

## Chapter 12

$$1. \quad E[a^{-1}(n)] = E\left[\prod_{t=1}^n (1+i_t)^{-1}\right]$$

$$= \prod_{t=1}^n E[1+i_t]^{-1} \quad \text{from independence}$$

$$= (1+\bar{i})^{-n}.$$

$$2. \quad E[a_{\overline{n}|}] = E\left[\sum_{t=1}^n \prod_{s=1}^t (1+i_s)^{-1}\right]$$

$$= \sum_{t=1}^n \prod_{s=1}^t E[1+i_s]^{-1} \quad \text{from independence}$$

$$= \sum_{t=1}^n (1+\bar{i})^{-t} = a_{\overline{n}|\bar{i}}.$$

3. (a) Year 1: 8% given.

Year 2:  $.5(.07 + .09) = .08$ , or 8%.

Year 3:  $.25[.06 + 2(.08) + .10] = .08$ , or 8%.

(b) Year 1:  $\sigma = 0$ , no variance.

Year 2:  $\sigma^2 = .5[(.07 - .08)^2 + (.09 - .08)^2] = .0001$

$$\sigma = \sqrt{.0001} = .01.$$

Year 3:  $\sigma^2 = .25[(.06 - .08)^2 + 2(.08 - .08)^2 + (.10 - .08)^2]$

$$= .0002$$

$$\sigma = \sqrt{.0002} = .01\sqrt{2}$$

(c)  $1000(1.08)(1.09)(1.10) = \$1294.92$ .

(d)  $1000(1.08)(1.07)(1.06) = \$1224.94$ .

(e)  $1000(1.08)^3 = \$1259.71$ .

(f)  $.25(1000)[(1.08)(1.09)(1.10) + (1.08)(1.09)(1.08)$

$$+ (1.08)(1.07)(1.08) + (1.08)(1.07)(1.06)]$$

$$= .25[1294.92 + 1271.38 + 1248.05 + 1224.94]$$

$$= \$1259.82$$

$$\begin{aligned}
 (g) \quad \sigma^2 &= .25 \left[ (1294.92 - 1259.82)^2 + (1271.38 - 1259.82)^2 \right. \\
 &\quad \left. + (1248.05 - 1259.82)^2 + (1224.94 - 1259.82)^2 \right] \\
 &= 2720.79 \\
 \sigma &= \sqrt{2720.79} = 52.16.
 \end{aligned}$$

$$\begin{aligned}
 4. (a) \quad E \left[ (1+i_t)^{-1} \right] &= \frac{1}{.09 - .07} \int_{.07}^{.09} \frac{1}{1+t} dt \\
 &= \frac{1}{.09 - .07} \ln(1+t) \Big|_{.07}^{.09} = .925952.
 \end{aligned}$$

Then set  $(1+\bar{i})^{-1} = .925952$  and solve  $\bar{i} = .07997$ .

$$(b) \text{ We have } a^{-1}(3) = (1.07997)^{-3} = .79390.$$

$$\begin{aligned}
 (c) \quad E \left[ (1+i_t)^{-2} \right] &= \frac{1}{.09 - .07} \int_{.07}^{.09} \frac{1}{(1+t)^2} dt \\
 &= \left[ \frac{-1}{.09 - .07} \cdot \frac{1}{1+t} \right]_{.07}^{.09} = .857412.
 \end{aligned}$$

Then set  $(1+\bar{k})^{-1} = .857412$  and solve  $\bar{k} = .16630$ .

(d) Applying formula (12.10), we have

$$\text{Var}[a^{-1}(3)] = (.857412)^3 - (.925952)^6 = .0000549$$

and the standard deviation is  $\sqrt{.0000549} = .00735$ .

5. (b) Applying formula (12.11), we have

$$E[a_{\overline{3}|}] = a_{\overline{3}|i} = a_{\overline{3}|.07997} = 2.5772.$$

(d) Applying formula (12.14), we have

$$\begin{aligned}
 \text{Var}[a_{\overline{3}|}] &= \frac{m_2^a + m_1^a}{m_2^a - m_1^a} a_{\overline{3}|\bar{k}} - \frac{2m_2^a}{m_2^a - m_1^a} a_{\overline{3}|\bar{i}} - (a_{\overline{3}|i})^2 \\
 &= \frac{.857412 + .925952}{.857412 - .925952} (2.2229) - \frac{(2)(.857412)}{.857412 - .925952} (2.5772) - (2.5772)^2 \\
 &= .005444
 \end{aligned}$$

and the standard deviation is  $\sqrt{.005444} = .0735$ .

6. The random variable  $i_t^{(2)}/2$  will be normal with  $\mu = 3\%$  and  $\sigma = .25\%$ .

(a) Applying formula (12.1), we have

$$E[100a(4)] = 100(1.03)^4 = 112.55.$$

Applying formula (12.3), we have

$$\begin{aligned} \text{Var}[100a_{\overline{4}|}] &= 10,000 \left[ (1 + 2\bar{i} + \bar{i}^2 + s^2)^4 - (1 + \bar{i})^8 \right] \\ &= 10,000 \left[ \{1 + (2)(.03) + (.03)^2 + .0025\}^4 - (1.03)^8 \right] \\ &= 119.828 \end{aligned}$$

and the standard deviation is  $\sqrt{119.828} = 10.95$ .

(b) Applying formula (12.5), we have

$$E[100\ddot{s}_{\overline{4}|}] = 100\ddot{s}_{\overline{4}|.03} = 430.91.$$

Applying formula (12.8), we have

$$m_1^s = 1.03$$

$$m_2^s = 1 + 2(.03) + (.03)^2 + .0025 = 1.0634$$

and

$$\begin{aligned} \text{Var}[100\ddot{s}_{\overline{4}|}] &= 10,000 \left[ \frac{1.0634 + 1.03}{1.0634 - 1.03} (4.67549) - \frac{(2)(1.0634)}{1.0634 - 1.03} (4.3091) - (4.3091)^2 \right] \\ &= 944.929 \end{aligned}$$

and the standard deviation is  $\sqrt{944.929} = 30.74$ .

7. (a)  $E[s_{\overline{n}|}] = E[\ddot{s}_{\overline{n+1}|} - 1] = \ddot{s}_{\overline{n+1}|\bar{i}} - 1.$

(b)  $\text{Var}[s_{\overline{n}|}] = \text{Var}[\ddot{s}_{\overline{n+1}|} - 1] = \text{Var}[\ddot{s}_{\overline{n+1}|}].$

(c)  $E[\ddot{a}_{\overline{n}|}] = E[1 + a_{\overline{n-1}|}] = 1 + a_{\overline{n-1}|\bar{i}}.$

(d)  $\text{Var}[\ddot{a}_{\overline{n}|}] = \text{Var}[1 + a_{\overline{n-1}|}] = \text{Var}[a_{\overline{n-1}|}].$

8. We know that  $1+i$  is lognormal with  $\mu = .06$  and  $\sigma^2 = .01$ . From the solution to Example 12.3(1), we have  $\bar{i} = .067159$  and then

$$\begin{aligned} s^2 &= e^{2\mu + \sigma^2} (e^{\sigma^2} - 1) = e^{2(.06) + .01} (e^{.01} - 1) \\ &= e^{.13} (e^{.01} - 1) = .011445. \end{aligned}$$

We then apply formula (12.4a) to obtain

$$\begin{aligned}\text{Var}[a(n)] &= (1 + 2\bar{i} + \bar{i}^2 + s^2)^n - (1 + \bar{i})^{2n} \\ &= [1 + 2(.067159) + (.067159)^2 + .011445]^5 - (1.067159)^{10} \\ &= .09821\end{aligned}$$

and the standard deviation =  $\sqrt{.09821} = .3134$  agreeing with the other approach.

9. (a) Formula (12.5) with  $\bar{i} = e^{\mu + \sigma^2/2} - 1$ .  
 (b) Formulas (12.6), (12.7) and (12.8) with  $\bar{j} = e^{2\mu + 2\sigma}$ .  
 (c) Formula (12.11) with  $\bar{i} = e^{\mu - \sigma^2/2} - 1$ .  
 (d) Formulas (12.12), (12.13) and (12.14) with  $\bar{k} = e^{-2\mu + 2\sigma^2}$ .

10. (a)  $E[1 + i_t] = e^{.06 + .0001/2} = 1.06189$   
 mean =  $E[a(10)] = (1.06189)^{10} = 1.823$ .  
 $\text{Var}[a(10)] = e^{(2)(10)(.06) + (10)(.0001)} (e^{(10)(.0001)} - 1)$   
 $= e^{1.201} (e^{.001} - 1) = .003325$   
 and s.d. =  $\sqrt{.003325} = .058$ .

(b) Mean =  $E[\ddot{s}_{\overline{10}|}] = \ddot{s}_{\overline{10}|.06189} = 14.121$   
 s.d. using formula (12.8) = .297.

(c)  $E[(1 + i_t)^{-1}] = e^{-.06 + .0001/2} = .941812$   
 mean =  $E[a^{-1}(10)] = (.941812)^{10} = .549$   
 $\text{Var}[a^{-1}(10)] = e^{-1.2 + .001} (e^{.001} - 1) = .000302$   
 and s.d. =  $\sqrt{.000302} = .017$ .

(d) We have  $(1 + \bar{i})^{-1} = .941812$  or  $\bar{i} = .06178$   
 and  $(1 + \bar{k})^{-1} = e^{-.12 + .0001} e^{.0001} = e^{-.1198}$   
 $= .887098$  or  $\bar{k} = .12727$ .

Mean =  $E[a_{\overline{10}|}] = a_{\overline{10}|.06178} = 7.298$ .  
 s.d. using formula (12.14) = .134.

$$11. E[1 + i_t] = e^{\mu + \sigma^2/2} = 1.067.$$

$$\text{Var}[1 + i_t] = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1) = .011445.$$

Solving two equations in two unknowns gives

$$\mu = .06 \quad \sigma^2 = .01$$

Therefore  $\delta_{[t]}$  follows a normal distribution with mean = .06 and var = .01.

$$12. E[1 + i_t] = 1.08 = e^{\mu + \sigma^2/2} = e^{\mu + .0001/2} \text{ so that } \mu = .07691.$$

$$\text{Var}[1 + i_t] = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1) = (1.08)^2 (e^{.0001} - 1) = .00011665.$$

$$E[a(3)] = (1.08)^3 = 1.25971.$$

$$\begin{aligned} \text{Var}[a(3)] &= [1 + 2(.08) + (.08)^2 + .00011665]^3 - (1.08)^6 \\ &= .0004762 \text{ and s.d.} = \sqrt{.0004762} = .02182. \end{aligned}$$

The 95% confidence interval is

$$1.25971 \pm 1.96(.02182) \text{ or } (1.21693, 1.30247).$$

$$13. E[s_{\overline{3}|}] = E[\ddot{s}_{\overline{4}|} - 1] = \ddot{s}_{\overline{4}|.08} - 1 = s_{\overline{3}|.08} = 3.246 = \text{mean.}$$

$$\text{Var}[s_{\overline{3}|}] = \text{Var}[\ddot{s}_{\overline{4}|} - 1] = \text{Var}[\ddot{s}_{\overline{4}|}].$$

$$\text{Var} = 65.62 \text{ using formula (12.8).}$$

$$14. E[\ln(1 + i_t)] = \frac{.07 + .09}{2} = .08 = \mu.$$

$$\text{Var}[\ln(1 + i_t)] = \frac{(.09 - .07)^2}{2} = \frac{.0001}{3} = \sigma^2.$$

$$E[\ln a^{-1}(30)] = -30\mu = -30(.08) = -2.4.$$

$$\text{Var}[\ln a^{-1}(30)] = 30\sigma^2 = 30\left(\frac{.0001}{3}\right) = .001.$$

The 95<sup>th</sup> percentile of  $\ln a^{-1}(30)$  is

$$-2.4 + 1.645\sqrt{.001} = -2.34798.$$

Thus,  $100,000e^{-2.34798} = \$9556.20$ .

15. Continuing Example 12.7:

$$\delta_{[6]} = .08 + .6(.091 - .08) + .2(.095 - .08) = .0896$$

$$\delta_{[7]} = .08 + .6(.0896 - .08) + .2(.091 - .08) = .0880$$

$$\delta_{[8]} = .08 + .6(.0880 - .08) + .2(.0896 - .08) = .0867.$$

16. (a) Formula (12.33)

$$\begin{aligned} \text{Var}[\delta_{[t]}] &= \frac{1 - k_2}{1 + k_2} \cdot \frac{\sigma^2}{(1 - k_2)^2 k_1^2} \\ &= \frac{\sigma^2}{1 - k_1} \quad \text{if } k_2 = 0 \end{aligned}$$

which is formula (12.30) with  $k_1 = k$ .

(b) Formula (12.34)

$$\text{Cov}[\delta_{[s]}, \delta_{[t]}] = \text{Var}[\delta_{[t]}] [\tau g_1^{t-s} + (1 - \tau) g_2^{t-s}].$$

We set  $k_2 = 0$ , so that

$$\tau = 1 \quad g_1 = k_1 \quad g_2 = 0$$

from formula (12.35). We also substitute the result from part (a).

$$\text{Thus, } \text{Cov}[\delta_{[s]}, \delta_{[t]}] = \frac{\sigma^2}{1 - k_1^2} k_1^{t-s}$$

which is formula (12.31) with  $k_1 = k$ .

17. Use formula (12.33) with  $k_1 = .6$  and  $k_2 = .2$ . Find the empirical estimate for  $\text{Var}[\delta_{[t]}]$  based upon the sample data for  $\delta_{[t]}$  given in Example 12.6. This will result in one equation in one unknown that can be solved for  $\sigma^2$ .

18. (a) Applying formula (12.33)

$$\begin{aligned} \text{Var}[\delta_{[t]}] &= \frac{1 - k_2}{1 + k_2} \cdot \frac{\sigma^2}{(1 - k_2)^2 - k_1^2} \\ &= \frac{1 - .2}{1 + .2} \cdot \frac{.0002}{(1 - .2)^2 - (.6)^2} = .0004762. \end{aligned}$$

(b) Applying formulas (12.34), (12.35) and (12.36) with  $k_1 = .6$  and  $k_2 = .2$  and with  $t - s = 2$  gives the answer .0001300.

19. (a) Applying formula (12.29) twice, we have

$$.096 = \delta + k(.100 - \delta)$$

$$.100 = \delta + k(.105 - \delta).$$

Solving these two equations in two unknowns, we have

$$k = .08 \quad \text{and} \quad \delta = .08.$$

Therefore

$$\delta_{[4]}^E = .08 + .8(.095 - .08) = .092.$$

(b) Applying formula (12.31), we have

$$\text{Cov}[\delta_{[s]}, \delta_{[t]}] = \frac{\sigma^2}{1 - k_2} k^{t-s} = (.0001)(.8)^{6-3} = .0000512.$$

20. There are 9 paths each with probability 1/9:

$$.06/.02/.02 - .04k \quad .06/.06/.02 \quad .06/.10/.02 + .04k$$

$$.06/.02/.06 - .04k \quad .06/.06/.06 \quad .06/.10/.06 + .04k$$

$$.06/.02/.10 - .04k \quad .06/.06/.10 \quad .06/.10/.10 + .04k$$

$$\begin{aligned} (a) \quad E[a(2)] &= \frac{1}{9} [(1.02)(1.02 - .04k) + (1.02)(1.06 - .04k) + 7 \text{ more terms}] \\ &= \frac{1}{3} [(1.02)(1.06 - .04k) + (1.06)^2 + (1.10)(1.06 - .04k)] \\ &= (1.06)^2 + \frac{1}{3}(.0032)k. \end{aligned}$$

$$\begin{aligned} (b) \quad E[a(2)^2] &= \frac{1}{9} [(1.02)^2(1.02 - .04k)^2 + (1.02)^2(1.06 - .04k)^2 + 7 \text{ more terms}] \\ &= \frac{1}{9} [(1.02)^2 + (1.06)^2 + (1.10)^2] + \frac{(1.10)^2 - (1.02)^2}{3} (.08)(1.06)k \\ &\quad + \frac{(1.10)^2 + (1.02)^2}{3} (.0016)k^2 \\ &= \frac{1}{9}(11.383876 + .04314624k + .01080192k^2) \end{aligned}$$

and

$$\begin{aligned} \text{Var}[a(2)] &= E[a(2)^2] - E[a(2)]^2 \\ &= \frac{1}{9}(.02158336 + .02157312k + .01079168k^2). \end{aligned}$$

21. At time  $t = 2$ :

$$i = .144 \quad V = \frac{(.5)(1000) + (.5)(1000)}{1.144} = 874.126$$

$$i = .10 \quad V = \frac{(.5)(1000) + (.5)(1000)}{1.1} = 909.091$$

$$i = .06944 \quad V = \frac{(.5)(1000) + (.5)(1000)}{1.06944} = 935.069$$

At time  $t = 1$ :

$$i = .12 \quad V = \frac{(.5)(874.126) + (.5)(909.091)}{1.12} = 796.079$$

$$i = .08333 \quad V = \frac{(.5)(909.091) + (.5)(935.069)}{1.08333} = 851.153$$

At time  $t = 0$ :

$$i = .10 \quad V = \frac{(.5)(796.079) + (.5)(851.153)}{1.1} = 748.74$$

obtaining the same answer as obtained with the other method.

22. (a)

Path	Probability	PV	PV <sup>2</sup>
10/11/12	.25	.73125	.53473
10/11/10	.25	.74455	.55435
10/9/10	.25	.75821	.57488
10/9/8	.25	.77225	.59637

Value of the bond is

$$1000(.25)[.73125 + .74455 + .75821 + .77225] = 751.57.$$

$$(b) \text{ Var} = (1000)^2 (.25)[.53473 + .55435 + .57488 + .59637] \\ = 232.5664$$

$$\text{and the s.d.} = \sqrt{232.5664} = 15.25.$$

(c) The mean interest rate is  $i = .10$  so the value is  $1000(1.1)^{-3} = 751.31$ .

23. (a) At time  $t = \frac{1}{2}$ :

$$i^{(2)} = .10 \quad V = \frac{(.3)(1045) + (.7)(1045)}{1.05} = 995.238$$

$$i^{(2)} = .08 \quad V = \frac{(.3)(1045) + (.7)(1045)}{1.04} = 1004.808.$$

At time  $t = 0$ :

$$i^{(2)} = .09 \quad V = \frac{(.3)(995.238 + 45) + (.7)(1004.808 + 45)}{1.045} \\ = \$1001.85.$$

(b) The equation of value is

$$1001.854 = 45v + 1045v^2$$

and solving the quadratic  $v = .95789$ . Then we have  $v = .95789 = e^{-.5\delta}$  and  $\delta = .0861$ , or 8.61%.

24. If the interest rate moves down, then call the bond, which gives

$$V = \frac{(.3)(995.238 + 45) + (.7)(1000 + 45)}{1.045} = \$998.63.$$

25. At time  $t = \frac{1}{2}$ :

$$j = .0288 \quad V = \frac{(.4)(1038/1.03458) + (.6)(1038/1.024)}{1.0288} = 981.273$$

$$j = .02 \quad V = \frac{(.4)(1038/1.024) + (.6)(1038/1.01667)}{1.02} = 998.095$$

$$j = .01389 \quad V = \frac{(.4)(1038/1.01667) + (.6)(1038/1.011575)}{1.01389} = 1010.036$$

At time  $t = \frac{1}{4}$ :

$$j = .024 \quad V = \frac{(.4)(981.273 + 38) + (.6)(998.095 + 38)}{1.024} = 1005.24$$

$$j = .01667 \quad V = \frac{(.4)(998.095 + 38) + (.6)(1010.036 + 38)}{1.01667} = 1026.1536.$$

At time  $t = 0$ :

$$j = .02 \quad V = \frac{(.4)(1005.24) + (.6)(1026.1536)}{1.02} = \$997.83.$$

26.

<u>Path</u>	<u>Probability</u>	<u>PV</u>	<u>CV</u>	<u>CV from time 1</u>
10/12/14.4	.16	.7095	1.4094	1.28128
10/12/10	.24	.7379	1.3552	1.23200
10/8.333/10	.24	.7629	1.3108	1.19170
10/8.333/6.944	.36	.7847	1.2744	1.15860

$$(a) E[a(3)] = (.16)(1.4094) + (.24)(1.3552) + (.24)(1.3108) + (.36)(1.2744) = 1.326.$$

$$(b) E[a^{-1}(3)] = .749.$$

$$(c) E[a(3)] = E[a^{-1}(3)] + E[a^{-1}(2)] + E[a^{-1}(1)] \\ = .749 + [(.4)(1.12)^{-1}(1.1)^{-1} + (.6)(1.08333)^{-1}(1.1)^{-1}] + (1.1)^{-1} \\ = 2.486.$$

$$(d) E[\ddot{s}_{\overline{3}|}] = 1.326 + 1.2038 + 1.096 = 3.626.$$

27. Rendleman – Bartter:

mean

$$E[\delta_t] = E[\delta_t - \delta_0 + \delta_0] = E[\Delta\delta_0] + E[\delta_0] \\ = a\delta_0 t + \delta_0 = \delta_0(1 + at)$$

variance

$$\text{Var}[\delta_t] = a^2 \delta_0^2 t$$

Vasicek:

mean

$$E[\delta_t] = E[\Delta\delta_0] + E[\delta_0] \\ = c(b - \delta_0) + \delta_0 = cb + (1 - c)\delta_0$$

variance

$$\text{Var}[\delta_t] = \sigma^2 t$$

Cox – Ingersoll – Ross:

mean

$$E[\delta_t] = cb + (1 - c)\delta_0$$

variance

$$\text{Var}[\delta_t] = \sigma^2 \delta_0 t, \text{ since the process error is proportional to } \sqrt{\delta} \text{ which} \\ \text{squares in computing variances.}$$

28. (a) We have

$$d\delta = c(b - \delta)dt + \sigma dz \\ = 0 + \delta dz \text{ if } c = 0 \\ = a dt + \sigma dz \text{ where } a = 0$$

which is the process for a random walk.

(b) We have

$$\begin{aligned} d\delta &= c(b - \delta)dt + \sigma dz \\ &= (b - \delta)dt + \sigma dz \text{ if } c = 1 \end{aligned}$$

which is the process for a normal distribution with  $\mu = b$ .

29. For the random walk model

$$\Delta\delta = a\Delta t + \sigma\Delta z$$

and for the Rendleman-Bartter model

$$\Delta\delta = a\delta\Delta t + \sigma\delta\Delta z$$

Random walk		Rendleman - Bartter	
$\delta_0 = .06$			$\delta_0 = .06$
$\delta_{.5} = .0675$	$\Delta\delta_{.5} = .0075$	$\Delta\delta_{.5} = (.0075)(.06)$	$\delta_{.5} = .06045$
$\delta_1 = .065$	$\Delta\delta_1 = -.0025$	$\Delta\delta_1 = (-.0025)(.06045)$	$\delta_1 = .06030$
$\delta_{1.5} = .063$	$\Delta\delta_{1.5} = -.0020$	$\Delta\delta_{1.5} = (-.002)(.06030)$	$\delta_{1.5} = .06018$
$\delta_2 = .0685$	$\Delta\delta_2 = .0055$	$\Delta\delta_2 = (.0055)(.06018)$	$\delta_2 = .06051$

30. (a) We have  $\delta_0 = .08$

$$E[\delta_{.5}] = \delta_0 + at = .08 + (.006)(.5) = .083$$

and

$$P = 39e^{-(.08)(.5)} + 1039e^{-(.08)(.5) - (.083)(.5)} = \$995.15.$$

(b) We have

$$995.151 = 39v + 1039v^2$$

and solving the quadratic

$$i^{(2)}/2 = .0606 \text{ so that } i^{(2)} = .1212.$$

(c) We have

$$\delta_{.5} = .08 + .006(.5) + (.01)(.5)\sqrt{.5} = .08654$$

and

$$P = 39e^{-(.08)(.5)} + 1039e^{-(.08)(.5) - (.08654)(.5)} = \$993.46.$$

31. Rework Examples 12.11-12.14 using  $\pm 2$  standard deviations. The following results are obtained:

Random walk

<u>Max</u>	<u>Min</u>
$\delta_{.25} = .0790$	$\delta_{.25} = .0590$
$\delta_{.50} = .0880$	$\delta_{.50} = .0480$
$\delta_{.75} = .0970$	$\delta_{.75} = .0370$
$\delta_1 = .106$	$\delta_1 = .026$

Rendleman – Bartter

<u>Max</u>	<u>Min</u>
$\delta_{.25} = .0790$	$\delta_{.25} = .0590$
$\delta_{.50} = .0892$	$\delta_{.50} = .0497$
$\delta_{.75} = .1007$	$\delta_{.75} = .0419$
$\delta_1 = .114$	$\delta_1 = .035$

Vasicek

<u>Max</u>	<u>Min</u>
$\delta_{.25} = .0790$	$\delta_{.25} = .0590$
$\delta_{.50} = .0876$	$\delta_{.50} = .0486$
$\delta_{.75} = .0957$	$\delta_{.75} = .0386$
$\delta_1 = .103$	$\delta_1 = .029$

Cox-Ingersoll-Ross

<u>Max</u>	<u>Min</u>
$\delta_{.25} = .0790$	$\delta_{.25} = .0590$
$\delta_{.50} = .0923$	$\delta_{.50} = .0494$
$\delta_{.75} = .1017$	$\delta_{.75} = .0410$
$\delta_1 = .111$	$\delta_1 = .034$

32. (a)  $(.08)(1.1)^{10} = .2075$ , or 20.75%.

(b)  $(.08)(.9)^{10} = .0279$ , or 2.79%.

(c)  $(.08)(1.1)^5(.9)^5 = .0761$ , or 7.61%.

(d) A 10% increase followed by a 10% decrease results in a result that is  $(1.1)(.9) = 99\%$  of the starting value.

(e)  $\binom{10}{5}(.5)^{10} = .2461$  using the binomial distribution.

$$(f) \text{ 10 increases } (.08)(1.1)^{10} = .2075$$

$$\text{9 increases } (.08)(1.1)^9 (.9) = .1698$$

$$\text{Probability} = \left[ \binom{10}{10} + \binom{10}{9} \right] (.5)^{10} = 11(.5)^{10} = .0107.$$

33. One year spot rates  $s_1$ :

$$i_0 = .070000$$

$$i_1 = .070000e^{1.65(.1)} = .082558$$

$$i_2 = .082558e^{-.26(.1)} = .080439$$

$$i_3 = .080439e^{.73(.1)} = .086530$$

$$i_4 = .086530e^{1.17(.1)} = .097270$$

$$i_5 = .097270e^{.98(.1)} = .1073, \text{ or } 10.73\%.$$

Five year spot rates  $s_5$ :

$$i_0 = .080000$$

$$i_1 = .080000e^{1.65(.05)} = .086880$$

$$i_2 = .086880e^{-.26(.05)} = .085758$$

$$i_3 = .085758e^{.73(.05)} = .088946$$

$$i_4 = .088946e^{1.17(.05)} = .094304$$

$$i_5 = .094304e^{.98(.05)} = .0990, \text{ or } 9.90\%.$$

The yield curve became inverted, since  $10.73\% > 9.90\%$ .