

Rosen, Discrete Mathematics and Its Applications, 7th edition
Extra Examples
Section 5.5—Program Correctness



— Page references correspond to locations of Extra Examples icons in the textbook.

p.373, icon at Example 1

#1. Show that the program segment S

```
a := 5
c := a + 2b
```

is correct with respect to the initial assertion $p: b = 3$ and the final assertion $q: c = 11$.

Solution:

Suppose p is true. Therefore $b = 3$ at the beginning of the program. As the program runs, 5 is assigned to a and then $5 + 2 \cdot 3$, or 11, is assigned to c . Therefore, $p\{S\}q$ is true.

p.375, icon at Example 4

#1. Use a loop invariant to prove that this program segment for computing nx (x a real number), where n is a positive integer, is correct:

```
multiple := 0
i := 1
while i ≤ n
begin
  multiple := multiple + x
  i := i + 1
end
```

Solution:

We will show that

$$p: \text{multiple} = (i - 1)x \text{ and } i \leq n + 1$$

is a loop invariant.

Initially p is true because $i = 1$ and $\text{multiple} = 0 = (1 - 1)x$. Now suppose that p is true and $i \leq n$ after the loop is executed. We must show that p is true after another execution of the loop. Because $i \leq n$, after one more execution of the loop, i will be incremented by 1 and we have $i \leq n + 1$. Also, multiple becomes $\text{multiple} + x$, or $(i - 1)x + x = ix$. Hence p remains true. Therefore, p is a loop invariant.

Finally, the loop terminates with $i = n + 1$ after n traversals of the loop because $i = 1$ prior to the loop and each traversal of the loop adds 1 to n . Thus, at termination $\text{multiple} = nx$.
