Chapter 9

1. A: A direct application of formula (9.7) for an investment of X gives

$$A = X\,\frac{(1.08)^{10}}{(1.05)^{10}} = X\left(\frac{1.08}{1.05}\right)^{10} = 1.32539X.$$

B: A direct application of formula (9.3a) for the same investment of X gives

$$1+i' = \frac{1.08}{1.05} = 1.028571$$

and the accumulated value is

$$B = X\,(1.028571)^{10} = 1.32539X.$$

The ratio $A/B = 1.00$.

2. Proceeding similarly to Exercise 1 above:

$$A = \frac{\ddot{s}_{\overline{10}|.08}}{(1.05)^{10}} = 9.60496.$$

$$B = \ddot{s}_{\overline{10}|.028571} = 11.71402.$$

The ratio $A/B = .82$.

3. Again applying formula (9.7) per dollar of investment

$$\frac{(1.07)^5}{(1.10)^5} = .87087$$

so that the loss of purchasing power over the five-year period is

$$1-.87087 = .129, \quad \text{or } 12.9\%.$$

4. The question is asking for the summation of the “real” payments, which is

$$18{,}000\left[\frac{1}{1.032}+\frac{1}{(1.032)^2}+\cdots+\frac{1}{(1.032)^{15}}\right]$$

$$= 18{,}000\,a_{\overline{15}|.032} = \$211{,}807 \text{ to the nearest dollar.}$$

5. The last annuity payment is made at time $t = 18$ and the nominal rate of interest is a level 6.3% over the entire period. The “real” rate over the last 12 years is

$$i' = \frac{i-r}{1+r} = \frac{.063-.012}{1+.012} = .0504.$$

Thus, the answer is

$$\begin{aligned} X &= 50(1.063)^{-6}\, a_{\overline{12}|.0504} \\ &= (50)(.693107)(8.84329) = \$306 \text{ to the nearest dollar.} \end{aligned}$$

6. The profitability index (PI) is computed using nominal rates of interest. From formula (9.3a)

$$1.04 = \frac{1+i}{1.035} \quad \text{and} \quad i = (1.04)(1.035) - 1 = .0764.$$

The profitability index is defined in formula (7.20)

$$\text{PI} = \frac{\text{NPV}}{\text{I}} = \frac{2000 a_{\overline{8}|.0764}}{10,000} = 1.17.$$

7. (*a*) Coupon 1 = $10,000(1.04)(.05) = \$520.$

Coupon 2 = $10,000(1.04)(1.05)(.05) = \$546.$

Maturity value = $10,000(1.04)(1.05) = \$10,920.$

(*b*) Nominal yield: The equation of value is

$$-10,500 + 520(1+i)^{-1} + (546 + 10,920)(1+i)^{-2} = 0$$

and solving the quadratic we obtain .0700, or 7.00%.

Real yield: The equation of value is

$$-10,500 + 500(1+i)^{-1} + (500 + 10,000)(1+i)^{-2} = 0$$

and solving the quadratic we obtain .0241, or 2.41%.

8. Bond A: Use a financial calculator and set

$$\text{N} = 5 \quad \text{PV} = -950 \quad \text{PMT} = 40 \quad \text{FV} = 1000 \quad \text{and} \quad \text{CPT I} = 5.16\%.$$

Bond B: The coupons will constitute a geometric progression, so

$$\begin{aligned} P &= 40\left[\left(\frac{1.05}{1.0516}\right) + \left(\frac{1.05}{1.0516}\right)^2 + \cdots + \left(\frac{1.05}{1.0516}\right)^5\right] + 1000(1.05)^5(1.0516)^{-5} \\ &= 40(1.05)\frac{1 - \left(\frac{1.05}{1.0516}\right)^5}{.0516 - .05} + 992.416 = \$1191.50 \end{aligned}$$

9. (*a*) The final salary in the 25th year will have had 24 increases, so that we have

$$10,000(1.04)^{24} = \$25,633 \text{ to the nearest dollar.}$$

(*b*) The final five-year average salary is

$$\frac{10,000}{5}\left[(1.04)^{20} + (1.04)^{21} + \cdots + (1.04)^{24}\right] = \$23,736 \text{ to the nearest dollar.}$$

(*c*) The career average salary is

$$\frac{10,000}{25}\left[1 + (1.04) + (1.04)^2 + \cdots + (1.04)^{24}\right] = \$16,658 \text{ to the nearest dollar.}$$

10. The annual mortgage payment under option A is

$$R_A = \frac{240,000 - 40,000}{a_{\overline{10}|.10}} = 32,549.08.$$

The annual mortgage payment under option B is

$$R_B = \frac{240,000 - 40,000}{a_{\overline{10}|.08}} = 29,805.90.$$

The value of the building in 10 years is

$$240,000(1.03)^{10} = 322,539.93.$$

Thus, the shared appreciation mortgage will result in a profit to Lender B

$$.50(322,539.93 - 240,000) = 41,269.97.$$

(*a*) The present value of payments under Option A is

$$PV_A = 40,000 + 32,549.08a_{\overline{10}|.08} = \$258,407 \text{ to the nearest dollar.}$$

The present value of payments under Option B is

$$PV_B = 40,000 + 29,805.90a_{\overline{10}|.08} + 41,269.97(1.08)^{-10}$$
$$= \$259,116 \text{ to the nearest dollar.}$$

Thus, at 8% choose Option A.

(*b*) Similar to (*a*) using 10%, we have

$$PV_A = 40,000 + 32,549.08a_{\overline{10}|.10} = \$240,000$$

which is just the original value of the property. Then,

$$PV_B = 40,000 + 29,805.90a_{\overline{10}|.10} + 41,269.97(1.10)^{-10}$$
$$= \$223,145 \text{ to the nearest dollar.}$$

Thus, at 10% choose Option B.

11. The price of the 3-year bond is

$$P = 1000(1.03)^{-6} + 40a_{\overline{6}|.03} = 1054.17.$$

The investor will actually pay $P + 12 = 1066.17$ for the bond. Solving for the semiannual yield rate *j* with a financial calculator, we have

$$1066.17 = 1000(1+j)^{-6} + 40a_{\overline{6}|j} \quad \text{and} \quad j = .027871.$$

The yield rate then is $2j = .0557$, or 5.57% compared to 5.37% in Example 9.3. The yield rate is slightly higher, since the effect of the expense is spread over a longer period of time.

12. The actual yield rate to A if the bond is held to maturity is found using a financial calculator to solve

$$910.00 = 1000(1+i)^{-5} + 60a_{\overline{5}|\,i}$$

which gives $i = .0827$, or 8.27%. Thus, if A sells the bond in one year and incurs another $10 commission, the price to yield 8.27% could be found from

$$910.00 = (P + 60 - 10)(1.0827)^{-1}$$

which gives $P = \$935.26.$

13. With no expenses the retirement accumulation is

$$10,000(1.075)^{35} = 125,688.70.$$

With the 1.5 expense ratio the retirement accumulation becomes

$$10,000(1.06)^{35} = 76,860.87.$$

The percentage reduction is

$$\frac{125,688.70 - 76,850.87}{125,688.70} = .389, \quad \text{or } 38.9\% \text{ compared to } 34.4\%.$$

14. The expense invested in the other account in year *k* is

$$10,000(1.06)^{k-1}(.01) \quad \text{for } k = 1, 2, \ldots, 10.$$

The accumulated value of the account after 10 years will be

$$100\left[(1.09)^9 + (1.06)(1.09)^8 + \cdots + (1.09)^9\right]$$

$$= 100\left[\frac{(1.09)^{10} - (1.06)^{10}}{.09 - .06}\right] = \$1921.73$$

by a direct application of formula (4.34).

15. The daily return rate j is calculated from

$$(i+j)^{365} = 1.075 \text{ or } j = .000198.$$

The daily expense ratio r is calculated from

$$(1+.000198-r)^{365} = 1.075 - .015 = 1.06$$
$$\text{or } r = .00003835.$$

Thus the nominal daily expense ratio is

$$(.00003835)(365) = .0140, \text{ or } 1.40\%.$$

16. Let the underwriting cost each year be $X million. The present value of the cash flows to the corporation equals zero at a 7% effective rate of interest. The equation of value (in millions) at time $t = 0$ is given by

$$10 - X - X(1.07)^{-1} - (.06)(10)a_{\overline{10}|.07} - 10(1.07)^{-10} = 0$$

and

$$X = \frac{10 - .6a_{\overline{10}|.07} - 10(1.07)^{-10}}{1+(.107)^{-1}}$$
$$= .363 \text{ million, or } \$363{,}000 \text{ to the nearest } \$1000.$$

17. Basis A:

The interest income $= (1.08)^{20} - 1 = 3.66096$ and the after-tax accumulated value is

$$A = 1 + (.75)(3.66096) = 3.74572.$$

Basis B:

The after-tax accumulated value is

$$B = [1+(.75)(.08)]^{20} = (1.06)^{20} = 3.20714.$$

The ratio $A/B = 1.168$, or 116.8%.

18. The tax deduction is 35% of the depreciation charge.

Year	Depreciation charge	Tax deduction
1	33,330	11,666
2	44,450	15,558
3	14,810	5,184
4	7,410	2,594

The after-tax yield rate is $(.12)(.65) = .078.$

Thus, the present value of the tax deductions is

$$\frac{11,666}{1.078}+\frac{15,558}{(1.078)^2}+\frac{5184}{(1.078)^3}+\frac{2594}{(1.078)^4}=\$30,267 \text{ to the nearest dollar.}$$

19. (*a*) The semiannual before-tax yield rate j^b can be found from

$$670=700\left(1+j^b\right)^{-10}+35a_{\overline{10}|j^b}$$

which gives $j^b=.05571.$ The before-tax effective yield rate is $(1.05571)^2-1=.1145,$ or 11.45%.

(*b*) The earnings on the bond are $(35)(10)+(700-670)=380$ and the tax on that amount is $(.25)(380)=95.$ Then, $670=(700-95)\left(1+j^a\right)^{-10}+35a_{\overline{10}|j^a}$ giving $j^a=.04432$ and an after-tax effective yield rate of $(1.04432)^2-1=.0906,$ or 9.06%.

20. Before-tax: The equation of value is

$$97.78\left(1+i^b\right)=10+102.50 \quad \text{and} \quad i^b=.151, \quad \text{or} \quad 15.1\%.$$

After-tax: The equation of value is

$$\begin{aligned}97.78\left(1+i^a\right)&=10+102.50-.40(10)-.20(102.50-97.78)\\&=107.556 \text{ and } i^a=.100, \text{ or } 10.0\%.\end{aligned}$$

21. (*a*) The installment payment is

$$R=\frac{10,000}{a_{\overline{30}|.05}}=650.51.$$

The income tax in the 10th year is 25% of the portion of the 10th installment that is interest, i.e.

$$(.25)(650.51)\left(1-v^{30-10+1}\right)=\$104.25.$$

(*b*) The total of the interest paid column in the amortization schedule is

$$650.51\left(30-a_{\overline{30}|.05}\right)=\$9515.37.$$

The total tax on this amount of interest is

$$(.25)(9515.37)=\$2378.84.$$

(c) The payment in the k^{th} year after taxes is

$$650.51\left[v^{30-k+1}+.75\left(1-v^{30-k+1}\right)\right]=650.51\left(.75+.25v^{31-k}\right) \text{ for } k=1,2,\ldots,30.$$

The present value of these payments at the before-tax yield rate is

$$\sum_{k=1}^{30} 650.51\left(.75+.25v^{31-k}\right)v^k \quad \text{at} \quad i=5\%$$

$$=(.75)(650.51)a_{\overline{30}|.05}+(.25)(650.51)(30)(1.05)^{-31}$$

$$=\$8575 \text{ to the nearest dollar.}$$

22. The annual installment payment is

$$10,000(.05)+\frac{10,000}{s_{\overline{30}|.04}}=500+178.30=678.30.$$

Total installment payments over 30 years are

$$30(678.30)=20,349.00.$$

Total interest on the sinking fund is

$$10,000-(30)(178.30)=4651.00.$$

Taxes on the sinking fund interest are

$$.25(4651.00)=1162.75.$$

Total cost of the loan to A is

$$20,349.00+1162.75=\$21,512 \text{ to the nearest dollar.}$$

23. The after-tax interest income on the fund is

$$800-(240+200+300)=60.$$

Since the income tax rate is 25%, the before-tax interest income on the fund is

$$60/.75=80.$$

Thus, the fund balance at the beginning of the third year before taxes is actually

$$800+(80-60)=820.$$

The equation of value for the before-tax yield rate is

$$240\left(1+i^b\right)^2+200\left(1+i^b\right)+300=820.$$

Solving the quadratic, we obtain

$$i^b=.113, \text{ or } 11.3\%.$$

24. (a) The NPV for the company is

$$62,000\ddot{a}_{\overline{6}|.08}=\$309,548.$$

(*b*) The NPV for the lessor reflecting taxes and depreciation is

$$(.65)(62,000)\ddot{a}_{\overline{6}|.08} + (.35)(250,000/5)a_{\overline{5}|.08} = \$271,079.$$

25. Applying formula (9.11)

$$1+i^d = (1+i^f)\frac{e^c}{e^e}$$

$$1+.075 = (1+.049)\frac{118}{e^e} \text{ and } e^e = 115.1.$$

26. The change in the real exchange rate is equal to the change in the nominal exchange rate adjusted for the different inflation rates, i.e.

$$\frac{1.5}{1.25}\times\frac{1.03}{1.02} - 1 = +.212, \text{ or } +21.2\%.$$

27. Line 1 - We have

$$1+i^d = (1+i^f)\frac{e^c}{e^e} \text{ or } 1+i^d = (1.0932)\frac{56.46}{60.99} \text{ and } i^d = .012, \text{ or } 1.2\%.$$

Line 2 - We have

$$(1.01)^{.25} = (1+i^f)^{.25}\,\frac{56.46}{57.61} \text{ and } i^f = .0948, \text{ or } 9.48\%.$$

Line 3 - We have

$$(1.01)^{1/12} = (1.0942)^{1/12}\,\frac{56.46}{e^e} \text{ and } e^e = 56.84.$$

28. (*a*) For a \$1000 maturity value, the price of the two-year coupon bond is

$$P_0 = 1000(1.0365)^{-2} = \$930.81.$$

The price one year later is

$$P_1 = 1000(1.03)^{-1} = \$970.87.$$

Thus, the one-year return is $\dfrac{970.87-930.81}{930.81} = .043,$ or 4.3%.

(*b*) Assume the person from Japan buys \$930.81 with yen, which costs

$$(930.81)(120.7) = ¥112,349.$$

After one year the person receives \$970.87 which is worth

$$(970.87)(115) = ¥111,650.$$

Thus, the one year return is

$$\frac{111,650 - 112,349}{112,349} = -.0062, \text{ or } -.62\%.$$

29. (*a*) $\text{NPV} = -80 + 10(1.06)^{-1} + 20(1.06)^{-2} + 23(1.06)^{-3} + 27(1.06)^{-4} + 25(1.06)^{-5}$
$= €6.61$ million.

(*b*) The expected exchange rate expressed in dollars per €1 (not in euros per \$1) at time $t = 1$ is $\frac{1.08}{1.06} = \frac{e^e}{1.2}$ and $e^e = 1.223.$ The cash flow at time $t = 1$ in dollars then is

$$(10)(1.223) = \$12.23 \text{ million}.$$

Using the same procedure to calculate the expected exchange rate and cash flow in dollars each year gives the following:

Time	0	1	2	3	4	5
e^e	1.2	1.223	1.246	1.269	1.293	1.318
\$ million	-96	12.23	24.91	29.19	34.92	32.94

(*c*) $\text{NPV} = -96 + 12.23(1.08)^{-1} + 24.91(1.08)^{-2} + 29.19(1.08)^{-3} + 34.92(1.08)^{-4}$
$+32.94(1.08)^{-5} = \$7.94$ million.

Interestingly, this answer could also be obtained from the answer in part (*a*), as $(6.613)(1.2) = 7.94$. This demonstrates the internal consistency in using interest rate parity.

30. The equation of value is

$$(1+i)^2 = .4(1+i) + .5 \text{ which simplifies to } i^2 + 1.6i + .1 = 0.$$

Solving the quadratic we obtain

$$i = -.0652, \text{ or } -6.52\%.$$

31. The equation of value is

$$10,000(1+i)^{10} = 1500 s_{\overline{10}|.08} = 21,729.84$$

and

$$i = (2.172984)^{1/10} - 1 = .0807, \text{ or } 8.07\%.$$

32. Use the basic formula for valuing bonds with an adjustment for the probability of default to obtain

$$\begin{aligned} P &= 80a_{\overline{10}|.12} + (.98)(1000)(1.12)^{-10} \\ &= 452.018 + 315.534 = \$767.55. \end{aligned}$$

33. (*a*) $\text{EPV} = \dfrac{(.90)(1000)+(.10)(0)}{1.25} = \$720.$

(*b*) $\text{E}(x^2) = .90\left(\dfrac{1000}{1.25}\right)^2 + .10(0)^2 = 576{,}000$

$\text{Var}(x) = 576{,}000 - (720)^2 = 57{,}600$

$\text{S.D.}(x) = \sqrt{57{,}600} = \$240.$

(*c*) $720 = 1000(1+i)^{-1}$ so that $i = .3889$. Thus, the risk premium is

$$.3889 - .25 = .1389, \text{ or } 13.89\%.$$

34. From formula (9.15) we have

$$\text{EPV} = \sum_{t=1}^{n} R_t \left(\frac{p}{1+i}\right)^t.$$

We can consider p to be the probability of payment that will establish equivalency between the two interest rates. Thus, we have

$$\frac{p}{1.0875} = \frac{1}{1.095} \quad \text{or} \quad p = .99315$$

and the annual probability of default is $1 - p = .00685$.

35. (*a*) $\text{EPV} = \displaystyle\sum_{t=1}^{n} R_t \left(\frac{p}{1+i}\right)^t = \sum_{t=1}^{n} R_t e^{-ct} e^{-\delta t}.$

(*b*) We can interpret the formula in part (*a*) as having force of interest δ, force of default c, and present values could be computed at the higher force of interest $\delta' = \delta + c$. The risk premium is $\delta' - \delta = c$.

(*c*) The probability of default is

$$1 - p = 1 - e^{-c}.$$

(*d*) The probability of non-default over *n* periods is $p^n = e^{-cn}$, so the probability of default is $1 - e^{-cn}$.

36. (*a*) Assume that the borrower will prepay if the interest rate falls to 6%, but not if it rises to 10%. The expected accumulated to the mortgage company is

$$.5[80,000(1.06) + 1,000,000(1.06)] + .5[580,000(1.10) + 500,000(1.08)]$$
$$= .5(1,144,800) + .5(1,178,000) = \$1,161,400.$$

(*b*) $\text{Var} = .5(1,144,800 - 1,161,400)^2 + .5(1,178,000 - 1,161,400)^2 = 275,560,000$ and $\text{S.D.} = \sqrt{275,560,000} = \$16,600.$

(*c*) We have $1,161,400 = 1,000,000(1+i)^2$ which solves for $i = .0777,$ or 7.77%.

(*d*) The option for prepayment by the borrower has a value which reduces the expected yield rate of 8% that the lender could obtain in the absence of this option.

37. (*a*) Form formula (9.15) we have

$$\text{EPV} = \sum_{t=1}^{n} R_t \left(\frac{p}{1+i} \right)^t$$

so that

$$150,000 = \sum_{t=1}^{n} R \left(\frac{.99}{1.12} \right)^t .$$

We can define an adjusted rate of interest i', such that

$$1 + i' = \frac{1.12}{.99} \quad \text{and} \quad i' = .131313.$$

We then obtain $R = \dfrac{150,000}{a_{\overline{15}|.131313}} = 23,368.91.$

If the probability of default doubles, we can define

$$1 + i'' = \frac{1.12}{.98} \quad \text{and} \quad i'' = .142857.$$

We then have

$$\text{EPV} = 23,368.91 a_{\overline{15}|.142857} = \$141,500 \quad \text{to the nearest \$100.}$$

(*b*) We now have

$$1 + i''' = \frac{1.14}{.98} \quad \text{and} \quad i''' = .163265$$

and

$$\text{EPV} = 23,368.91 a_{\overline{15}|.163265} = \$128,300 \quad \text{to the nearest \$100.}$$

38. If the bond is not called, at the end of the 10 years the investor will have

$$100 s_{\overline{10}|.07} + 1000 = 2381.65.$$

If the bond is called, at the end of 10 years, the investor will have

$$100 s_{\overline{5}|.07} (1.07)^5 + 1050(1.07)^5 = 2279.25.$$

Thus, the expected accumulated value (EAV) is

$$(.75)(2381.65) + (.25)(2279.25) = 2356.05.$$

The expected yield rate to the investor can be obtained from

$$1100(1+i)^{10} = 2356.05$$

and

$$i = \left(\frac{2356.05}{1100}\right)^{1/10} - 1 = .0791, \text{ or } 7.91\%.$$