CHAPTER 18

Investment Decisions

In this final chapter, we will address the question of how managers can make decisions about investment projects in order to maximize the value of the firm. Investment projects may involve purchasing new equipment for a plant, expanding the size of a production facility, adding a new product to the firm's product line, or even buying another firm.

Investment decisions involve cash flows over multiple periods of time in the future. These cash flows are inherently risky—they are obviously not known with certainty. To make investment decisions, managers must examine the present value of the stream of revenues and costs associated with the many investment projects available. As you will see, investment decision making requires using the analysis of present value presented in the appendix to Chapter 1, as well as the optimization theory from Chapter 4 and the risk analysis discussed in Chapter 17.

We begin the analysis of investment decisions by describing how managers can determine the value of a stream of risky cash flows received over a period of time. Then we develop the net present value rule for maximizing the value of a firm and apply it to the investment decision. As always, this analysis is an extension of the marginal benefit–marginal cost rule first set forth in Chapter 4. We discuss the various methods of finding the appropriate discount rate for making investment decisions. Next, we present some critiques and several alternative investment criteria—payback, return on investment, and the internal rate of return—that are sometimes used by managers to make investment decisions. Finally, we examine the manager's investment decision when the firm's investment funds are constrained by budgetary limits imposed either by banks or by the firm itself. This is the problem of capital rationing.

The topics covered in this chapter on investment decisions are covered in much greater detail in the finance courses you will take or may have taken already. Since investment decision making is a critical component of a manager's decision-making responsibilities, and since these decisions are typically made using the basic techniques of microeconomic analysis, managerial economics courses traditionally cover investment decision making. We follow tradition by including this chapter.

18.1 VALUING RISKY CASH FLOWS

The value of a *riskless* project (asset) is given by its present value. To obtain the present value of some specific project, j, the following valuation equation is used:

$$PV_j = \sum_{t=1}^{T} \frac{NCF_{j,t}}{(1 + r_t)^t}$$

where $NCF_{j,t}$ is the net cash flow generated by project j in year t and r_t is the riskless discount rate in year t (the interest rate on U.S. government securities).[1] We now examine the way this valuation equation changes when project j is a *risky* project.

Risky Cash Flows

When cash flows are risky, the numerator of the present value equation must change. The cash flows from project j are no longer known with certainty. Instead of a single, known value of $NCF_{j,t}$, there exists a probability distribution for the cash flow from project j in year t, an illustration of which is provided in Figure 18.1. Since the probability distribution for the NCF is known, the manager can calculate the expected value of the net cash flow in each time period t for project j, $E(NCF_{j,t})$. Recall from Chapter 17 that decisions involving risk may require calculating the expected value of random outcomes. For investment decisions, the manager is interested in the expected value of risky net cash flows.

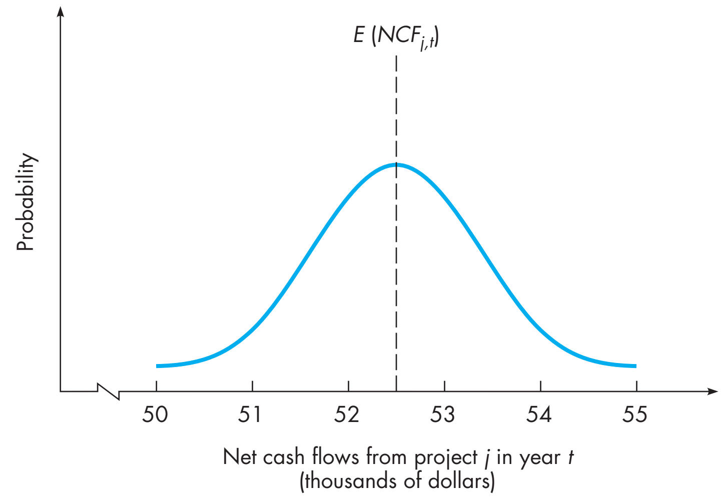
A Discount Rate Reflecting Risk

Another change in the valuation equation must be made when risk is present. A change is required in the denominator. For the riskless project, the appropriate discount rate is the riskless rate. Since there is no question about the size of the cash flow, the only thing that matters is the time at which the cash flows are to be received (paid).

However, as is clear from Figure 18.1, the probability distribution for the cash flows from the risky project j has a positive variance. Because the variance is a measure of risk, the cash flows from this project are risky in the sense that the size of the cash flow varies randomly. The larger the variance (or standard deviation) of the cash flows from a project, the riskier the project.

[1] The appendix to Chapter 1 reviews the mathematics of present value calculations.

FIGURE 18.1
Cash Flows from a Risky Project



Since the risk associated with the variance in the probability distribution for cash flows is not accounted for in the numerator of the valuation equation (*NCF*), it must be accounted for in the denominator. Thus, this risk must be reflected in the discount rate used. The riskless rate is clearly not appropriate; neither is some common or general discount rate for risky projects. Project *j* has some level of risk associated with the probability distribution of its cash flows, and this risk is not necessarily the same as the risk for some other project *k*—even if the two projects have the same expected net cash flows. Therefore, there will exist a specific discount rate that reflects the risk associated with project *j*. We denote this specific discount rate for cash flows received (paid) from project *j* in year *t* as $r_{j,t}$.

Combining the discussions of uncertain cash flows and a risk-adjusted discount rate, it should be clear that, for a risky project *j*, the *expected* present value is

$$E(PV_j) = \sum_{t=1}^{T} \frac{E(NCF_{j,t})}{(1 + r_{j,t})^t}$$

Just as the present value equation provides the value of a riskless project, this equation can be used to value a risky project.

It should be clear now how a manager could estimate a probability distribution for cash flows from a project, and then, using the appropriate discount rate, determine the expected net cash flows. Choosing the appropriate discount rate to reflect the riskiness of a project is not always a simple matter. We will now discuss the problem of finding the appropriate discount rate.

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18.2 THE APPROPRIATE DISCOUNT RATE FOR A RISKY PROJECT

To discount the expected net cash flows from a risky project j, a manager should use a discount rate that compensates the firm for bearing the additional risk. Thus, the discount rate should be higher than the riskless rate. The appropriate discount rate for cash flows from risky project j in year t could be expressed as

$$r_{j,t} = r_t + (\text{risk premium})_j$$

where r_t is the riskless rate and the second term is the appropriate risk premium for project j.[2]

We will now show how to determine the appropriate risk premium. Several methods exist. We will describe two that are used most often: risk-adjusted discount rates and the weighted average cost of capital. After defining and briefly discussing these two rules of thumb, we will present a simple numerical example to illustrate their use.

The Risk-Adjusted Discount Rate

risk-adjusted discount rate
The riskless rate plus a risk premium.

The preceding equation illustrates the discounting problem quite clearly: a risk premium must be attached to the riskless rate. The **risk-adjusted discount rate** approach does that directly, simply by adding a specific risk premium directly to the riskless rate. Apparently simple, the risk-adjusted discount rate methodology is complicated by an obvious problem: How does the manager determine the appropriate risk premium? Elegant mathematical formulas have been presented by Franco Modigliani and Merton Miller and by J. Miles and R. Ezzell. However, the risk-adjusted discount rates remain rules of thumb and are at the discretion of the manager.

The Weighted Average Cost of Capital

weighted average cost of capital
The rate at which the firm can borrow, weighted by the ratio of debt to net worth, plus the rate of return required by stockholders, weighted by the ratio of equity to net worth: $r_{WACC} = r_D[D/(D + E)] + r_E[E/(D + E)]$.

The weighted-average-cost-of-capital (WACC) approach to finding the appropriate risk premium reflects market conditions more than the risk-adjusted discount rate approach. In essence, the weighted-average-cost-of-capital method is based on the assumption that the appropriate discount rate for new projects is the interest rate currently paid by the firm in the market, that is, a weighted-average rate at which the firm can borrow funds and the rate of return required by the firm's shareholders. Following this approach, the **weighted average cost of capital** for a firm, denoted as r_{WACC}, is calculated as[3]

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[2]The risk discussed here deals with variation in the size of the cash flow. It should not be confused with default (performance) risk, which is embedded in the numerator: $E(NCF)$ will incorporate any probabilities of default.

[3]This formula neglects corporate taxes. If corporate taxes were included

$$r_{WACC} = (1 - CTR)\left[r_D\left(\frac{D}{D + E}\right) + r_E\left(\frac{E}{D + E}\right)\right]$$

where CTR is the marginal corporate income tax rate.

$$r_{WACC} = r_D\left(\frac{D}{D+E}\right) + r_E\left(\frac{E}{D+E}\right)$$

where D and E are, respectively, the current market values of the firm's outstanding debt and equity and r_D and r_E are, respectively, the current rate at which the firm can borrow and the rate of return on equity required by the firm's shareholders to induce them to hold the shares of stock.

In the above equation, D plus E is the net worth of the firm. Thus the first term is the rate at which the firm can borrow, weighted by the proportion of net worth that is represented by debt. The second term is the rate of required return on equity, weighted by the proportion of net worth represented by equity. Hence the term, weighted average cost of capital.

The weighted-average-cost-of-capital approach is intuitively appealing, and, in some cases, it actually works. Specifically, the WACC approach is appropriate (1) if the project being considered is just like the rest of the firm and (2) if the project is to be financed with the same mix of debt and equity prevailing in the rest of the firm. The WACC approach is not appropriate for projects that are more or less risky than the firm's existing portfolio of projects. Likewise, the WACC approach is not appropriate if acceptance of the project would cause the firm's debt/equity ratio to change.

Perhaps because it is more intuitively appealing than the risk-adjusted discount rate approach, the weighted-average-cost-of-capital approach is widely used. However, like the risk-adjusted discount rate, weighted average cost of capital is only a rule of thumb.

Estimating the Probability Distribution of Cash Flows and Discount Rates: A Numerical Example

We will now show how a firm can estimate a probability distribution for the cash flows from a project and determine the net cash flows. Suppose the managers of Zeus Manufacturing are considering the acquisition of machinery to produce a new product line. The machinery has a five-year time horizon. (At the end of the five years, the project has no scrap value.) The firm obtained low, best, and high estimates for the net cash flows from the project in each of its five years. The resulting table of outcomes is presented below:

	Net cash flow estimates ($, millions)		
Year	Low	Best	High
1	−2	0	4
2	1	3	5
3	4	5	6
4	4	5	6
5	2	4	5

Management subjectively assigned probabilities to these outcomes as low, 20 percent; best, 70 percent; high, 10 percent.

Using these probabilities, the expected net cash flow in year 1 is zero:

$$E(NCF_1) = (-2)(0.2) + (0)(0.7) + (4)(0.1)$$
$$= -0.4 + 0 + 0.4 = 0$$

Calculated in the same way, the expected net cash flows for years 2 through 5 are (in millions of dollars)

$$E(NCF_2) = 2.8$$
$$E(NCF_3) = 4.9$$
$$E(NCF_4) = 4.9$$
$$E(NCF_5) = 3.7$$

The managers of Zeus now want to determine the expected present value of this project. Their policy has been to attack a risk premium based on their evaluation of how risky the project is. The risk premiums they use are as follows:

Project riskiness	Risk premium (percent)
Low-risk project	3
Average-risk project	6
High-risk project	9

In their judgment, the project being considered is an average-risk project. Hence, the rates used to discount the expected cash flows are obtained by adding this risk premium to the riskless rate (the interest rate for U.S. government securities):

Years to maturity	Riskless rate (percent)	Risk premium (percent)	Risk-adjusted discount rate (percent)
1	5.75	6	11.75
2	6.00	6	12.00
3	6.25	6	12.25
4	6.50	6	12.50
5	6.75	6	12.75

Using these risk-adjusted discount rates, the expected present value of the project is

$$E(PV_j) = \frac{0}{(1.1175)} + \frac{2.8}{(1.12)^2}$$
$$+ \frac{4.9}{(1.1225)^3} + \frac{4.9}{(1.125)^4} + \frac{3.7}{(1.1275)^5}$$
$$= 0 + 2.232 + 3.464 + 3.059 + 2.031$$
$$= 10.786$$

That is, the expected present value of the new machinery is \$10,786,000, using a risk-adjusted discount rate.

Now suppose the managers of Zeus want to compare this expected present value with an expected present value calculated with a discount rate reflecting the WACC method. To calculate the weighted average cost of capital for Zeus Manufacturing, the managers first needed the current market value of Zeus's debt and equity. The market value of Zeus's equity was the easier of the two: there were 1,200,000 shares of stock currently selling at $27.25 per share, so

$$E = 1{,}200{,}000 \times \$27.25 = \$32{,}700{,}000$$

Zeus had issued debt with a face value of $95 million. Currently, these corporate bonds are being traded at 92 percent of their face (par) value, so

$$D = 0.92 \times \$95{,}000{,}000 = \$87{,}400{,}000$$

The managers also needed Zeus's current borrowing rate and the rate of return required by its shareholders. Using data on the general performance of the stock market and a subjective assessment of the riskiness of Zeus, the finance director estimated that Zeus's shareholders require a return of 18 percent:

$$r_E = 0.18$$

Using the market valuation of Zeus's debt issues and prevailing interest rates, the finance director calculated that the current yield on Zeus's debt—the interest rate Zeus would have to pay to borrow money today—is 7 percent:

$$r_D = 0.07$$

Hence, the weighted average cost of capital for Zeus Manufacturing is

$$\begin{aligned} r_{\text{WACC}} &= 0.07\left(\frac{87.4}{87.4 + 32.7}\right) + 0.18\left(\frac{32.7}{87.4 + 32.7}\right) \\ &= 0.05 + 0.05 = 0.10 \end{aligned}$$

Using this discount rate to discount the expected net cash flows of the project under consideration,

$$\begin{aligned} E(PV_j) &= \frac{0}{(1.1)} + \frac{2.8}{(1.1)^2} + \frac{4.9}{(1.1)^3} + \frac{4.9}{(1.1)^4} + \frac{3.7}{(1.1)^5} \\ &= 0 + 2.314 + 3.681 + 3.347 + 2.297 \\ &= 11.639 \end{aligned}$$

That is, the expected present value of the project is $11,639,000, using a WACC discount rate. The latter method yields an expected present value for the project that is $853,000 higher than the first method of calculation, because it discounts with a lower rate than the risk-adjusted rate.

18.3 MAKING INVESTMENT DECISIONS TO MAXIMIZE THE VALUE OF THE FIRM

As we demonstrated, present values are additive. The present value of the portfolio of two projects, *A* and *B*, is equal to the sum of the present values of projects *A* and *B*: $PV(A + B) = PV(A) + PV(B)$. Going a step further, a firm is really a portfolio of its projects (assets). As we emphasized in Chapter 1, the worth or the value of a firm, the price the firm would bring if it were sold, is the present value of the firm. The present value of the firm is the sum of the present values of all its projects or assets.

If, as noted in Chapter 1, the objective of management is to maximize the value of the firm, the following principle applies: The present value of a firm will increase if the present value of an additional project (the marginal benefit) exceeds the marginal cost of the project. If the present value of an additional project is less than its marginal cost, that project will decrease the present value of the firm. Therefore, the firm should undertake projects for which the present value is greater than the cost. It should reject projects for which the present value is less than the cost. This simple rule is the foundation of the theory of investment.

net present value (NPV)
The present value of an investment minus its cost.

We define **net present value (NPV)** as the present value of an investment minus the cost of the investment. Thus the general rule for a value-maximizing firm is summarized in the following:

Principle The net present value rule for maximizing the value of the firm is to accept projects (acquire assets) for which the net present value is positive and reject projects for which the net present value is negative.

The Expected Net Present Value Rule for Investment

As we stressed in Section 18.2, the risk associated with a particular project can be accounted for by discounting the *expected* net cash flow using a discount rate that includes a risk premium. This modification results in an expected net present value rule for investment decision making. Accordingly, we define net present value of investment project j as[4]

$$E(NPV_j) = E(PV_j) - \text{cost of investment project } j$$
$$= \sum_{t=1}^{T} \frac{E(NCF_{j,t})}{(1 + r_{j,t})^t} - C_0$$

where the numerator is the expected value of the net cash flow in time period t, $r_{j,t}$ is the risk-adjusted discount rate for project j in time period t, and C_0 is the cost of the investment project.

The expected net present value rule for investment projects is to accept projects for which the expected net present value is positive and reject those for which the net present value is negative:

$$E(NPV_j) > 0 \ldots \text{Accept}$$
$$E(NPV_j) < 0 \ldots \text{Reject}$$

Implementation of the expected net present value rule is a straightforward application of the techniques we have described in this text:

[4]We consider a simple investment project for which the only outlay for the project occurs in the current period (C_0). However, it would not be difficult to generalize this expression to incorporate an investment project that requires outlays in future periods as well as the current period.

ILLUSTRATION 18.1

Do Income Taxes Affect Managerial Decision Making?

Throughout this text we have essentially ignored any effects of income taxes or corporate taxes in our analysis of managerial decision making. Excise taxes raise prices and decrease quantities sold, the extent of which depends on elasticity and the size of the tax. But we have not yet mentioned the effect of an income tax or corporate profit tax on price and sales, even though these are the taxes that are typically foremost in many people's minds. When we discussed managerial decision making and optimization, there was no reason to consider the effect of income taxes—until now.

To put things into perspective, we will quote briefly from a column by Michael Kinsley, in *The New Republic,* September 6, 1993. Mr. Kinsley was commenting on congressional and media debate over the effect on small businesses of President Clinton's proposal to increase tax rates for upper-income taxpayers. He pointed out that many politicians and small-business owners had been complaining that the higher taxes would put their businesses at a disadvantage when competing with foreign companies—presumably because of resulting price increases—and force them to eliminate jobs—presumably because of reduced production.

Mr. Kinsley commented, "Neither [complaint] makes economic sense. The income tax is levied on a businessperson's net profits. [The rate of the tax] has no effect on the question of how best to maximize those profits: how much to produce, what prices to charge, how many people to hire, etc. To be sure, higher tax rates can reduce the *incentive* to work and invest for small business people, like any other people."

Now we will take a look at these different effects to explain why we have ignored income taxes thus far. First, consider the effect on profit-maximizing decisions: output, price, and hiring. As we have emphasized throughout the text, profit is maximized in a given situation when price and output are chosen so that $MR = MC$ or when the usage of variable inputs is chosen so the MRP = Price of the input. Suppose a firm is choosing an output and price to maximize profit, but there is no income tax. If an income tax of t percent is levied, the firm would still choose the price and output that maximize profit—$MR = MC$—because the owners prefer to receive $(100 - t)$ percent of the maximum possible profit to $(100 - t)$ percent of any lower profit at which MR is not equal to MC. There is no incentive to change output, price, or, for that matter, the usage of any input. For example, suppose the income tax rate is 25 percent and the maximum before-tax profit is $1 million. The firm pays $250,000 in taxes and keeps $750,000. If the tax rate rises to 35 percent, paying taxes of $350,000 on the maximum before-tax profit of $1 million and keeping $650,000 is better than any alternative that would reduce before-tax profit.

1. Forecast demand to obtain estimates of expected revenues from the project, $E(R_{j,t})$.
2. Forecast (estimate) costs to provide estimates of the expected future costs associated with the project, $E(C_{j,t})$.
3. Combine the expected revenues and costs to obtain estimates of expected net cash flows for the project:

$$E(NCF_{j,t}) = E(R_{j,t}) - E(C_{j,t})$$

4. Determine the appropriate discount rate, $r_{j,t}$.
5. Discount the expected net cash flows to obtain the expected present value of the project.

Therefore, the reason that we have ignored the discussion of income taxes until now is that we have been concerned with the way firms maximize profit under given conditions and the tax rates have no effect on decision making under these circumstances. But now we are considering the investment decision, and, as Mr. Kinsley pointed out, taxes can have an effect on the incentives to work and invest. We will consider here only the effect on the incentive to invest and will ignore the incentive to work.

As we have stressed in this chapter, the *NPV* rule provides the foundation for investment decision making. If *E*(*PV*) is greater than the cost of a project, *E*(*NPV*) is positive and the project should be undertaken. But the return that investors are interested in is the after-tax expected net present value. In an extreme case, suppose there is a 99 percent tax on yearly cash flows. This would presumably reduce the after-tax *NPV* of most prospective investments below their costs and substantially reduce the number of investments with a positive *E*(*NPV*). Alternatively, when choosing price and quantity, an owner would prefer 1 percent of maximum profit over 1 percent of a lesser amount, as we discussed. So the tax would probably have a large effect on investment and no effect on price.

As the tax rate is reduced, the after-tax expected cash flows from investment projects would increase; and more and more projects would change from negative *E*(*NPV*) to positive *E*(*NPV*)—assuming the cost of the project does not change. However, since the cost of an investment can be a tax deduction when financed with retained earnings, the costs may change when the tax rate changes. Nonetheless, the basic conclusion is the same. Increases in the tax rate reduce investment by reducing after-tax expected net cash flows, and decreases in the tax rate increase investment by increasing after-tax expected net cash flows. There are some additional factors influencing *E*(*NPV*), so the extent of the effect on tax rates is basically an empirical question and depends to some extent on the characteristics of individual investors.

There is another way in which the tax structure, combined with inflation, may have a negative effect on investment. Under the present tax structure, nominal, not real, income is subject to taxation. Therefore, if someone purchases an asset and sells it later, all gains are subject to taxation, even though most, or even all, of the gain could be due to inflation. For example, suppose a firm purchases an asset for $100,000. The value of the asset increases during a year at the same rate as the rate of inflation, 5 percent. If the firm sells the asset for $105,000, realizing a net gain of $5,000, which, for sake of illustration, is taxed at a 34 percent rate, the after-tax return is $3,300. The firm, in real terms, has lost $1,700 (or .34 × $5,000), because the $105,000 is worth only $100,000 in year-1 dollars. In order to receive $105,000 after inflation and taking taxes into account, the rate of return must be about 7.6 percent.

6. Subtract the current cost of the project to obtain expected net present value.

To show how this might be accomplished, we present the following example.

Investment Decision Making at Trenton Enterprises: An Example

The manager of Trenton Enterprises is considering purchasing a new production facility for a price of $5.3 million. The manager expects to use the production facility for five years, then resell it. Investment analysts at Trenton determined expected revenues and costs and the expected resale value of the plant. Using these data, the manager obtained the following expected net cash flows for the firm's investment in a new production facility:

Year	Expected revenues*	Expected resale value*	Expected cost*	Expected net cash flow*
1	$10.2	—	$10.4	−$0.2
2	10.2	—	10.4	−0.2
3	14.2	—	11.6	2.6
4	16.3	—	13.2	3.1
5	16.3	$3.5	13.2	6.6

*In millions of dollars per year.

Using the weighted-average-cost-of-capital method of determining the appropriate discount rate, the manager obtained the following discount rates for each of the next five years:

Year	Discount rate (percent)
1	13.13
2	13.38
3	13.63
4	13.88
5	14.13

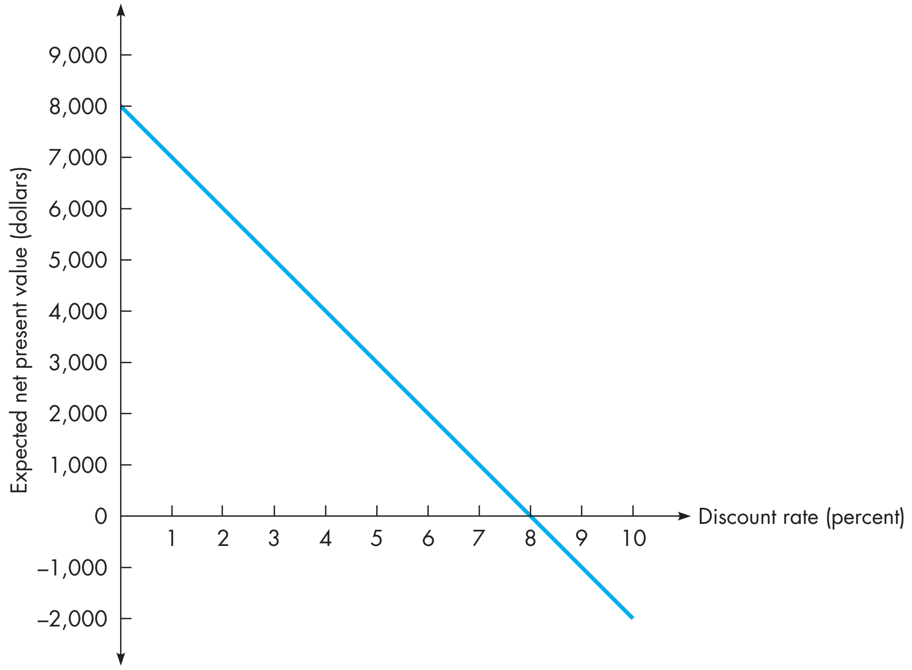
Discounting each of the expected net cash flows by the appropriate discount rate and summing, the manager obtained the expected present value of the new plant:

Year	Expected net cash flow $, millions, (per year)	Discount rate (percent)	Expected present value ($, millions, per year)
1	−$0.2	13.13	−$0.18
2	−0.2	13.38	−0.16
3	2.6	13.63	1.77
4	3.1	13.88	1.84
5	6.6	14.13	3.41
		Total	$6.68

Subtracting the cost of the project, $5.3 million, from the expected present value of the project, $6.68 million, the expected net present value of the plant is $1.38 million. Since the expected net present value of the project is positive, the project should be undertaken.

As should be obvious, the discount rate plays a key role in determining the expected net present value and, therefore, in the investment decision. The expected net present value equation indicates the inverse relation between the expected net present value and the discount rate: As the discount rate rises, the expected net present value of the project will fall.

FIGURE 18.2
An Expected Net Present Value Profile



For example, consider a one-year investment project that currently costs \$100,000 and will generate an expected net cash flow of \$108,000 at the end of one year. With a risk-adjusted discount rate of 7 percent, the expected net present value of this project is \$935. If the discount rate falls to 6 percent, the expected net present value of the project rises to \$1,887. If the discount rate rises to 8 percent, the expected net present value of the project falls to zero. And if the discount rate rises further, to 9 percent, the net present value of the project becomes negative, −\$917. This relation between the expected net present value of the project and the discount rate—sometimes referred to as the *expected net present value profile*—is illustrated in Figure 18.2. This profile clearly shows that the expected net present value falls as the discount rate rises.

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Principle When evaluating risky investment projects, the firm should accept projects with a positive expected net present value and reject projects with a negative expected net present value.

18.4 ALTERNATIVES TO THE EXPECTED NET PRESENT VALUE RULE

Although economists would argue that the expected net present value rule is the correct investment criterion, it is not the only criterion available to managers. Three of the most widely cited are payback, return on investment, and internal rate of return. We will now discuss these alternative criteria to show how they compare with the net present value rule.

Payback Period

payback period
Time required for the firm to recover its initial investment.

The **payback period** for an investment project is the time required for the firm to recover its initial investment. For example, if a project costs $1 million and it is expected to return $250,000 per year, the payback period is four years; if expected returns are $500,000 per year, the payback period is two years.

Using the payback criterion, the payback period for the investment project is calculated and compared with some maximum payback period set by the firm. If the project's payback period is less than this maximum, the project is accepted.

For example, returning to Trenton Enterprises, suppose the manager has set the maximum payback period for investment projects as three years. The payback period for the previously discussed prospective production facility is calculated from the cumulative expected net cash flows from the project:

Year	Expected net cash flow*	Cumulative expected net cash flow*
1	−$0.2	−$0.2
2	−0.2	−0.4
3	2.6	2.2
4	3.1	5.3
5	6.6	11.9

*In millions of dollars.

The cumulative expected net cash flows equal (or exceed) the cost of the investment ($5.3 million) at the end of the fourth year. Hence, the payback period for this project is four years. And since the payback period is longer than the maximum set by the firm, the project would be judged unacceptable using this criterion.

As this example makes clear, the major problem with the payback criterion is that it can lead to the rejection of positive net present value projects—projects that will increase the value of the firm. Conversely, this rule could lead to accepting negative net present value projects.

As should be clear from the discussion to this point, the reason the payback rule can lead to this value-reducing situation is that the cash flows are not discounted. Hence, the payback criterion gives too much weight to near returns and too little weight to distant returns: with the payback rule, net cash flows received after the maximum payback period have no value. This criterion ignores the time value of money and the time pattern of the cash flows generated by the investment project.

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One critic noted that a survey of investors revealed that many users of the payback criterion thought of the payback period as a measure of risk. He pointed out that gambling at the tables in Las Vegas may have a shorter payback period than purchasing a U.S. government security but that doesn't mean the crap tables in Las Vegas are less risky than T-bills.

Return on Investment (*ROI*)

return on investment (*ROI*)
Average return from an investment divided by the average investment in a project.

The average **return on an investment (*ROI*)** project is defined as average returns from the investment divided by the average investment in the project. Then, using the *ROI* criterion, the decision of whether or not to invest in the project is made by comparing the *ROI* for the project with the firm's target return.

For example, suppose Trenton Enterprises requires a rate of return on investment of 60 percent. The manager wanted to look at the prospective investment again with this criterion in mind. As shown in previous calculations, the cumulative net cash flows from the project amounted to $11.9 million for the five years of the project's lifetime. Hence, the average net cash flow was $11.9/5 = $2.38 million. Dividing this average income by the amount the firm would invest in the project, $5.3 million, the average return on the investment is

$$2.38/5.3 = 45\%$$

Since this *ROI* is less than the firm's target return of 60 percent, the project would be rejected using the rate of return on investment as the criterion.

As in the case of payback, the *ROI* criterion could result in positive net present value projects not being undertaken. And, also like the payback criterion, the problem with the return on investment criterion is that the cash flows are not discounted. However, unlike the payback criterion, which gives *too little* weight to distant cash flows, the *ROI* criterion gives distant cash flows *too much* weight. With the *ROI* criterion, distant cash flows are treated as equivalent to current cash flows.

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Internal Rate of Return (*IRR*)

internal rate of return (*IRR*)
The discount rate that makes the net present value of a project equal to zero.

The **internal rate of return (*IRR*)** for an investment project is the discount rate that makes the net present value of the project equal to zero. In order to understand the concept of the internal rate of return, consider again the single-period investment project we discussed earlier and illustrated in Figure 18.2:

Cost of investment	$100,000
Net cash flow at end of year 1	$108,000

The rate of return on this investment project is 8 percent:

$$\frac{108{,}000 - 100{,}000}{100{,}000} = 0.08 = 8\%$$

Hence, for this single-period investment project, we have a criterion that is equivalent to the net present value rule: Accept the project if the discount rate for the project is less than 8 percent; reject the project if the discount rate is more than 8 percent.

Indeed, for any single-period investment project, the *NPV* rule is implemented by comparing the project's rate of return with its discount rate:

$$\text{Rate of return} > \text{Discount rate} \rightarrow NPV > 0 \rightarrow \text{Accept project}$$
$$\text{Rate of return} < \text{Discount rate} \rightarrow NPV < 0 \rightarrow \text{Reject project}$$

From this, it follows that the internal rate of return on the project is 8 percent, which makes the net present value of the project equal to zero:

$$NPV = 0 \rightarrow \text{Rate of return} = \text{Discount rate}$$

For investment projects with longer lifetimes (multiple-period projects), the internal rate of return becomes more difficult to determine. Nonetheless, the definition of the internal rate of return (*IRR*) is simply a generalization of the preceding relation. However, solving for the *IRR* is not an easy arithmetic problem in such cases, since it involves solving for the discount rate at which the *NPV* of the project is zero. Operationally, that means that the equation

$$NPV = \sum_{t=1}^{T} \frac{NCF_t}{(1 + IRR)^t} - C_0 = 0$$

is solved for *IRR*. Given (1) the complexity of this solution and (2) the wide acceptance of the *IRR* criterion, it is probably not surprising that most business calculators are preprogrammed to calculate this value.

From the discussion of the single-period investment project, it should be clear that the investment criterion associated with the internal rate of return is to accept the project if the cost of capital to the firm is less than the *IRR* and reject the project if the cost of capital to the firm exceeds the *IRR*.

The *IRR* criterion can be illustrated graphically by looking at the net present value profile for an investment project. A generalized profile is presented in Figure 18.3. As long as the internal rate of return exceeds the discount rate—the opportunity cost of capital—the *NPV* of the project is positive and the project should be undertaken. However, if the *IRR* is less than the discount rate, the *NPV* of the project is negative and the project should be rejected.

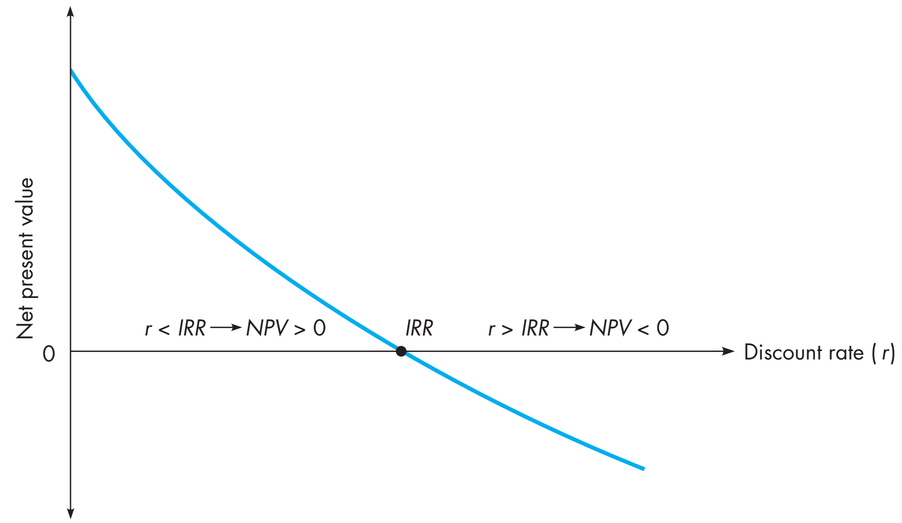
As an example of the use of the internal rate of return as an investment criterion, we return once again to Trenton Enterprises and its investment decision. The manager determined that the cost to Trenton of raising additional capital is 13.5 percent. That is, to raise money to finance investment projects, Trenton will have to pay 13.5 percent annually. He then reevaluated the proposed acquisition of the facility by looking at the project's rate of return relative to Trenton's opportunity cost of capital.

The internal rate of return for the project is the single discount rate (the *IRR*) that would make the *NPV* of the project equal to zero. This is found by solving the following for the *IRR:*

$$NPV = \frac{-0.2}{(1 + IRR)} + \frac{-0.2}{(1 + IRR)^2} + \frac{2.6}{(1 + IRR)^3} + \frac{3.1}{(1 + IRR)^4} + \frac{6.6}{(1 + IRR)^5} - 5.3 = 0$$

FIGURE 18.3

A Generalized Net Present Value Profile



The resulting value for the *IRR* is 20.2 percent. Therefore, since the *IRR* for the project exceeds the firm's opportunity cost of capital, the project should be undertaken.

Note that, in contrast to the payback and return on investment criteria, the *IRR* criterion led to acceptance of the hypothetical investment project. In the context of this simple example, it appears as though the *IRR* criterion and the *NPV* criterion are equivalent rules. And they are: as long as the *NPV* of the project declines smoothly as the discount rate rises—as is illustrated in Figure 18.3—the two criteria are functionally equivalent (for evaluating single projects).

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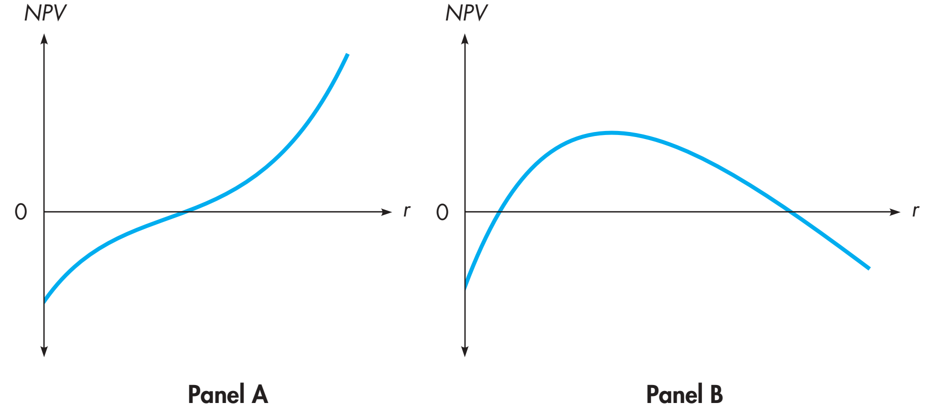
Thus, as long as the relation between *NPV* and the discount rate is smooth and negative, the *NPV* rule and the *IRR* rule will give the same results. However, there are times when the *IRR* rule does not work.

Nonequivalence of *IRR* and *NPV* Rules

If the net present value profile does not look like that in Figure 18.3, the *IRR* rule and *NPV* rule may no longer be equivalent. Most investment projects are like lending money; an original outflow is made in return for a stream of inflows, thereby generating downsloping net present value profiles like that in Figure 18.3. However, this need not always be the case; it is not always the case that the largest expenditures on the project occur in the initial period. It is possible that the inflows occur earlier than the outflows—the investment project could look more like borrowing than lending. In this case the net present value profile would look like that illustrated in Panel A of Figure 18.4, where the discount rate r is plotted along the horizontal axis. Or it could be the case that the investment project will require net

FIGURE 18.4

Net Present Value Profiles That Do Not Decline Smoothly as the Discount Rate Rises



cash outflows both initially and in some subsequent period; for example, the project may require a retrofit at some date in the future. In this case, the net present value profile will look like that illustrated in Panel B of Figure 18.4. In either case, the *IRR* criterion is no longer equivalent to the *NPV* criterion.

Another case in which *IRR* and *NPV* rules do not necessarily provide the same recommendation occurs when the decision concerns mutually exclusive projects. Consider a firm deciding whether to replace or refit a machine—decisions that are clearly mutually exclusive. Suppose the net cash flows from these two projects are as presented in the following table:

	Refit	Replace
Current cost	$100,000	$250,000
Net cash flow, year 1	75,000	125,000
Net cash flow, year 2	50,000	175,000

Looking at the internal rates of return,

	Refit	Replace
IRR	17.5%	12.3%

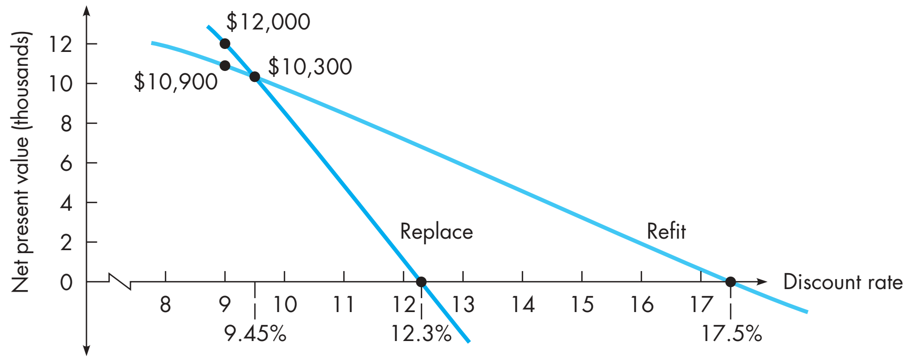
it seems as if refitting is the better choice. And if the firm's cost of capital is greater than 12.3 percent but less than 17.5 percent, refitting is the correct decision. But suppose the firm's opportunity cost of capital is less than 12.3 percent. Is refitting always the best choice? Suppose the discount rate is 9 percent. Looking at the net present values using a 9 percent discount rate,

	Refit	Replace
NPV (r = 9%)	$10,900	$12,000

the choice is reversed; the better choice now is to replace the machine.

FIGURE 18.5

Net Present Value Profiles for Mutually Exclusive Projects



The reason for the inconsistency is illustrated in Figure 18.5. For discount rates in excess of 9.45 percent, the *IRR* criterion and the *NPV* criterion will be consistent. That is, with the discount rate in excess of 9.45 percent, the project with the higher *IRR* will have the higher *NPV.* However, for discount rates below 9.45 percent, the *IRR* is no longer a useful criterion. With a discount rate less than 9.45 percent, the project with the higher *IRR* has the lower *NPV.* And, with these lower discount rates, the *IRR* criterion would lead to selecting the project with the lower, not higher, *NPV.*

Finally, you may have noted a methodological difference. When we worked with net present value, we used different discount rates for different periods; payments received early were discounted at a different rate than those received late. The internal rate of return approach does not permit that differentiation. With the *IRR* approach, there is a single discount rate: payments received early are discounted using the same discount rate as those received late.

18.5 CAPITAL RATIONING

In the broadest perspective, capital rationing by the manager of a firm should not occur; there is no *external* constraint on a number of projects a firm can undertake. If the firm has available a project that will increase the value of the firm, the project should be undertaken. The capital constraint (the limit of available monies to finance the project) can always be eliminated by the credit market. If going to the credit market means that the firm will have to pay a higher and higher price for its capital (if the opportunity cost of capital is rising), the net present value of the project will decline. Indeed, if the opportunity cost of capital rises, some projects will no longer have a positive net present value. But in this case we are simply looking at the investment decision, not capital rationing.

From a more pragmatic perspective, most firms ration capital; most firms are subject to a constraint on the number of investment projects they can undertake. However, it is important to recognize that the constraint is, by and large, a *self-imposed constraint*. For some reason, the managers must believe that they will be unable to fund all the investment projects available. And if this is the case, they need some way to determine which projects to undertake.

When confronted by this constraint, many managers first think of the internal rate of return as a means of ranking the competing projects. However, we hope we have convinced you of the problems involved in using the *IRR* to choose among competing projects.

Then why not just rank projects according to their net present values? Suppose the firm had available the three projects ranked below by their net present values:

Project	*NPV* ($, millions)	Rank
A	$10	1
B	7	2
C	5	3

Project *A* has the highest *NPV.* But if the combination of projects *B* and *C* costs less than *A* alone, it would be preferable to invest in the combination, since investing in both *B* and *C* would give a higher total *NPV* than would *A* alone. It is not sufficient to think simply about the project's net present value; a manager should think about the *NPV* per dollar spent on the project.

This shouldn't be at all surprising. This is a constrained optimization problem: the manager wants to maximize the value of the firm subject to the capital limitation. The rule for constrained optimization is to allocate so that the marginal benefit (the *NPV*) per dollar spent is equal among the competing activities. Hence, the most straightforward approach to the problem is to determine a **profitability index:** the ratio of the present value of the investment project to its cost. Presented below are additional data for the three projects introduced above:

profitability index
The ratio of the present value of an investment project to its cost.

Project	*NPV* ($, millions)	Cost ($, millions)	Present value ($, millions)	Profitability index
A	$10	$5	$15	3.0
B	7	3	10	3.3
C	5	3	8	2.7

Given the values of the profitability index, the firm would allocate the first $3 million to project *B,* since it has the largest ratio of marginal benefits to cost. The second project to be undertaken would be project *A.* Project *C* would be undertaken only if the capital constraint is lifted.

The profitability index is, however, not without its own limitations. This approach fails if there is more than one constraint, for example, if the capital constraint is imposed in more than one period. It is also unreliable when the projects are mutually exclusive or when one project is dependent on another's being undertaken. For such cases, more complicated techniques, including a linear programming approach, have been developed.

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18.6 SUMMARY

The investment decision combines decision making over time and under conditions of risk. A fundamental problem faced by all firms making investment decisions is the valuation of risky net cash flows generated by risky projects, assets, or investments. When dealing with risky cash flows, a manager must consider the *expected present value* of the project:

$$E(PV_j) = \sum_{t=1}^{T} \frac{E(NCF_{j,t})}{(1 + r_{j,t})^t}$$

where $E(NCF_{j,t})$ is the expected net cash flow generated by this jth project in period t and $r_{j,t}$ is the discount rate in period t, which reflects the riskiness of project j.

The primary difficulty in evaluating the expected present value of a project is the determination of the appropriate discount rate for a risky project. Put another way, the problem is the determination of the *risk premium* for the jth project,

$$r_{j,t} = r_t + (\text{risk premium})_j$$

We first considered two rules of thumb. With a *risk-adjusted discount rate*, some premium associated with the (total) variability in returns to the project is used. With the weighted-average-cost-of-capital approach, the firm's borrowing and equity costs are weighted by their relative shares to provide the discount rate for the firm:

$$r_{WACC} = r_D\left(\frac{D}{D+E}\right) + r_E\left(\frac{E}{D+E}\right)$$

The investment decision-making rule consistent with maximizing the value of the firm is simply the expected net present value rule. The expected net present value rule for investment projects is to accept those projects for which the expected net present value is positive and reject those for which the expected net present value is negative:

$$E(NPV_j) > 0 \ldots \text{Accept}$$
$$E(NPV_j) < 0 \ldots \text{Reject}$$

The expected net present value of a project is calculated as

$$\begin{aligned} E(NPV) &= E(PV) - \text{cost of the project} \\ &= \sum_{t=1}^{T} \frac{E(NCF_t)}{(1 + r_t)^t} - C_0 \end{aligned}$$

where r_t is the risk-adjusted discount rate for the project in time period t.

To implement the expected net present value rule, the manager must (1) forecast revenues, (2) forecast (estimate) operating costs for the project, and (3) determine the appropriate discount rate for the project. Particular emphasis was placed on the relation between expected net present value and the discount rate, a relation depicted graphically by the expected net present value profile.

While the expected net present value rule is the theoretically and analytically correct investment criterion, other rules continue to be used. Two rules of thumb were considered:

- *The payback rule:* The payback period is the time required for the firm to recover its initial investment. The investment criterion is to accept only those projects that have a payback period less than some arbitrary maximum set by the firm.
- *The return on investment* (ROI) *rule:* The average return on investment is the ratio of average net cash flows to average investment. If the *ROI* for the project is larger than the firm's arbitrary target return, the investment project will be undertaken.

The primary shortcoming with these two ad hoc rules is that the net cash flows are not discounted; these rules ignore the time value of money. The payback rule gives too much weight to cash flows that will be received early; the *ROI* rule gives too much weight to distant cash flows. A preferable alternative investment criterion is the internal rate of return rule:

- *The internal rate of return* (IRR) *rule:* The *IRR* is the discount rate that makes the *NPV* of the investment project equal to zero. The project should be accepted if the *IRR* is greater than the firm's opportunity cost of capital.

As long as the expected net present value profile is a smooth, downward-sloping function, the *IRR* and expected *NPV* rules are functionally equivalent. However, the *IRR* rule can provide erroneous recommendations when mutually exclusive projects are considered. And the *IRR* methodology requires that all net cash flows be discounted at the same rate: early and late cash flows are discounted using the same discount rate.

The constraint of limited funding for investment projects and the resulting capital rationing problem are, by and large, self-imposed by the management of the firm. Nonetheless, this is yet another constrained optimization problem: the solution must involve the ratio of marginal benefits to marginal costs (dollar expenditures). In this case, the profitability index is the ratio of the present value of the investment project (the marginal benefit from undertaking the project) to its cost. In order to maximize the value of the firm, managers will respond to the capital rationing problem by undertaking investment projects in the order of their profitability indexes.

TECHNICAL PROBLEMS

1. A manager is considering a risky investment project with a three-year life that will generate the following expected net cash flows:

Year	Expected net cash flow
1	$275,000
2	425,000
3	300,000

The manager determines that the risk of the project is appropriately accounted for by a project-specific discount rate of 8.2 percent.

a. Calculate the expected present value of the risky project.

New information about the project causes the manager to revise upward to 9.2 percent the appropriate discount rate for the project.

b. Calculate the expected present value of the risky project in light of this new information.

c. Did the higher discount rate increase or decrease the expected present value of the project? Why?

2. Reconsider the example in Section 18.2 of Zeus Manufacturing's acquisition of a new machine. The old manager is fired, and a new manager is asked to evaluate the project. The new manager subjectively assigns different probabilities to the three outcomes: 10 percent chance of the low outcome, 60 percent chance of the best outcome, and 30 percent chance of the high outcome.
   a. Using the new probabilities and the net cash flow estimates presented in the example, calculate the new expected net cash flows for each of the five years.
   b. The new manager assesses the risk of the project to be low. Using the new risk-adjusted discount rates, calculate the expected present value of the project.
   c. The new manager also found that shareholders demand only a 14 percent return and Zeus would still have to pay 7 percent to borrow money. What is the new weighted average cost of capital? What is the expected present value using the new weighted average cost of capital?
3. Consider an investment project costing $162,500 that is expected to generate net cash flows of $100,000 in years 1 and 2.
   a. Calculate the expected *NPV* for this project using discount rates of 10 percent, 15 percent, and 20 percent.
   b. Sketch the expected *NPV* profile for this project.
4. Consider an investment project with the following expected net cash flows:

Year	Expected net cash flow
1	$20,000
2	15,000
3	10,000
4	10,000
5	5,000

The investment will cost the firm $45,000. The appropriate discount rate is 10 percent.

a. What is the expected net present value?
b. Should the firm undertake this project? Briefly explain.
c. Suppose the appropriate discount rate is 18 percent. What is the expected net present value now?
d. At 18 percent, will this investment project increase the present value of the firm?
e. Compute the expected net present value for the following discount rates:

Discount rate (percent)	*E*(*NPV*)
9.5	________
11.0	________
13.25	________

5. Your supervisor has asked you to evaluate two potential investment projects. Both of these projects cost $2 million. The net cash flows from the two projects are presented below:

	Net cash flows ($, millions)	
Year	Project *A*	Project *B*
1	2	1.0
2	0	0.8
3	0	0.6
4	0	0.4

The firm's policy is that the maximum payback period for an investment project is two years.

a. Evaluate projects *A* and *B* using the criterion that two years is the maximum payback period.
b. Evaluate the two projects using the expected *NPV* criterion with a discount rate of 10 percent.
c. Compare the recommendations in parts *a* and *b*.

6. Reconsider the investment project in problem 4 using the 10 percent discount rate.

a. What is the payback period for this project?
b. If corporate policy is to accept only those projects with payback periods shorter than 30 months, will management accept this project?
c. Under what circumstances will management's decision under the payback rule be consistent with maximization of the firm's present value?

7. Explain why the payback rule gives too little weight to distant net cash flows and the *ROI* rule gives them too much weight.
8. Your firm has a target rate of return on investment of 35 percent. Using this criterion, you have been asked to evaluate two investment projects, both of which cost $20.5 million and have four-year lifetimes. The net cash flows from the two projects are provided below:

	Net cash flows ($, millions)	
Year	Project *A*	Project *B*
1	$10	$ 4
2	8	6
3	6	9
4	4	11

a. Which, if either, of the two projects would be accepted using the *ROI* criterion?
b. Reevaluate the two projects using the expected *NPV* criterion and a discount rate of 15 percent.
c. Compare the two recommendations. What report would you forward on these two projects?

9. Calculate the internal rate of return (*IRR*) for the project in
a. Problem 3.
b. Problem 4.
10. In general, when would the *IRR* and the expected *NPV* rules give conflicting recommendations?
11. The capital rationing problem is, by and large, self-imposed. Explain why this is so. What could the management of a firm do to eliminate the constraint? Why do so many firms ration capital?
12. Your firm has available four investment projects, the cost and expected net present values of which are presented below:

Project	Expected *NPV*	Cost
A	$20	$10
B	17	10
C	12	5
D	8	5

a. Calculate the profitability index for each of the projects.
b. Which projects will be undertaken if the firm has an expenditure (funding) constraint of $5? $10? $15? $20?

APPLIED PROBLEMS

1. Consider a risky project under consideration by Sharp Investments that will produce a single net cash flow at the end of two years. The probability distribution for the net cash flow is

Net cash flow ($, thousands)	Probability (percent)
25	5
30	5
35	20
40	30
45	30
50	10

a. Calculate the expected net cash flow for this project.
b. Suppose the interest rates on U.S. government securities are currently

Maturity	Interest rate (percent)
6 months	6.0
1 year	6.5
2 years	7.0
3 years	7.78

Compare the expected value for this project using the appropriate risk-free interest rate. Is the risk-free rate the appropriate discount rate to use? Why or why not?
c. Suppose Sharp Investments' current borrowing rate is 8.5 percent and its current outstanding debt is $20 million. Shareholders expect to earn 10 percent on equity, which amounts, in total, to $40 million. Compute the weighted average cost of capital (r_{WACC}). Using r_{WACC} for the discount rate, compute the expected present value of this project.

2. The fact that you are attending a college or university indicates that you have made an investment decision. What kind of investment decision is this? What factors did you (at least implicitly) evaluate when making this decision? In what way would the decision to go to graduate school differ?
3. In this chapter we concentrated on investment projects, implicitly talking about investments in plant and equipment. However, the same techniques could be used to evaluate other investment projects, including new products.

   Down-Home Eatin' is considering the introduction of Diet Grits. The proposal is to test market the new project for one year in two regions: Macon, Georgia, and northwest Bergen County, New Jersey. If the test markets are successful, the product will be introduced nationwide.

   How would the investment decision be structured? What data are required to make the decision? How would the necessary data be obtained?

For problems 4 through 7, use the following data: Argonaut Enterprises had available four potential investment projects that would all begin in 2002. The characteristics of these projects are summarized in the table below:

Project:	*A*	*B*	*C*	*D*
Cost:	$123,000	$89,200	$56,600	$55,800
Net cash flow*				
2002	30,000	50,000	20,000	40,000
2003	30,000	50,000	20,000	20,000
2004	30,000	0	20,000	10,000
2005	30,000	0	20,000	0
Scrap or resale value at end of 2005:	50,000	0	10,000	0

*At year-end.

4. Evaluate these projects using the expected *NPV* criterion and a discount rate of 15 percent.
5. The expected *NPV* evaluation did not sit very well with the vice president for operations, for whom project *A* was a particular favorite. He argued that the discount rate used in the calculation of the net present values was too high. In response, the board of directors asked to see the internal rates of return for each of the projects. Provide these values and an evaluation based on these values.
6. The vice president for operations came back to the board of directors with another argument: He believes that the resale value for project *A* was underestimated and that the resale value should be $70,000 rather than $50,000. If this is true, should this project be undertaken, using 15 percent as the relevant discount rate? Use the net present value of the project to support your answer.
7. It turns out that project *B* also has a supporter. The director of new product development argues that the capital outlay necessary for project *B* is $86,800 rather than $89,200. If this is true, what would be the internal rate of return for project *B?* Would this project be undertaken, using 15 percent as the relevant opportunity cost of capital?