

Rosen, Discrete Mathematics and Its Applications, 7th edition
Extra Examples
Section 7.3—Bayes' Theorem



— Page references correspond to locations of Extra Examples icons in the textbook.

p.469, icon at Example 1

#1. It is estimated that a certain disease occurs in 0.1% of the U.S. population. A test that attempts to detect the disease has been developed with the following results: 99.7% of people with the disease test positive for the disease and 0.2% of people without the disease test positive for the disease. (A result that says that a person has the disease when in reality the person does not have the disease is called a “false positive”.)

Find the probability that a person actually has the disease, given that the person tests positive for the disease.

Solution:

Let

A be the event that the person tests positive for the disease

and

B be the event that the person actually has the disease.

We want $p(B | A)$. According to Bayes' Theorem, we have

$$p(B | A) = \frac{p(B) \cdot p(A | B)}{p(B) \cdot p(A | B) + p(\overline{B}) \cdot p(A | \overline{B})} = \frac{\frac{1}{1000} \cdot \frac{997}{1000}}{\frac{1}{1000} \cdot \frac{997}{1000} + \frac{999}{1000} \cdot \frac{2}{1000}} = \frac{997}{997 + 1998} \approx 0.333 = 33.3\%.$$

That is, only about one third of the people who test positive for the disease actually have the disease and about two thirds of the people who test positive for the disease are really disease-free. (How would you weigh the advantage of having such a test to detect the disease against the anxiety that it would cause two thirds of the people who take the test?)
