Chapter 13

1. <u>Stock</u> <u>Option</u> (a) $\frac{84-80}{80} = +5\%$ $\frac{0-2}{2} = -100\%$ (b) $\frac{80-80}{80} = 0\%$ $\frac{0-2}{2} = -100\%$ (c) $\frac{78-80}{80} = -2.5\%$ $\frac{2-2}{2} = 0\%$ (d) $\frac{76-80}{80} = -5\%$ $\frac{4-2}{2} = +100\%$ (e) \$78, from part (c) above (f) TVP = P - IVP = 2 - 0 = \$2

2. (a)
$$IVC = S - E = 100 - 98 = $2$$

(b) $TVC = C - IVC = 6 - 2 = 4
(c) $IVP = 0 since $S \ge E$

- (*d*) TVP = P IVP = 2 0 = \$2
- 3. Profit position = Cost of \$40 call + Cost of \$45 call
 + Value of \$40 call Value of \$45 call
 - (a) -3 + 1 + 0 0 = -\$2
 - (b) -3 + 1 + 0 0 = -\$2
 - $(c) \ -3 + 1 + 2.50 0 = \$.50$
 - (d) -3 + 1 + 5 0 = \$3
 - (e) -3 + 1 + 10 5 = \$3
- 4. See answers to the Exercises on p. 623.
- 5. (a) Break-even stock prices = E + C + P and E C P.
 (b) Largest amount of loss = C + P

- 6. (*a*) The shorter-term option should sell for a lower price than the longer-term option. Thus, sell one \$5 option and buy one \$4 option. Adjust position in 6 months.
 - (*b*) If $S \le 50$ in 6 months, profit is:

\$1 if S = 48 in one year. \$1 if S = 50 in one year. \$3 if S = 52 in one year. If S > 50 is 6 months, profit is: \$3 if S = 48 in one year. \$1 if S = 50 in one year.

- \$1 if S = 52 in one year.
- 7. See answers to the Exercises on p. 623.
- 8. *P* increases as *S* decreases, the opposite of calls.

P increases as *E* increases, the opposite of calls.

P increases at t increases, since longer-term options are more valuable than shorter-term options.

P increases as σ increases, since all option values increase as volatility increases.

P increases as *i* decreases, the opposite of calls. The replicating transaction for calls involves lending money, while the replicating transaction for puts involves borrowing money.

- 9. Figure 13.5 provides the explanation.
- 10. (*a*) 0 from Figure 13.5.
 - (b) $S Ee^{-\delta n}$ from Figure 13.5.
 - (c) S, since the call is equivalent to the stock.
 - (d) 0, since the option is far "out of the money."
 - (e) S E, if $S \ge E$
 - 0, if S < E, the *IVC*.
 - (f) S from Figure 13.5.

11. Using put-call parity, we have

$$S + P = v^t E + C$$
 or $C = S + P - v^t E$.

In the limit as $S \rightarrow \infty$, $P \rightarrow 0$, so that

$$C = S + 0 - v^t E = S - v^t E.$$

12. Using put-call parity, we have

$$S + P = v^t E + C$$

 $49 + P = \left(1 + \frac{.09}{12}\right)^{-3} (50) + 1 \text{ and } P = \$.89.$

- 13. Buy the call. Lend \$48.89. Sell the stock short. Sell the put. Guaranteed profit of -1+48.89+49+2=\$1.11 at inception.
- 14. See Answers to the Exercises on p. 624.
- 15. (*a*) At S = 45, profit is

$$(2)(4) - 3 - 6 + 0 + 0 = -\$1$$

At S = 50, profit is

$$(2)(4) - 3 - 6 + 5 + 0 + 0 = +$$
\$4

At S = 55, profit is

$$(2)(4) - 3 - 6 + 10 - (5)(2) + 0 = -\$1$$

(b) See Answers to the Exercises on p. 624.

16. (a) The percentage change in the stock value is +10% in an up move, and -10% in a down move. The risk-free rate of interest is i = .06. Let p be the probability of an up move. We have

$$p(.10) + (1-p)(-.10) = .06$$

or .20p = .16 and p = .8.

(*b*) Using formula (13.12)

$$C = \frac{p \cdot V_U + (1 - p)V_D}{1 + i} = \frac{(.8)(10) + (.2)(0)}{1.06} = \$7.55.$$

17. (*a*) Using formula (13.8)

$$\Delta = \frac{V_U - V_D}{S_U - S_D} = \frac{10 - 0}{110 - 90} = \frac{1}{2}.$$

(b) Bank loan = Value of stock – Value of 2 calls = 100 - 2(7.55) = 84.906 for 2 calls. For one call the loan would be $\frac{84.906}{2} = 42.45 .

Year 1	Year 2	Probability	Stock Value
Up	Up	(.8)(.8) = .64	$100(1.1)^2 = 121$
Up	Down	(.8)(.2) = .16	100(1.1)(.9) = 99
Down	Up	(.2)(.8) = .16	100(.9)(1.1) = 99
Down	Down	(.2)(.2) = .04	$100(.9)^2 = 81$
	Up Up Down	UpUpUpDownDownUp	UpUpUp $(.8)(.8) = .64$ UpDown $(.8)(.2) = .16$ DownUp $(.2)(.8) = .16$

We then have

$$C = \frac{(.64)(121 - 100)}{(1.06)^2} = \$11.96$$

19. (a) Using the formula (13.7)

 $k = e^{\sigma\sqrt{h}} - 1 = e^{.3\sqrt{.125}} - 1 = .11190.$

(b) Up move: 90(1+k) = 100.071

Down move:
$$90(1+k)^{-1} = 80.943$$

Now

$$100.071p + 80.943(1-p) = 90e^{.125(.1)} = 91.132$$

and solving, we obtain p = .5327.

- (c) Applying formula (13.13) with the values of k and p obtained in parts (a) and (b) above together with n = 8, we obtain C = \$10.78. This, compare with the answer of \$10.93 in Example 13.7.
- 20. Using formula (13.12) together with the stock values obtained in Exercise 18, p = .8 and i = .06 we obtain

$$P = \frac{(.16)(100 - 99) + (.16)(100 - 99) + (.04)(100 - 81)}{(1.06)^2} = \$.96.$$

21. The value of a put = 0 if $S(1+k)^{n-2t} \ge E = E - S(1+k)^{n-t}$ if $S(1+k)^{n-2t} < E$ or $\max\left[0, E - S(1+k)^{n-2t}\right]$. Thus, the value of an European put becomes

$$P = \frac{1}{(1+i)^n} \sum_{t=0}^n \binom{n}{t} p^{n-t} (1-p)^t \max\left[0, E-S(1+k)^{n-2t}\right].$$

22. We are asked to verify that formulas (13.14) and (13.16) together satisfy formula (13.5). We have

$$S + P = S + Ee^{-\delta n} \left[1 - N(d_2) \right] - S \left[1 - N(d_1) \right]$$

$$S + P = v^n E - Ee^{-\delta n} N(d_2) + SN(d_1)$$

$$= v^n E + C \text{ validating the result.}$$

23. Applying formula (13.16) directly, we have

 $P = 100e^{-.1}(1 - .4333) - 90(1 - .5525) = \$11.00.$

The result could also be obtained using put-call parity with formula (13.5).

- 24. Applying formulas (13.14) and (13.15) repeatedly with the appropriate inputs gives the following:
 - (*a*) 5.76
 - (*b*) 16.73
 - (*c*) 8.66
 - (*d*) 12.58
 - (*e*) 5.16
 - (*f*) 15.82
 - (g) 5.51
 - (*h*) 14.88
- 25. We modify the final equation in the solution for Example 13.8 to obtain

$$C = (90 - 360e^{-.1})(.5525) - (100e^{-.1})(.4333) = \$8.72.$$

26. The price of the noncallable bond is $B^{nc} = 100$ since the bond sells at par. The price of the callable bond can be obtained from formula (13.17) as

$$B^c = B^{nc} - C$$

Thus, the problem becomes one of estimating the value of the embedded option using the Black Scholes formula. This formula places a value of 2.01 on the embedded call. The answer is then 100.00 - 2.01 = \$97.99.

27. We modify the put-call parity formula to obtain

$$S - PV$$
 dividends $+ P = v^{t}E + C$
 $49 - .50a_{\overline{3}|.0075} + P = (1.0075)^{-3}(50) + 1$

and solving for *P* we obtain

$$P = 2.37$$
.

28. The average stock price is

$$\frac{10.10 + 10.51 + 11.93 + 12.74}{4} = 11.32$$

and the option payoff is 11.32 - 9 = \$2.32.