

Rosen, Discrete Mathematics and Its Applications, 7th edition
Extra Examples
Section 12.2—Representing Boolean Functions



— Page references correspond to locations of Extra Examples icons in the textbook.

p.820, icon at Example 3

#1. Find a Boolean function $f(x, y, z)$ that has the following element table:

x	y	z	$f(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

Solution:

Consider the second row of the table of values for f , where $x = 1, y = 1, z = 0$. In this case the product $xy\bar{z} = 1 \cdot 1 \cdot \bar{0} = 1 \cdot 1 \cdot 1 = 1$. Next, consider the third row of the table, where $x = 1, y = 0, z = 1$. In this case the product $x\bar{y}z = 1 \cdot \bar{0} \cdot 1 = 1 \cdot 1 \cdot 1 = 1$. In general, we can obtain a function value 1 in a particular row if we form the appropriate product of literals. We can then add these products to obtain any function table.

In this case, we form the sum as the formula for f :

$$f(x, y, z) = xy\bar{z} + x\bar{y}z + \bar{x}y\bar{z} + \bar{x}\bar{y}z,$$

obtaining a function that is equal to 1 in exactly rows 2, 3, 6, and 7.

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#2. Let $f(x, y, z) = \bar{z} + \bar{x}z$.

- (a) Find the sum-of-products expansion (disjunctive normal form) for f .
- (b) Find the product-of-sums expansion (conjunctive normal form) for f .

Solution:

The values of f are given in the following table:

x	y	z	$f(x, y, z) = \bar{z} + \bar{x}z$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

(a) The sum-of-products expansion for f is

$$f(x, y, z) = xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{z}\bar{y}\bar{z}.$$

(b) The product-of-sums expansion for f is

$$f(x, y, z) = (\bar{x} + \bar{y} + \bar{z})(\bar{x} + y + \bar{z}).$$
