{I_8}{I_{22}} = \frac{1-v^8}{1-v^{14}} = 1 + v^{14} = \frac{135}{108} = 1.25$$

so that $v^{14} = .25$.

Now, we can solve for $R$

$$R = \frac{108}{1-v^{14}} = \frac{108}{.75} = 144.$$

Finally,

$$I_{29} = R\left(1-v^7\right) = 144\left[1-(.25)^{.5}\right] = \$72.$$

15. We have

$$L = 1000 a_{\overline{10}|}.$$

and using the column total from Table 5.1

$$L = 1000\left(10 - a_{\overline{10}|}\right)$$

Equating the two we have $a_{\overline{10}|} = 5$ and solving for the unknown rate of interest using a financial calculator, we have $i = 15.0984\,\%$. Thus, the answer is

$$I_1 = iL = (.150984)(5000) = \$754.95.$$

16. We know that $X = Ra_{\overline{n}|.125}$.

From $(i)$ we have

$$R(1-v) = 153.86 \quad \text{so that} \quad R = 1384.74.$$

From $(ii)$ we have

$$\begin{aligned} X &= 6009.12 + (1384.74 - 153.86) = 7240.00 \\ &= 1384.74 a_{\overline{n}|.125}. \end{aligned}$$

Therefore, $a_{\overline{n}|.125} = 7240/1384.74 = 5.228$ and solving for the unknown $n$ using a financial calculator we obtain $n = 9$.

From $(iii)$ we have

$$Y = Rv^{9-1+1} = 1384.74(1.125)^{-9} = \$479.73.$$

17. ($a$) $.10(10,000) = \$1000.$

($b$) $1500 - 1000 = \$500.$

($c$) $1000 - .08(5000) = \$600.$

($d$) $1500 - 600 = \$900$.

($e$) $5000(1.08) + 500 = \$5900$.

As a check, note that $5900 - 5000 = 900,$ the answer to part ($d$).

18. (*a*) $B_5 = 1000(1.08)^5 - 120 s_{\overline{5}|}$ by the retrospective definition of the outstanding loan balance.

(*b*) $B_5 = 1000\left[1 + i s_{\overline{5}|}\right] - 120 s_{\overline{5}|} = 1000 + 80 s_{\overline{5}|} - 120 s_{\overline{5}|} = 1000 - 40 s_{\overline{5}|}$. The total annual payment 120 is subdivided into 80 for interest on the loan and 40 for the sinking fund deposit. After five years the sinking fund balance is $40 s_{\overline{5}|}$. Thus, $B_5$ is the original loan less the amount accumulated in the sinking fund.

19. We have $X \ddot{s}_{\overline{10}|.07} = 10,000$ so that $X = \dfrac{10,000}{\ddot{s}_{\overline{10}|.07}} = \dfrac{10,000}{14.7836} = \$676.43.$

20. Amortization payment: $\dfrac{.5L}{a_{\overline{10}|.05}}$.

Sinking fund payment: $(.05)(.5L) + \dfrac{.5L}{s_{\overline{10}|.04}}$.

The sum of the two is equal to 1000. Solving for $L$ we obtain

$$L = \frac{1000}{\frac{.5}{a_{\overline{10}|.05}} + .025 + \frac{.5}{s_{\overline{10}|.04}}} = \frac{1000}{.06475 + .025 + .04165} = \$7610 \quad \text{to the nearest dollar.}$$

21. The interest on the loan and sinking fund deposits are as follows:

| Years | Interest | SFD |
|---|---|---|
| 1 - 10 | $.06(12,000) = 720$ | $1000 - 720 = 280$ |
| 11 - 20 | $.05(12,000) = 600$ | $1000 - 600 = 400$ |

The sinking fund balance at time $t = 20$ is

$$\begin{aligned} 280 s_{\overline{10}|.04}(1.04)^{10} + 400 s_{\overline{10}|.04} &= (280)(12.00611)(1.48024) + (400)(12.00611) \\ &= 9778.57. \end{aligned}$$

Thus the shortage in the sinking fund at time $t = 20$ is $12,000 - 9778.57 = \$2221$ to the nearest dollar.

22. (*a*) The total payment is

$$3000(.04) + \frac{\frac{1}{3}(3000)}{s_{\overline{20}|.025}} + \frac{\frac{2}{3}(3000)}{s_{\overline{20}|.035}} = 120 + 39.15 + 70.72 = \$229.87\,.$$

(*b*) We have

$$\frac{1}{3}Ds_{\overline{20|}.025}+\frac{2}{3}Ds_{\overline{20|}.035}=3000$$

so that

$$D=\frac{3000}{8.5149+18.8531}=109.62\,.$$

and the total payment is

$$120+109.62=\$229.62$$

(*c*) In part (*a*) more than $1/3$ of the sinking fund deposit goes into the lower-earning sinking fund, whereas in part (*b*) exactly $1/3$ does. Therefore, the payment in part (*a*) must be slightly higher than in part (*b*) to make up for the lesser interest earned.

23. We have

$$36,000=400,000i+\frac{400,000}{s_{\overline{31|}.03}}=400,000i+8000$$

and

$$i=\frac{36,000-8000}{400,000}=.07, \quad \text{or} \quad 7\%.$$

24. We have $P=\dfrac{1000}{a_{\overline{10|}.10}}=\dfrac{1000}{6.14457}=162.745.$

The interest on the loan is $.10(1000)=100,$ so that $D=162.745-100=62.745$. The accumulated value in the sinking fund at time $t=10$ is

$$62.745s_{\overline{10|}.14}=(62.745)(19.3373)=1213.32\,.$$

Thus, the excess in the sinking fund at time $t=10$ is $1213.32-1000=\$213.32$.

25. Total interest = total payments minus the loan amount, so

$$\begin{aligned}&(500)(4)(10)-(500)(4)a^{(4)}_{\overline{10|}.08}=20,000-2000(6.90815)\\&=\$6184 \quad \text{to the nearest dollar.}\end{aligned}$$

26. Semiannual interest payment = $(10,000)(.12/2) = 600$.

    Annual sinking fund deposit = $\dfrac{10,000}{s_{\overline{5|}.08}} = \dfrac{10,000}{5.8666} = 1704.56$.

    Total payments = $(600)(2)(5) + (1704.56)(5) = \$14,523$ to the nearest dollar.

27. The quarterly interest rate is $j = .10/4 = .025$. We are given $R = 3000$ and $I_3 = 2000$, so therefore $P_3 = 1000$. There are $3 \times 4 = 12$ interest conversion periods between $P_3$ and $P_6$. Therefore $P_6 = P_3(1+j)^{12} = 1000(1.025)^{12} = \$1344.89$.

28. The quarterly interest rate on the loan is $j_1 = .10/4 = .025$. The semiannual interest rate on the sinking fund is $j_2 = .07/2 = .035$. The equivalent annual effective rate is $i_2 = (1.035)^2 - 1 = .07123$. Thus, the required annual sinking fund deposit is

$$D = \frac{5000(1.025)^{40}}{s_{\overline{10|}.07123}} = \frac{5000(2.865064)}{13.896978} = \$966.08.$$

29. There are 17 payments in total. We have $B_3 = 300a_{\overline{14|}} + 50(Ia)_{\overline{14|}}$

    and

$$\begin{aligned} P_4 &= 350 - iB_3 \\ &= 350 - 300(1 - v^{14}) - 50(\ddot{a}_{\overline{14|}} - 14v^{14}) \\ &= 50 + 1000v^{14} - 50\ddot{a}_{\overline{14|}} \\ &= 50 + 1000(.577475) - 50(10.9856) \\ &= \$78.20. \end{aligned}$$

30. The semiannual loan interest rate is $j_1 = .06/2 = .03$. Thus, the semiannual interest rate payments are $30, 27, 24, \ldots, 3$. The semiannual yield rate is $j_2 = .10/2 = .05$. The price is the present value of all the payments at this yield rate, i.e.

$$\begin{aligned} &100a_{\overline{10|}.05} + 3(Da)_{\overline{10|}.05} \\ &= 100a_{\overline{10|}.05} + (3)(20)(10 - a_{\overline{10|}.05}) \\ &= 600 + 40a_{\overline{10|}.05} = 600 + 40(7.7217) = \$908.87. \end{aligned}$$

31. (*a*) Retrospectively, we have

$$B_3 = 2000(1.1)^3 - 400\left[(1.1)^2 + (1.04)(1.1) + (1.04)^2\right] = \$1287.76\,.$$

(*b*) Similarly to part (*a*)

$$B_2 = 2000(1.1)^2 - 400(1.1 + 1.04) = 1564.00$$

so that

$$P_3 = B_2 - B_3 = 1564.00 - 1287.76 = \$276.24.$$

32. A general formula connecting successive book values is given by

$$B_t = B_{t-1}(1+i) - (1.625t)\left(i \cdot B_{t-1}\right).$$

Letting $t = 16,$ we have

$$B_{16} = B_{15}(1+i) - 26iB_{15} = 0$$

since the fund is exactly exhausted. Therefore $1 + i - 26i = 0$ and $i = \dfrac{1}{25}$ or 4%.

33. Under option (*i*)

$$P = \frac{2000}{a_{\overline{10}|.0807}} = \frac{2000}{6.68895} = 299$$

and total payments $= 299(10) = 2990.$

Under option (*ii*) the total interest paid needs to be $2990 - 2000 = 990$. Thus, we have

$$990 = i(2000 + 1800 + 1600 + \cdots + 200) = 11{,}000i$$

so that

$$i = \frac{990}{11{,}000} = .09, \text{ or } 9\%.$$

34. There are a total of 60 monthly payments. Prospectively $B_{40}$ must be the present value of the payments at times 41 through 60. The monthly interest rate is $j = .09/12 = .0075$. Payments decrease 2% each payment, so we have

$$\begin{aligned} B_{40} &= 1000\left[(.98)^{40}(1.0075)^{-1} + (.98)^{41}(1.0075)^{-2} + \cdots + (.98)^{59}(1.0075)^{-20}\right] \\ &= 1000(.98)^{40}(1.0075)^{-1}\frac{1-(.98/1.0075)^{20}}{1-(.98/1.0075)} \\ &= \$6889 \text{ to the nearest dollar upon summing the geometric progression.} \end{aligned}$$

35. We have $B_0 = 1000$. For the first 10 years only interest is paid, so we have $B_{10} = 1000$. For the next 10 years each payment is equal to 150% of the interest due. Since the lender charges 10% interest, 5% of the principal outstanding will be used to reduce the principal each year. Thus, we have $B_{20} = 1000(1-.05)^{10} = 598.74$. The final 10 years follows a normal loan amortization, so

$$X = \frac{598.74}{a_{\overline{10}|.10}} = \frac{598.74}{6.14457} = \$97.44.$$

36. We have $B_t = \bar{a}_{\overline{25-t}|}$ and the interest paid at time $t$ is $\delta B_t\, dt$ by applying formulas (5.12) and (5.14). Thus, the interest paid for the interval $5 \le t \le 10$ is

$$\int_5^{10} \delta \bar{a}_{\overline{25-t}|} dt = \int_5^{10} \left(1 - v^{25-t}\right) dt = \left[ t - \frac{v^{25-t}}{\delta} \right]_5^{10} = (10-5) - \frac{1}{\delta}\left(v^{15} - v^{20}\right).$$

Evaluating this expression for $i = .05,$ we obtain

$$5 - \frac{1}{\ln(1.05)}\left[(1.05)^{-15} - (1.05)^{-20}\right] = 2.8659.$$

37. (*a*) $$(1+i)^t - \frac{\bar{s}_{\overline{t}|}}{\bar{a}_{\overline{n}|}} = (1+i)^t - \frac{(1+i)^t - 1}{1-v^n} = \frac{(1+i)^t - v^{n-t} - (1+i)^t + 1}{1-v^n} = \frac{1-v^{n-t}}{1-v^n} = \frac{\bar{a}_{\overline{n-t}|}}{\bar{a}_{\overline{n}|}}.$$

(*b*) The LHS is the retrospective loan balance and the RHS is the prospective loan balance for a loan of 1 with continuous payment $1/\bar{a}_{\overline{n}|}$.

38. The loan is given by

$$L = \int_0^n t v^t dt = (\bar{I}\,\bar{a})_{\overline{n}|}.$$

(*a*) $$B_k^p = \int_k^n t v^{t-k} dt = \int_0^{n-k} (k+s) v^s ds = k\bar{a}_{\overline{n-k}|} + (\bar{I}\,\bar{a})_{\overline{n-k}|}.$$

(*b*) $$B_k^r = L(1+i)^k - \int_0^k t(1+i)^{k-t}\, dt = (\bar{I}\,\bar{a})_{\overline{n}|}(1+i)^k - (\bar{I}\,\bar{s})_{\overline{k}|}.$$

39. (*a*) Since $B_0 = 1$ and $B_{10} = 0$ and loan balances are linear, we have

$$B_t = 1 - t/10 \quad \text{for} \quad 0 \le t \le 10.$$

The principal repaid over the first 5 years is $B_0 - B_5 = 1 - .5 = .5.$

(*b*) The interest paid over the first 5 years is

$$\int_0^5 \delta B_t\, dt = \int_0^5 \delta\left(1 - \frac{t}{10}\right) = \delta\left[t - \frac{t^2}{20}\right]_0^5 = .10\left(5 - \frac{25}{20}\right) = .375.$$

40. (*a*) The undiscounted balance is given by

$$B_t = \int_t^\infty P(s)\,ds = \alpha e^{-Bt}.$$

The rate of payment is the rate of change in $B_t$, i.e.

$$P(t) = -\frac{d}{dt}B_t = -\frac{d}{dt}\alpha e^{-Bt} = \alpha\beta e^{-Bt}.$$

(*b*) This is $B_0 = \alpha e^{-Bt}\big|_{t=0} = \alpha.$

(*c*) The present value of the payment at time $t=0$ is

$$\int_0^\infty v^t P(t)\,dt = \int_0^\infty e^{-\delta t}\alpha\beta e^{-Bt}dt = \alpha\beta\int_0^\infty e^{-(\beta+\delta)t}dt = \frac{\alpha\beta}{\beta+\delta}.$$

(*d*) Similarly to part (*c*)

$$\int_t^\infty v^{s-t}P(s)\,ds = \alpha\beta\int_t^\infty e^{-\delta(s-t)}e^{-Bs}ds = \alpha\beta\int_t^\infty e^{\delta t}e^{-(\beta+\delta)s}ds = \frac{\alpha\beta}{\beta+\delta}e^{-\beta t}.$$

41. The quarterly interest rate is $.16/4=.04$ on the first 500 of loan balance and $.14/4=.035$ on the excess. Thus, the interest paid at time $t$ is $I_t=(.04)(500)+.035(B_{t-1}-500)=2.50+.035B_{t-1}$ as long as $B_t \geq 500$. We can generate values recursively as follows:

$$I_1 = 2.50+.035(2000)=72.50$$
$$P_1 = P - I_1 = P - 72.50$$
$$B_1 = B_0 - P_1 = 2072.50 - P$$

$$I_2 = 2.50+.035(2072.50-P)=75.04-.035P$$
$$P_2 = P - I_2 = 1.035P - 75.04$$
$$B_2 = B_1 - P_2 = 2147.54 - 2.035P$$

$$I_3 = 2.50+.035(2147.54-2.035P)=77.664-.071225P$$
$$P_3 = P - I_3 = 1.071225P - 77.664$$
$$B_3 = B_2 - P_3 = 2225.204 - 3.106225P$$

$$I_4 = 2.50+.035(2225.204-3.106225P)=80.382-.108718P$$
$$P_4 = P - I_4 = 1.108718P - 80.382$$
$$B_4 = B_3 - P_4 = 2305.586 - 4.214943P = 1000.00$$

Solving for $P = \dfrac{2305.586-1000.00}{4.214943} = \$310$ to the nearest dollar.

42. The quarterly interest rate is $.12/4=.03$ on the first 500 of loan balance and $.08/4=.02$ on the excess.

    (*a*) For each payment of 100, interest on the first 500 of the loan balance is $.03(500)=15$. Thus, the remaining loan balance of $1000-500=500$ is amortized with payments of $100-15=85$ at 2% interest. Retrospectively,

$$B_3 = 500(1.02)^3 - 85s_{\overline{3}|.02} = 270.46$$
$$I_4 = .02(270.46) = 5.41$$
$$P_4 = 85 - I_4' = 85 - 5.41 = \$79.59.$$

    (*b*) Prior to the crossover point, the successive principal repayments form a geometric progression with common ratio 1.02 (see Table 5.5 for an illustration).

43. We have $B_0 = 3000.$

    Proceeding as in Exercise 41, we find that

$$B_5 = 3191.289 - 5.101005P.$$

    Proceeding further, we find that

$$B_9 = 3364.06 - 9.436502P.$$

    However, prospectively we also know that

$$B_9 = Pa_{\overline{3}|.015}.$$

    Equating the two expressions for $B_9$, we have

$$P = \frac{3364.06}{9.436502 + a_{\overline{3}|.015}} = \$272.42.$$

44. (*a*) We have

$$a_{\overline{n}|} + i\sum_{t=0}^{n-1} a_{\overline{n-t}|} = a_{\overline{n}|} + \sum_{t=0}^{n-1}\left(1-v^{n-t}\right) = a_{\overline{n}|} + n - a_{\overline{n}|} = n.$$

    (*b*) For a loan $L = a_{\overline{n}|}$, then $a_{\overline{n}|}$ is the sum of the principal repaid column. The summation of $ia_{\overline{n-t}|}$ is the sum of the interest paid column. The two together sum to the total installment payments which is *n*.

45. (*a*) Prospectively, we have

$$B_t = Ra_{\overline{n-t|}} = \frac{R}{i}\left(1-v^{n-t}\right) = \frac{R}{i}\left(1-v^n - v^{n-t} + v^n\right)$$
$$= \frac{R}{i}\left[\left(1-v^n\right) - v^n\left\{(1+i)^t - 1\right\}\right] = R\left(a_{\overline{n|}} - v^n s_{\overline{t|}}\right).$$

(*b*) The outstanding loan balance $B_t$ is equal to the loan amount $Ra_{\overline{n|}}$ minus the sum of the principal repaid up to time *t*.

46. The initial fund is $B_0 = 10,000a_{\overline{10|}.035}$. After 5 years, fund balance is retrospectively

$$B_5' = 10,000a_{\overline{10|}.035}(1.05)^5 - 10,000s_{\overline{5|}.05}.$$

The outstanding balance on the original schedule is

$$B_5 = 10,000a_{\overline{5|}.035}.$$

Thus, the excess interest at time $t=5$ is

$B_5' - B_5 = 10,000\left[(8.3166)(1.27628) - 5.5256 - 4.5151\right] = \$5736$ to the nearest dollar.

47. (*a*) The original deposit is

$$D_1 = \frac{10,000}{\ddot{s}_{\overline{10|}.05}} = \frac{10,000}{13.20679} = \$757.19.$$

(*b*) After 5 years the balance is

$$D_1\ddot{s}_{\overline{5|}.05} = (757.19)(5.80191) = 4393.14\,.$$

Then, the revised deposit is

$$D_2 = \frac{10,000 - 4393.14(1.04)^5}{\ddot{s}_{\overline{5|}.04}} = \$826.40.$$

48. We have $R_L = \dfrac{L}{a_{\overline{30|}.04}}$.

The payment for loan *M* is $L/30$ in principal plus a declining interest payment. The loan balances progress linearly as

$$L, \frac{29L}{30}, \frac{28L}{30}, \cdots, \frac{31-k}{30}L$$

in year $k$. We have $P_L = P_M$ , so that

$$\frac{L}{a_{\overline{30}|.04}} = \frac{L}{30} + \frac{.04(31-k)L}{30}$$

$$\text{or} \quad \frac{L}{a_{\overline{30}|.04}} = \frac{2.24-.04k}{30}$$

and solving, we obtain $k = 12.63$ . Thus, $P_L$ first exceeds $P_M$ at time $t = 13$.

49. ($a$) We have on the original mortgage

$$R = \frac{80,000}{a_{\overline{20}|.08}} \quad \text{and} \quad B_9 = \left(\frac{80,000}{a_{\overline{20}|.08}}\right) a_{\overline{11}|.08}$$

and on the revised mortgage

$$B_9' = B_9 - 5000 = R' a_{\overline{9}|.09}.$$

Thus,

$$R' = \frac{\left(\frac{80,000}{a_{\overline{20}|.08}}\right) a_{\overline{11}|.08} - 5000}{a_{\overline{9}|.09}}.$$

($b$) We have at issue

$$80,000 = \left(\frac{80,000}{a_{\overline{20}|.08}}\right) a_{\overline{9}|.09} + 5000 v_{.09}^9 + R' v_{.09}^9 a_{\overline{9}|.09}$$

so that

$$R' = \frac{80,000(1.09)^9 - \left(\frac{80,000}{a_{\overline{20}|.08}}\right) s_{\overline{9}|.09} - 5000}{a_{\overline{9}|.09}}.$$

50. The regular payment is

$$R = \frac{1000}{a_{\overline{10}|.05}} = \frac{1000}{7.72173} = 129.50.$$

The penalties are:

$$.02(300 - 129.50) = 3.41 \text{ at } t = 1$$

$$\text{and } .02(250 - 129.50) = 2.41 \text{ at } t = 2.$$

Thus, only 296.59 and 247.59 go toward principal and interest. Finally,

$B_3 = 1000(1.05)^3 - 296.59(1.05)^2 - 247.59(1.05) = \$571$ to the nearest dollar.