



**Solution:**

(a)  $R$  is reflexive:  $(a, b)$  and  $(a, b)$  lie on the same line through the origin, namely on the line  $y = bx/a$  (if  $a \neq 0$ ), or else on the line  $x = 0$  (if  $a = 0$ ).

(b)  $R$  is symmetric: if  $(a, b)$  and  $(c, d)$  lie on the same line through the origin, then  $(c, d)$  and  $(a, b)$  lie on the same line through the origin.

(c)  $R$  is not antisymmetric:  $(1, 1)$  and  $(2, 2)$  lie on the same line through the origin. Therefore,  $(1, 1)R(2, 2)$  and  $(2, 2)R(1, 1)$ .

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**p.577, icon at Example 10**

#4. Let  $A = \{(x, y) \mid x, y \text{ integers}\}$ . Define a relation  $R$  on  $A$  by the rule

$$(a, b)R(c, d) \leftrightarrow a \leq c \text{ and } b \leq d.$$

Determine whether  $R$  is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

**Solution:**

(a)  $R$  is reflexive:  $(a, b)R(a, b)$  for all elements  $(a, b)$  because  $a \leq a$  and  $b \leq b$  is always true.

(b)  $R$  is not symmetric: For example,  $(1, 2)R(3, 7)$  (because  $1 \leq 3$  and  $2 \leq 7$ ), but  $(3, 7) \not R(1, 2)$ .

(c)  $R$  is antisymmetric: Suppose  $(a, b)R(c, d)$  and  $(c, d)R(a, b)$ . Therefore  $a \leq c$ ,  $c \leq a$ ,  $b \leq d$ ,  $d \leq b$ . Therefore  $a = c$  and  $b = d$ , or  $(a, b) = (c, d)$ .

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**p.577, icon at Example 10**

#5. Let  $A = \{(x, y) \mid x, y \text{ integers}\}$ . Define a relation  $R$  on  $A$  by the rule

$$(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d.$$

Determine whether  $R$  is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

**Solution:**

(a)  $R$  is reflexive:  $(a, b)R(a, b)$  for all elements  $(a, b)$  because  $a = a$  and  $b = b$  are always true.

(b)  $R$  is symmetric: Suppose  $(a, b)R(c, d)$ . Therefore  $a = c$  or  $b = d$ . Therefore  $c = a$  or  $d = b$ . Therefore  $(c, d)R(a, b)$ .

(c)  $R$  is not antisymmetric: For example,  $(1, 2)R(1, 3)$  and  $(1, 3)R(1, 2)$  because  $1 = 1$ , but  $(1, 2) \neq (1, 3)$ .

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**p.578, icon at Example 13**

#1. Let  $R$  be the following relation defined on the set  $\{a, b, c, d\}$ :

$$R = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)\}.$$

Determine whether  $R$  is transitive.

**Solution:**

The relation  $R$  is not transitive because, for example,  $(a, c) \in R$  and  $(c, b) \in R$ , but  $(a, b) \notin R$ .

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**p.578, icon at Example 13**

**#2.** Let  $R$  be the following relation on the set of real numbers:

$$aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor, \text{ where } \lfloor x \rfloor \text{ is the floor of } x.$$

Determine whether  $R$  is transitive.

**Solution:**

$R$  is transitive: suppose  $\lfloor a \rfloor = \lfloor b \rfloor$  and  $\lfloor b \rfloor = \lfloor c \rfloor$ ; from transitivity of equality of real numbers, it follows that  $\lfloor a \rfloor = \lfloor c \rfloor$ .

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**p.578, icon at Example 13**

**#3.** Let  $A$  be the set of all points in the plane with the origin removed. That is,

$$A = \{(x, y) \mid x, y \in \mathbf{R}\} - \{(0, 0)\}.$$

Define a relation on  $A$  by the rule:

$$(a, b)R(c, d) \leftrightarrow (a, b) \text{ and } (c, d) \text{ lie on the same line through the origin.}$$

Determine if  $R$  is transitive.

**Solution:**

$R$  is transitive: suppose  $(a, b)$  and  $(c, d)$  lie on the same line  $L$  through the origin and  $(c, d)$  and  $(e, f)$  lie on the same line  $M$  through the origin. Then  $L$  and  $M$  both contain the two distinct points  $(0, 0)$  and  $(c, d)$ . Therefore  $L$  and  $M$  are the same line, and this line contains  $(a, b)$  and  $(e, f)$ . Therefore  $(a, b)$  and  $(e, f)$  lie on the same line through the origin.

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**p.578, icon at Example 13**

**#4.** Let  $A = \{(x, y) \mid x, y \text{ integers}\}$ . Define a relation  $R$  on  $A$  by the rule

$$(a, b)R(c, d) \leftrightarrow a \leq c \text{ and } b \leq d.$$

Determine whether  $R$  is transitive.

**Solution:**

$R$  is transitive: Suppose  $(a, b)R(c, d)$  and  $(c, d)R(e, f)$ . Therefore  $a \leq c$  and  $c \leq e$ , and  $b \leq d$  and  $d \leq f$ . Therefore,  $a \leq e$  and  $b \leq f$ , or  $(a, b)R(e, f)$ .

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**p.578, icon at Example 13**

**#5.** Let  $A = \{(x, y) \mid x, y \text{ integers}\}$ . Define a relation  $R$  on  $A$  by the rule

$$(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d.$$

Determine whether  $R$  is transitive.

**Solution:**

$R$  is not transitive: For example,  $(1, 2)R(1, 3)$  because  $1 = 1$ , and  $(1, 3)R(4, 3)$  because  $3 = 3$ . But  $(1, 2) \neq (4, 3)$  because  $1 \neq 4$  and  $2 \neq 3$ .

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