



— Page references correspond to locations of Extra Examples icons in the textbook.

**p.346, icon at Example 1**

#1. Suppose  $f(n+1) = \left\lfloor \frac{n^2 f(n) + 2}{n+1} \right\rfloor$  and  $f(0) = 2$ . Find  $f(1)$ ,  $f(2)$ ,  $f(3)$ ,  $f(4)$ .

**Solution:**

To find  $f(1)$ , we use  $n = 0$ :  $f(1) = f(0+1) = \left\lfloor \frac{0^2 f(0) + 2}{0+1} \right\rfloor = \left\lfloor \frac{0^2 \cdot 2 + 2}{0+1} \right\rfloor = \left\lfloor \frac{2}{1} \right\rfloor = 2$ .

To find  $f(2)$ , we use  $n = 1$ :  $f(2) = f(1+1) = \left\lfloor \frac{1^2 f(1) + 2}{1+1} \right\rfloor = \left\lfloor \frac{1^2 \cdot 2 + 2}{1+1} \right\rfloor = \left\lfloor \frac{4}{2} \right\rfloor = 2$ .

To find  $f(3)$ , we use  $n = 2$ :  $f(3) = f(2+1) = \left\lfloor \frac{2^2 f(2) + 2}{2+1} \right\rfloor = \left\lfloor \frac{2^2 \cdot 2 + 2}{2+1} \right\rfloor = \left\lfloor \frac{10}{3} \right\rfloor = 3$ .

To find  $f(4)$ , we use  $n = 3$ :  $f(4) = f(3+1) = \left\lfloor \frac{3^2 f(3) + 2}{3+1} \right\rfloor = \left\lfloor \frac{3^2 \cdot 3 + 2}{3+1} \right\rfloor = \left\lfloor \frac{29}{4} \right\rfloor = 7$ .

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#2. Suppose

$$f(n) = \begin{cases} f(n-2) & \text{if } n \text{ is even} \\ f(n-2) + 3 & \text{if } n \text{ is odd.} \end{cases}$$

Also suppose that  $f(0) = 1$  and  $f(1) = 4$ . Find  $f(7)$ .

**Solution:**

Using the recurrence relation, we obtain  $f(3) = 7$ ,  $f(5) = 10$ , and  $f(7) = 13$ .

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#3. Prove that the following proposed recursive definition of a function on the set of nonnegative integers fails to produce a well-defined function.

$$f(n) = \begin{cases} f(n-2) & \text{if } n \text{ is even} \\ 3f(n-2) & \text{if } n \text{ is odd} \end{cases}$$

with  $f(0) = 4$ .

**Solution:**

The value  $f(1)$  cannot be computed because  $f(1) = 3f(-1)$ , but  $f(-1)$  is not defined.

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**#4.** Prove that the following proposed recursive definition of a function on the set of nonnegative integers fails to produce a well-defined function.

$$f(n) = f(f(n-1)) + 5, f(0) = 1.$$

**Solution:**

When we try to compute  $f(1)$ , we obtain

$$f(1) = f(f(0)) + 5 = f(1) + 5,$$

which cannot happen.

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**p.347, icon at Example 4**

**#1.** For the sequence of Fibonacci numbers  $f_0, f_1, f_2, \dots$  ( $0, 1, 1, 2, 3, 5, 8, 13, \dots$ ), prove that

$$f_0 + f_2 + f_4 + f_6 + \dots + f_{2n} = f_{2n+1} - 1$$

for all  $n \geq 0$ .

**Solution:**

Let  $P(n)$  be:  $f_0 + f_2 + f_4 + f_6 + \dots + f_{2n} = f_{2n+1} - 1$ .

*BASIS STEP:*  $P(0)$  states that  $f_0 = f_1 - 1$ , which is true because  $f_0 = 0$  and  $f_1 - 1 = 1 - 1 = 0$ .

*INDUCTIVE STEP:*  $P(k) \rightarrow P(k+1)$ : Suppose that  $P(k)$  is true; i.e.,  $f_0 + f_2 + f_4 + f_6 + \dots + f_{2k} = f_{2k+1} - 1$ . We must show that  $f_0 + f_2 + f_4 + f_6 + \dots + f_{2(k+1)} = f_{2(k+1)+1} - 1$ , i.e.,  $f_0 + f_2 + f_4 + f_6 + \dots + f_{2k+2} = f_{2k+3} - 1$ :

$$\begin{aligned} f_0 + f_2 + f_4 + f_6 + \dots + f_{2k} + f_{2k+2} &= (f_0 + f_2 + f_4 + f_6 + \dots + f_{2k}) + f_{2k+2} \\ &= (f_{2k+1} - 1) + f_{2k+2} \\ &= f_{2k+1} + f_{2k+2} - 1 \\ &= f_{2k+3} - 1. \end{aligned}$$

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**#2.** For the sequence of Fibonacci numbers  $f_0, f_1, f_2, \dots$  ( $0, 1, 1, 2, 3, 5, 8, 13, \dots$ ), prove for all nonnegative integers  $n$ :

$$f_0^2 + f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1}.$$

**Solution:**

Let  $P(n)$  be the proposition

$$f_0^2 + f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1}.$$

*BASIS STEP:*  $P(0)$  is the proposition  $f_0^2 = f_0 f_1$ . It is true because  $f_0^2 = 0^2 = 0$  and  $f_0 f_1 = 0 \cdot 1 = 0$ .

*INDUCTIVE STEP:* Suppose  $P(k)$  is true. Then

$$f_0^2 + f_1^2 + f_2^2 + \dots + f_k^2 = f_k f_{k+1}.$$

We need to show that  $P(k+1)$  is true:  $f_0^2 + f_1^2 + f_2^2 + \cdots + f_{k+1}^2 = f_{k+1}f_{k+2}$ . We take  $P(k)$  and add  $f_{k+1}^2$  to both sides of the equation, obtaining

$$\begin{aligned}(f_0^2 + f_1^2 + f_2^2 + \cdots + f_k^2) + f_{k+1}^2 &= f_k f_{k+1} + f_{k+1}^2 \\ &= f_{k+1}(f_k + f_{k+1}) \\ &= f_{k+1}f_{k+2} \quad (\text{using the Fibonacci sequence recurrence})\end{aligned}$$

Therefore  $P(k+1)$  follows from  $P(k)$ .

Therefore, by the Principle of Mathematical Induction,  $P(n)$  is true for all nonnegative integers  $n$ .

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**p.349, icon at Example 5**

**#1.** Give a recursive definition for the set  $S = \{4, 7, 10, 13, 16, 19, \dots\}$ .

**Solution:**

The set can be written starting from 4 and adding 3 over and over.

*BASIS STEP:*  $4 \in S$ .

*RECURSIVE STEP:*  $n \in S \rightarrow n + 3 \in S$ .

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