Chapter 7

1. The maintenance expense at time $t=6$ is $3000(1.06)^{6-0}=4255.56$. The projected annual return at time $t=6$ is $30,000(.96)^{6-1}=24,461.18$. Thus,

$$R_6 = 24,461.18 - 4255.56 = \$20,206 \text{ to the nearest dollar.}$$

2. (*a*) $P(i) = -7000 + 4000v_i - 1000v_i^2 + 5500v_i^3$.

   Thus, $P(.09) = 1000\left[-7 + 4(.91743) - (.91743)^2 + 5.5(.91743)^3\right] = 75.05.$

   (*b*) $P(.10) = 1000\left[-7 + 4(.90909) - (.90909)^2 + 5.5(.90909)^3\right] = -57.85.$

3. Net cash flows are:

Time	NCF
0	− 3000
1	2000 − 1000 = 1000
2	4000

   The IRR is found by setting $P(i)=0$, i.e.

$$-3000 + 1000v + 4000v^2 = 0$$
$$4v^2 + v - 3 = (4v-3)(v+1) = 0$$

   so that $v = \frac{3}{4}$, rejecting the root $v=-1$. Finally, $1+i = \frac{4}{3}$, and $i = \frac{1}{3}$, so $n=3$.

4. The equation of value equating the present values of cash inflows and cash outflows is

$$2,000,000 + Xv^5 = 600,000a_{\overline{10}|} - 300,000a_{\overline{5}|} \text{ at } i = 12\%.$$

   Therefore,

$$X = \left[600,000a_{\overline{10}|} - 300,000a_{\overline{5}|} - 2,000,000\right](1.12)^5$$
$$= \$544,037.$$

5. Project P: $P(i) = -4000 + 2000v + 4000v^2$.

   Project Q: $P(i) = 2000 + 4000v - Xv^2$.

   Now equating the two expressions, we have

$$(X + 4000)v^2 - 2000v - 6000 = 0$$
$$(X + 4000) - 2000(1.1) - 6000(1.1)^2 = 0$$

   and

$$X = 2200 + 7260 - 4000 = \$5460.$$

6. (*a*) This Exercise is best solved by using the NPV functionality on a financial calculator. After entering all the NCF's and setting $\text{I} = 15\%$, we compute $\text{NPV} = P(.15) = -\$498,666.$

   (*b*) We use the same NCF's as in part (*a*) and compute $\text{IRR} = 13.72\%$.

7. (*a*) The formula for $P(i)$ in Exercise 2 has 3 sign changes, so the maximum number of positive roots is 3.

   (*b*) Yes.

   (*c*) There are no sign changes in the outstanding balances, i.e.

$$7000 \quad \text{to} \quad 3000 \quad \text{to} \quad 4000 \quad \text{at} \quad i = 0.$$

   Taking into account interest in the range of 9% to 10 % would not be significant enough to cause any sign changes.

8. The equation of value at time $t = 2$ is

$$100(1+r)^2 - 208(1+r) + 108.15 = 0$$
$$(1+r)^2 - 2.08(1+r) + 1.0815$$

   which can be factored as

$$[(1+r) - 1.05][(1+r) - 1.03].$$

   Thus, $r = .05$ and .03, so that $|i - j| = .02.$

9. Using one equation of value at time $t = 2,$ we have

$$\begin{matrix} 1000(1.2)^2 + A(1.2) + B = 0 \\ 1000(1.4)^2 + A(1.4) + B = 0 \end{matrix} \quad \text{or} \quad \begin{matrix} 1.2A + B = -1440 \\ 1.4A + B = -1960. \end{matrix}$$

   Solving two equations in two unknowns gives $A = -2600$ and $B = 1680.$

10. (*a*) Adapting formula (7.6) we have:

    Fund A: 10,000

    Fund B: $600 s_{\overline{5}|.04}(1.04)^5 = (600)(5.416323)(1.216653) = 3953.87$

    Fund C: $600 s_{\overline{5}|.05} = (600)(5.525631) = 3315.38$.

    A+B+C = $10,000 + 3953.87 + 3315.38 = \$17,269$ to the nearest dollar.

(*b*) We then have the equation of value

$$10,000(1+i')^{10} = 17,269$$

so that

$$i' = (1.7269)^{1/10} - 1 = .0562, \text{ or } 5.62\%.$$

11. If the deposit is D, then the reinvested interest is $.08D,\ .16D,\ .24D, \ldots,\ .80D$. We must adapt formula (7.7) for an annuity-due rather than an annuity-immediate. Thus, we have the equation of value

$$10D + .08D(Is)_{\overline{10}|.04} = 1000$$

so that

$$D = \frac{1000}{10 + \frac{.08}{.04}\left(\ddot{s}_{\overline{10}|.04} - 10\right)} = \frac{1000}{2\ddot{s}_{\overline{10}|.04} - 10} = \frac{1000}{\ddot{s}_{\overline{11}|.04} - 12}.$$

12. The lender will receive a total accumulated value of $1000s_{\overline{20}|.05} = 33,065.95$ at the end of 20 years in exchange for the original loan of 10,000. Thus, we have the equation of value applying formula (7.9)

$$10,000(1+i')^{20} = 33,065.95$$

and

$$i' = (3.306595)^{1/20} - 1 = .0616, \quad \text{or} \quad 6.16\%.$$

13. From formula (7.7) the total accumulated value in five years will be

$$5(1000) + 40\frac{s_{\overline{5}|.03} - 5}{.03} = 5412.18\,.$$

The purchase price P to yield 4% over these five years is

$$P = 5412.18(1.04)^{-5} = \$4448 \text{ to the nearest dollar.}$$

14. Applying formula (7.10) we have

$$110(1+i')^{24} = 5s_{\overline{24}|.035} + 100 = 283.3326$$

so that

$$(1+i')^{24} = 2.57575 \quad \text{and} \quad i' = (2.57575)^{1/24} - 1 = .04021.$$

The answer is

$$2i' = 2(.04021) = .0804, \text{ or } 8.04\%.$$

15. The yield rate is an annual effective rate, while the bond coupons are semiannual. Adapting formula (7.10) for this situation we have

$$1000(1.07)^{10} = 30s_{\overline{20}|j} + 1000$$

and

$$s_{\overline{20}|j} = 32.23838.$$

We now use a financial calculator to solve for the unknown rate j to obtain $j = .047597$. The answer is the annual effective rate i equivalent to j, i.e. $i = (1+j)^2 - 1 = .0975$, or 9.75%.

16. The equation of value is

$$300\ddot{s}_{\overline{20}|.08} = (20)(300) + 300i(Is)_{\overline{20}|i/2}$$

or

$$14{,}826.88 = 6000 + 300i\left(\frac{s_{\overline{21}|i/2} - 21}{i/2}\right)$$

$$= 6000 + 600s_{\overline{21}|i/2} - 12{,}600$$

and

$$s_{\overline{21}|i/2} = 35.711467.$$

We now use a financial calculator to solve for the unknown rate $i/2$ to obtain $i/2 = .050$, so that $i = .100$, or 10.0%.

17. The loan is 25,000 and if it is entirely repaid at the end of one year the amount paid will be

$$25{,}000(1.08) = 27{,}000.$$

This money can be reinvested by the lender at only 6% for the next three years. Thus, over the entire four-year period we have a lender yield rate of

$$25{,}000(1+i')^4 = 27{,}000(1.06)^3 = 32{,}157.43$$

or

$$i' = (1.286)^{1/4} - 1 = .0649, \text{ or } 6.49\%.$$

18. The accumulated value of the 50,000 payments at time $t = 4$ is

$$50{,}000s_{\overline{3}|.08} = 162{,}300.$$

Thus we have

$$\text{NPV} = P(.1) = -100{,}000 + (1.1)^{-4}(162{,}300) = \$10{,}867 \text{ to the nearest dollar.}$$

19. We have

$$B = 1000(1.04) + 200\left[1 + \frac{3}{4}(.04)\right] - 300\left[1 + \frac{1}{4}(.04)\right]$$
$$= \$943.$$

20. First, we apply formula (7.11)

$$B = A + C + I$$
$$10,636 = 10,000 + 1800 - K + 900 + I$$

so that $I = K - 2064$. Next, we apply formula (7.15)

$$i = .06 = \frac{I}{A + \sum_t C_t (1-t)} = \frac{K - 2064}{10,000 + 1800\left(\frac{5}{6}\right) - K\left(\frac{1}{2}\right) + 900\left(\frac{1}{3}\right)} = \frac{K - 2064}{11,800 - \frac{1}{2}K}$$

and solving for K

$$.06\left(11,800 - \frac{1}{2}K\right) = K - 2064$$
$$1.03K = 2772 \quad \text{giving} \quad K = \$2691 \quad \text{to the nearest dollar.}$$

21. We have

$$2,000,000 = .08(25,000,000) + .04(X - 2,200,000 - 750,000)$$
$$= 1,882,000 + .04X$$
$$\text{and} \quad X = 2,950,000.$$

Now

$$B = 25,000,000 + 2,950,000 + 2,000,000 - 2,200,000 - 750,000 = 27,000,000.$$

Finally, we apply formula (7.16) to obtain

$$i = \frac{2I}{A + B - I} = \frac{(2)(2,000,000)}{25,000,000 + 27,000,000 - 2,000,000} = .08, \text{ or } 8\%.$$

22. Under compound interest theory

$$(1 + {}_t i_0)(1 + {}_{1-t} i_t) = 1 + i$$

without approximation.

(*a*) $${}_t i_0 = \frac{1+i}{1+(1-t)i} - 1 = \frac{ti}{1+(1-t)i}.$$

(*b*) $${}_{1-t} i_t = \frac{1+i}{1+ti} - 1 = \frac{(1-t)i}{1+ti}.$$

23. We combine formula (7.11)
$$B = A + C + I$$
and formula (7.15) with one term in the denominator to obtain
$$i \approx \frac{I}{A + \sum_t C_t(1-t)} = \frac{I}{A + C(1-k)}$$
$$= \frac{I}{A + (B - A - I)(1-k)} = \frac{I}{kA + (1-k)B - (1-k)I}.$$

24. (*a*) Yes. The rate changes because the new dates change the denominator in the calculation of i^{DW}.
(*b*) No. The rate does not change because the calculation of i^{TW} depends on the various fund balances, but not the dates of those balances.

25. (*a*) The equation of value is
$$1000(1+i)^2 + 1000(1+i) = 2200$$
$$\text{or } (1+i)^2 + (1+i) - 2.2.$$
Solving the quadratic $1 + i = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2.2)}}{(2)(1)} = 1.06524$ rejecting the negative root. Thus, $i^{DW} = i = .0652$, or 6.52%.
(*b*) Over the two-year time period formulas (7.18) and (7.19) give
$$1 + i^{TW} = \left(\frac{1200}{1000}\right)\left(\frac{2200}{2200}\right) = 1.2.$$
The equivalent annual effective rate is
$$i = (1 + i^{TW})^{½} - 1 = (1.2)^{.5} - 1 = .0954, \text{ or } 9.54\%.$$

26. Dollar-weighted calculation:
$$2000(1+i) + 1000\left(1 + \frac{1}{2}i\right) = 3200$$
$$i^{DW} = i = \frac{200}{2500} = .08.$$
Time-weighted calculation:
$$i^{TW} = i^{DW} + .02 = .08 + .02 = .10$$
$$\text{and } 1 + i = 1.1 = \frac{X}{2000} \cdot \frac{3200}{X + 1000} = 1.6\frac{X}{X + 1000}.$$
Solving for *X* we obtain $X = \$2200$.

27. (*a*) The equation of value is

$$2000(1+i)+1000(1+i)^{1/2}=3213.60$$

$$2+(1+i)+(1+i)^{1/2}-3.2136=0$$

which is a quadratic in $(1+i)^{1/2}$. Solving the quadratic

$$(1+i)^{1/2}=\frac{-1\pm\sqrt{1^2-(4)(2)(-3.2136)}}{(2)(2)}=1.042014$$

rejecting the negative root. Finally, $i^{DW}=i=(1.042014)^2-1=.0857$, or 8.57%.

(*b*) $1+i=\left(\frac{2120}{2000}\right)\left(\frac{3213.60}{3120}\right)=1.0918$

so $i^{TW}=.0918$, or 9.81%.

28. The 6-month time-weighted return is

$$i^{TW}=\left(\frac{40}{50}\right)\left(\frac{80}{60}\right)\left(\frac{157.50}{160}\right)-1=.05.$$

The equivalent annual rate is

$$(1.05)^2-1=.1025.$$

The 1-year time-weighted return is

$$i^{TW}=\left(\frac{40}{50}\right)\left(\frac{80}{60}\right)\left(\frac{175}{160}\right)\left(\frac{X}{250}\right)-1=.1025.$$

and solving, we obtain $X=236.25$.

29. Time-weighted return:

$$i^{TW}=0 \quad \text{means} \quad \frac{12}{10}\cdot\frac{X}{12+X}=1 \quad \text{so} \quad X=60.$$

Dollar-weighted return:

$$I=X-X-10=-10$$

so that

$$i^{DW}=Y=\frac{-10}{10+(.5)(60)}=-25\%.$$

30. (*a*) Dollar-weighted:

$$A\left(1+i^{DW}\right)=C \quad \text{and} \quad i^{DW}=\frac{C}{A}-1=\frac{C-A}{A}.$$

Time-weighted:

$$1+i^{TW}=\left(\frac{B}{A}\right)\left(\frac{C}{B}\right) \quad \text{and} \quad i^{DW}=\frac{C}{A}-1=\frac{C-A}{A}.$$

(*b*) Dollar-weighted:

The interest earned is $I=C-A-D$

and the "exposure" is $A+\frac{1}{2}D$, so $i^{DW}=\dfrac{C-A-D}{A+\frac{1}{2}D}.$

Time-weighted:

$$i^{TW}=\left(\frac{B}{A}\right)\left(\frac{C}{B+D}\right)-1.$$

(*c*) Dollar-weighted:

same as part (*b*), so $i^{DW}=\dfrac{C-A-D}{A+\frac{1}{2}D}.$

Time-weighted:

$$i^{TW}=\left(\frac{B-D}{A}\right)\left(\frac{C}{B}\right)-1$$

(*d*) Dollar-weighted calculations do not involve interim fund balances during the period of investment. All that matters are cash flows in or out of the fund and the dates they occur.

(*e*) Assume $i_b^{TW}\le i_c^{TW}$, then

$$\left(\frac{B}{A}\right)\left(\frac{C}{B+D}\right)\le\left(\frac{B-D}{A}\right)\left(\frac{C}{B}\right) \quad \text{or} \quad \frac{B}{B+D}\le\frac{B-D}{B}$$

which implies that $B^2\le B^2-D^2$, a contradiction. Therefore we must have $i_b^{TW}>i_c^{TW}$.

31. We have

$$B_2=10,000(1.0825)(1.0825)=11,718.06$$
$$B_6=10,000(1.0825)(1.0825)(1.0840)(1.0850)(1.0850)(1.0835)=16,202.18$$

so the amount of interest earned is

$$B_6-B_2=16,202.18-11,718.06=\$4484.12.$$

32. Deposit in $z+3$ $(1.090)(1.090)(1.091)(1.091)(1.092)=1.54428$
Deposit in $z+4$ $(1.090)(1.091)(1.092)(1.093)=1.41936$
Deposit in $z+5$ $(1.0925)(1.0935)(1.095)=1.30814$
Deposit in $z+6$ $(1.095)(1.095)=1.19903$
Deposit in $z+7$ $1.100=\underline{1.10000}$
$6.5708.$

33. $P=1000(1.095)(1.095)(1.096)=1314.13$
$Q=1000(1.0835)(1.086)(1.0885)=1280.82$
$R=1000(1.095)(1.10)(1.10)=1324.95$
Thus, $R>P>Q$.

34. Let $i=.01j$. Interest earned on:
Deposit in z $100(1.1)(1.1)(1+i)(.08)=9.68(1+i)$
Deposit in $z+1$ $100(1.12)(1.05)(.10)=11.76$
Deposit in $z+2$ $100(1.08)(i-.02)=108(i-.02)$.
Thus, total interest is
$$9.68+9.68i+11.76+108i-2.16=28.40$$
$$117.68i=9.12 \quad \text{and} \quad i=.0775.$$
The answer is $j=100i=7.75\%$.

35. The accumulated value is
$$1000\left(1+i_1^5\right)\left(1+i_2^5\right)\left(1+i_3^5\right)=1000(1.085)^{1.05}(1.090)^{1.05}(1.095)^{1.05}$$
$$=1000(1.31185).$$
The equivalent level effective rate is
$$i=(1.31185)^{1/3}-1=.0947, \quad \text{or } 9.47\%.$$

36. (*a*) $\delta_{s,t}=\dfrac{\frac{\partial}{\partial t}a(s,t)}{a(s,t)}=\dfrac{\partial}{\partial t}\ln a(s,t).$

(*b*) $a(s,s)=1$ and $a(s,t)=e^{\int_0^t \delta_{s,r}dr}$.

(*c*) Using an average portfolio rate
$$a(s)a(s,t)=a(t) \quad \text{and} \quad a(s,t)=\frac{a(t)}{a(s)}.$$

(*d*) $a(0,t) = (1+i)^t$.

(*e*) $a(t,t) = 1$, since no interest has yet been earned.

37. The margin is $1000m$ and the interest on it is $(.08)(1000m) = 80m$. The net profit is $200 + 80m - 60 = 140 + 80m$ on a deposit of $1000m$. Thus, the yield rate is

$$\frac{140+80m}{1000m} = \frac{7+4m}{50m}.$$

38. The margin is $(.08)(50) = 40$

Interest on margin $= 4$

Dividend on stock $= 2$

Profit on short sale $= 50 - X$

Thus, $.2 = \frac{(50-X)+4-2}{40}$ and $X = 44$.

39. The margin is $(.40)(25{,}000) = 10{,}000$

Interest on margin $= (.08)(10{,}000) = 800$

Profit on short sale $= 25{,}000 - X$

Thus, $.25 = \frac{(25{,}000-X)+800}{10{,}000}$ and $X = \$23{,}300$.

40. A's transaction:

The margin is $(.50)(1000) = 500$
Interest on margin $= (.60)(500) = 30$
Dividend on the stock $= X$
Profit on short sale $= 1000 - P$
Thus, $.21 = \frac{(1000-P)+30-X}{500}$.

B's transaction:

$$.21 = \frac{(1000-P+25)+30-2X}{500}.$$

Solving the two equations in two unknowns gives

$$X = \$25 \quad \text{and} \quad P = \$900.$$

41. Earlier receipt of dividends. Partial release of margin.

42. The yield rate in Exercise 2 is between 9% and 10% and thus less than the interest preference rate of 12%. Thus, the investment should be rejected.

43. The yield rate of the financing arrangement can be determined from the equation of value

$$5000 = 2400 + 1500v + 1500v^2$$
$$\text{or } 1.5v + 1.5v - 2.6 = 0.$$

Solving the quadratic, we have

$$v = \frac{-1.5 \pm \sqrt{(1.5)^2(-4)(1.5)(-2.6)}}{(2)(1.5)} = .90831$$

rejecting the negative root. Thus, $i = .10095$. Since the buyer would be financing at a rate higher than the interest preference rate of 10%, the buyer should pay cash.

44. (*a*) In Example 7.4 we have

$$P(i) = -100 + 200v - 101v^2 = 0$$
$$\text{or } P(i) = 100(1+i)^2 - 200(1+i) + 101$$
$$= 1 + 100i^2 = 0$$

The graph has a minimum at (0,1) and is an upward quadratic in either direction.

(*b*) There are no real roots, since the graph does not cross the *x*-axis.

45. Option (*i*):

$$800(1+i) = 900 \qquad i = \frac{900}{800} - 1 = .125.$$

Option (*ii*):

$$1000(1+i) = 1125 \qquad i = \frac{1125}{1000} - 1 = .125.$$

Thus they are equivalent, but both should be rejected. They both exceed the borrower's interest preference rate of 10%.

46. We have

$$P(i) = -100 + 230(1+i)^{-1} - 132(1+i)^{-2}$$

and

$$P'(i) = -230(1+i)^{-2} + 264(1+i)^{-3} = 0$$

so that

$$(1+i)^{-1} = \frac{230}{264} \qquad i = \frac{264}{230} - 1 = .1478, \text{ or } 14.78\%.$$

47. The following is an Excel spreadsheet for this Exercise.

Year	Contributions	Returns	PV Factors	PV Contributions	PV Returns
0	10,000	0	1.0000000	10,000.00	0.00
1	5,000	0	0.9090909	4,545.45	0.00
2	1,000	0	0.8264463	826.45	0.00
3	1,000	0	0.7513148	751.31	0.00
4	1,000	0	0.6830135	683.01	0.00
5	1,000	0	0.6209213	620.92	0.00
6	1,000	8,000	0.5644739	564.47	4,515.79
7	1,000	9,000	0.5131581	513.16	4,618.42
8	1,000	10,000	0.4665074	466.51	4,665.07
9	1,000	11,000	0.4240976	424.10	4,665.07
10		12,000	0.3855433	0.00	4,626.52
	23,000	50,000		19,395.39	23,090.88
				PI =	1.191

48. We have

$$100 + 132(1.08)^{-2} = 230(1+i)^{-1}$$
$$213.16872 = 230(1+i)^{-1}$$

so that

$$1 + i = \frac{230}{213.16872} = 1.0790.$$

Thus, the $\text{MIRR} = 7.90\%$, which is less than the required return rate of 8%. The project should be rejected.

49. The investor is in lender status during the first year, so use $r = .15$. Then $B_1 = 100(1.15) - 230 = -115$. The investor is now in borrower status during the second year, so use *f*. Then $B_2 = 0 = -115(1+f) + 132$ and $f = \dfrac{132}{115} - 1 = .1478,$ or 14.78%.

50. We compute successive balances as follows:

$$B_0 = 1000$$
$$B_1 = 1000(1.15) + 2000 = 3150$$
$$B_2 = 3150(1.15) - 4000 = -377.50$$
$$B_3 = -377.50(1.1) + 3000 = 2584.75$$
$$B_4 = 2584.75(1.15) - 4000 = -1027.54$$
$$B_5 = -1027.54(1.1) + 5000 = \$3870 \text{ to the nearest dollar.}$$

51. The price of the bond is

$$1000(1.03)^{-20} + 40a_{\overline{20}|.03} = 1148.77.$$

Thus, the loan and interest paid is

$$1148.77(1.05)^{10} = 1871.23.$$

The accumulated bond payments are

$$1000 + 40s_{\overline{20}|.02}1971.89.$$

Thus, the net gain is $1971.89 - 1871.23 = \$100.66.$

52. A withdrawal of $W = 1000$ would exactly exhaust the fund at $i = .03$. We now proceed recursively:

$$F_0 = 1000a_{\overline{10}|.03}$$
$$F_1 = F_0(1.04) = 1000\ddot{a}_{\overline{10}|.03}(1.04)$$
$$W_1 = \frac{1000a_{\overline{10}|.03}(1.04)}{\ddot{a}_{\overline{10}|.03}} = \frac{1000(1.04)}{1.03}$$
$$F_1 - W_1 = 1000(1.04)\left[a_{\overline{10}|.03} - v_{.03}\right] = \frac{1000(1.04)a_{\overline{9}|.03}}{1.03}$$
$$F_2 = \frac{1000(1.04)^2 a_{\overline{9}|.03}}{1.03}$$
$$W_2 = \frac{1000(1.04)^2 a_{\overline{9}|.03}}{1.03\ddot{a}_{\overline{9}|.03}} = \frac{1000(1.04)^2}{(1.03)^2}$$

Continuing this recursive process 8 more times and reflecting the interest rate change at time $t = 4,$ we arrive at

$$W_{10} = \frac{1000(1.04)^4(1.05)^6}{(1.03)^{10}} = \$1167 \text{ to the nearest dollar.}$$

53. We are given:

$$A = 273{,}000 \quad B = 372{,}000 \quad I = 18{,}000$$

so that

$$C = B - A - I = 81{,}000.$$

Using the simple interest approximation $273{,}000(1.06) + 81{,}000(1 + .06t) = 372{,}000$ which can be solved to give $t = 1/3$. Thus, the average date for contributions and withdrawals is September 1, i.e. the date with four months left in the year.

54. The accumulation factor for a deposit made at time t evaluated at time n, where $0 \le t \le n,$ is

$$e^{\int_t^n \delta_r dr} = e^{\int_t^n \frac{dr}{1+r}} = e^{\ln(1+n) - \ln(1+t)}$$
$$= \frac{1+n}{1+t}.$$

Then, the accumulated value of all deposits becomes

$$1 \cdot \left(\frac{1+n}{1+0}\right) \int_0^n (1+t)\left(\frac{1+n}{1+t}\right) dt = (1+n) + n(1+n) = (n+1)^2.$$