Appendix

B Time Value of Money

Learning Objectives

CAP

Conceptual

C1 Describe the earning of interest and the concepts of present and future values. *(p. B-2)*

Procedural

P1 Apply present value concepts to a single amount by using interest tables. *(p. B-3)*

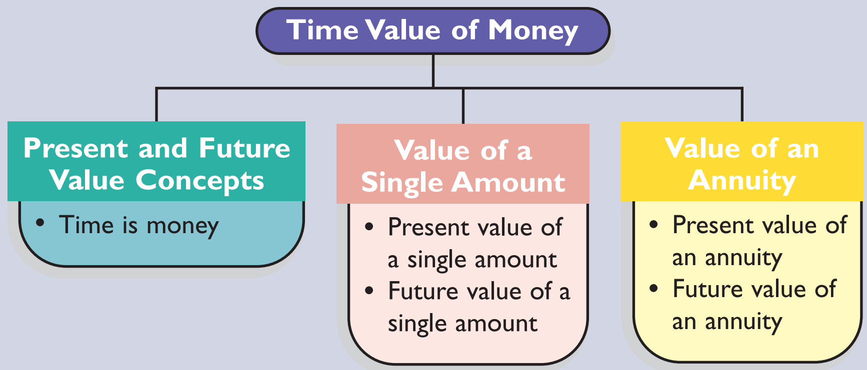
P2 Apply future value concepts to a single amount by using interest tables. *(p. B-5)*

P3 Apply present value concepts to an annuity by using interest tables. *(p. B-6)*

P4 Apply future value concepts to an annuity by using interest tables. *(p. B-7)*

Appendix Preview

The concepts of present and future values are important to modern business activity. The purpose of this appendix is to explain, illustrate, and compute present and future values. This appendix applies these concepts with reference to both business and everyday activities.



Present and Future Value Concepts

C1 Describe the earning of interest and the concepts of present and future values.

The old saying "Time is money" reflects the notion that as time passes, the values of our assets and liabilities change. This change is due to *interest,* which is a borrower's payment to the owner of an asset for its use. The most common example of interest is a savings account asset. As we keep a balance of cash in the account, it earns interest that the financial institution pays us. An example of a liability is a car loan. As we carry the balance of the loan, we accumulate interest costs on it. We must ultimately repay this loan with interest.

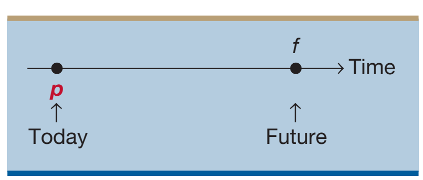
Present and future value computations enable us to measure or estimate the interest component of holding assets or liabilities over time. The present value computation is important when we want to know the value of future-day assets *today.* The future value computation is important when we want to know the value of present-day assets *at a future date.* The first section focuses on the present value of a single amount. The second section focuses on the future value of a single amount. Then both the present and future values of a series of amounts (called an *annuity*) are defined and explained.

Present Value of a Single Amount

We graphically express the present value, called p, of a single future amount, called f, that is received or paid at a future date in Exhibit B.1.

Exhibit B.1

Present Value of a Single Amount Diagram



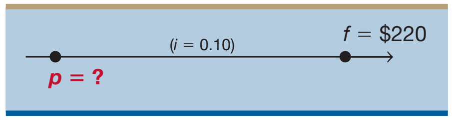
The formula to compute the present value of a single amount is shown in Exhibit B.2, where p = present value; f = future value; i = rate of interest per period; and n = number of periods. (Interest is also called the *discount,* and an interest rate is also called the *discount rate.*)

Exhibit B.2

Present Value of a Single Amount Formula

$$p = \frac{f}{(1 + i)^n}$$

To illustrate present value concepts, assume that we need \$220 one period from today. We want to know how much we must invest now, for one period, at an interest rate of 10% to provide for this \$220. For this illustration, the *p*, or present value, is the unknown amount—the specifics are shown graphically as follows:



Conceptually, we know *p* must be less than \$220. This is obvious from the answer to this question: Would we rather have \$220 today or \$220 at some future date? If we had \$220 today, we could invest it and see it grow to something more than \$220 in the future. Therefore, we would prefer the \$220 today. This means that if we were promised \$220 in the future, we would take less than \$220 today. But how much less? To answer that question, we compute an estimate of the present value of the \$220 to be received one period from now using the formula in Exhibit B.2 as follows:

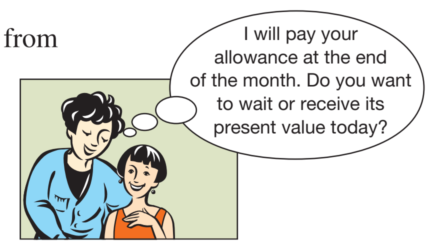
$$p = \frac{f}{(1 + i)^n} = \frac{\$220}{(1 + 0.10)^1} = \$200$$

We interpret this result to say that given an interest rate of 10%, we are indifferent between \$200 today or \$220 at the end of one period.

We can also use this formula to compute the present value for *any number of periods.* To illustrate, consider a payment of \$242 at the end of two periods at 10% interest. The present value of this \$242 to be received two periods from now is computed as follows:

$$p = \frac{f}{(1 + i)^n} = \frac{\$242}{(1 + 0.10)^2} = \$200$$

Together, these results tell us we are indifferent between \$200 today, or \$220 one period from today, or \$242 two periods from today given a 10% interest rate per period.



The number of periods (n) in the present value formula does not have to be expressed in years. Any period of time such as a day, a month, a quarter, or a year can be used. Whatever period is used, the interest rate (i) must be compounded for the same period. This means that if a situation expresses n in months and i equals 12% per year, then i is transformed into interest earned per month (or 1%). In this case, interest is said to be *compounded monthly.*

A present value table helps us with present value computations. It gives us present values (factors) for a variety of both interest rates (i) and periods (n). Each present value in a present value table assumes that the future value (f) equals 1. When the future value (f) is different from 1, we simply multiply the present value (p) from the table by that future value to give us the estimate. The formula used to construct a table of present values for a single future amount of 1 is shown in Exhibit B.3.

$$p = \frac{1}{(1 + i)^n}$$

Exhibit B.3

Present Value of 1 Formula

This formula is identical to that in Exhibit B.2 except that f equals 1. Table B.1 at the end of this appendix is such a present value table. It is often called a **present value of 1 table**. A present value table involves three factors: p, i, and n. Knowing two of these three factors allows us to compute the third. (A fourth is f, but as already explained, we need only multiply the 1 used in the formula by f.) To illustrate the use of a present value table, consider three cases.

P1 Apply present value concepts to a single amount by using interest tables.

Case 1 (solve for p when knowing i and n). To show how we use a present value table, let's look again at how we estimate the present value of $220 (the f value) at the end of one period ($n = 1$) where the interest rate (i) is 10%. To solve this case, we go to the present value table (Table B.1) and look in the row for 1 period and in the column for 10% interest. Here we find a present value (p) of 0.9091 based on a future value of 1. This means, for instance, that $1 to be received one period from today at 10% interest is worth $0.9091 today. Since the future value in this case is not $1 but $220, we multiply the 0.9091 by $220 to get an answer of $200.

Case 2 (solve for n when knowing p and i). To illustrate, assume a $100,000 future value ($f$) that is worth $13,000 today ($p$) using an interest rate of 12% (i) but where n is unknown. In particular, we want to know how many periods (n) there are between the present value and the future value. To put this in context, it would fit a situation in which we want to retire with $100,000 but currently have only $13,000 that is earning a 12% return. How long will it be before we can retire? To answer this, we go to Table B.1 and look in the 12% interest column. Here we find a column of present values (p) based on a future value of 1. To use the present value table for this solution, we must divide $13,000 ($p$) by $100,000 ($f$), which equals 0.1300. This is necessary because *a present value table defines* f *equal to 1, and* p *as a fraction of 1.* We look for a value nearest to 0.1300 (p), which we find in the row for 18 periods (n). This means that the present value of $100,000 at the end of 18 periods at 12% interest is $13,000 or, alternatively stated, we must work 18 more years.

Decision Insight

Keep That Job Lottery winners often never work again. Kenny Dukes, a recent Georgia lottery winner, doesn't have that option. He is serving parole for burglary charges, and Georgia requires its parolees to be employed (or in school). Dukes had to choose between $31 million in 30 annual payments or $16 million in one lump sum ($10.6 million after-tax); he chose the latter.

Case 3 (solve for i when knowing p and n). In this case, we have, say, a $120,000 future value ($f$) worth $60,000 today ($p$) when there are nine periods (n) between the present and future values, but the interest rate is unknown. As an example, suppose we want to retire with $120,000, but we have only $60,000 and hope to retire in nine years. What interest rate must we earn to retire with $120,000 in nine years? To answer this, we go to the present value table (Table B.1) and look in the row for nine periods. To use the present value table, we must divide $60,000 ($p$) by $120,000 ($f$), which equals 0.5000. Recall that this step is necessary because a present value table defines f equal to 1 and p as a fraction of 1. We look for a value in the row for nine periods that is nearest to 0.5000 (p), which we find in the column for 8% interest (i). This means that the present value of $120,000 at the end of nine periods at 8% interest is $60,000 or, in our example, we must earn 8% annual interest to retire in nine years.

Quick Check Answer—p. B-8

1. A company is considering an investment expected to yield $70,000 after six years. If this company demands an 8% return, how much is it willing to pay for this investment?

Future Value of a Single Amount

We must modify the formula for the present value of a single amount to obtain the formula for the future value of a single amount. In particular, we multiply both sides of the equation in Exhibit B.2 by $(1 + i)^n$ to get the result shown in Exhibit B.4.

Exhibit B.4

Future Value of a Single Amount Formula

$$f = p \times (1 + i)^n$$

The future value (f) is defined in terms of p, i, and n. We can use this formula to determine that $200 ($p$) invested for 1 ($n$) period at an interest rate of 10% (i) yields a future value of

$220 as follows:

$$f = p \times (1 + i)^n$$
$$= \$200 \times (1 + 0.10)^1$$
$$= \$220$$

This formula can also be used to compute the future value of an amount for *any number of periods* into the future. To illustrate, assume that $200 is invested for three periods at 10%. The future value of this $200 is $266.20, computed as follows:

$$f = p \times (1 + i)^n$$
$$= \$200 \times (1 + 0.10)^3$$
$$= \$266.20$$

P2 Apply future value concepts to a single amount by using interest tables.

A future value table makes it easier for us to compute future values (f) for many different combinations of interest rates (i) and time periods (n). Each future value in a future value table assumes the present value (p) is 1. As with a present value table, if the future amount is something other than 1, we simply multiply our answer by that amount. The formula used to construct a table of future values (factors) for a single amount of 1 is in Exhibit B.5.

Exhibit B.5

Future Value of 1 Formula

$$f = (1 + i)^n$$

Table B.2 at the end of this appendix shows a table of future values for a current amount of 1. This type of table is called a **future value of 1 table**.

There are some important relations between Tables B.1 and B.2. In Table B.2, for the row where $n = 0$, the future value is 1 for each interest rate. This is so because no interest is earned when time does not pass. Also notice that Tables B.1 and B.2 report the same information but in a different manner. In particular, one table is simply the *inverse* of the other. To illustrate this inverse relation, let's say we invest $100 annually for a period of five years at 12% per year. How much do we expect to have after five years? We can answer this question using Table B.2 by finding the future value (f) of 1, for five periods from now, compounded at 12%. From that table we find $f = 1.7623$. If we start with $100, the amount it accumulates to after five years is $176.23 ($100 × 1.7623). We can alternatively use Table B.1. Here we find that the present value (p) of 1, discounted five periods at 12%, is 0.5674. Recall the inverse relation between present value and future value. This means that $p = 1/f$ (or equivalently, $f = 1/p$). We can compute the future value of $100 invested for five periods at 12% as follows: $f = \$100 \times (1/0.5674) = \$176.24$.

A future value table involves three factors: f, i, and n. Knowing two of these three factors allows us to compute the third. To illustrate, consider these three possible cases.

Case 1 (solve for f when knowing i and n). Our preceding example fits this case. We found that $100 invested for five periods at 12% interest accumulates to $176.24.

Case 2 (solve for n when knowing f and i). In this case, we have, say, $2,000 ($p$) and we want to know how many periods (n) it will take to accumulate to $3,000 ($f$) at 7% ($i$) interest. To answer this, we go to the future value table (Table B.2) and look in the 7% interest column. Here we find a column of future values (f) based on a present value of 1. To use a future value table, we must divide $3,000 ($f$) by $2,000 ($p$), which equals 1.500. This is necessary because *a future value table defines* p *equal to 1, and* f *as a multiple of 1.* We look for a value nearest to 1.50 (f), which we find in the row for six periods (n). This means that $2,000 invested for six periods at 7% interest accumulates to $3,000.

Case 3 (solve for i when knowing f and n). In this case, we have, say, $2,001 ($p$) and in nine years ($n$), we want to have $4,000 ($f$). What rate of interest must we earn to accomplish this? To answer that, we go to Table B.2 and search in the row for nine periods. To use a future value table, we must divide $4,000 ($f$) by $2,001 ($p$), which equals 1.9990. Recall that this is necessary because a future value table defines p equal to 1 and f as a multiple of 1. We look for a value nearest to 1.9990 (f), which we find in the column for 8% interest (i). This means that $2,001 invested for nine periods at 8% interest accumulates to $4,000.

Quick Check Answer—p. B-8

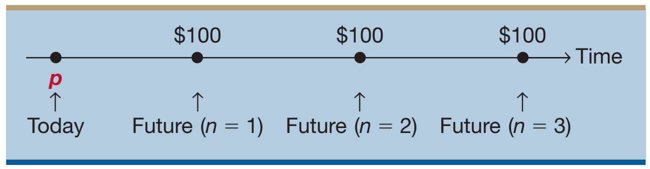
2. Assume that you win a $150,000 cash sweepstakes. You decide to deposit this cash in an account earning 8% annual interest, and you plan to quit your job when the account equals $555,000. How many years will it be before you can quit working?

Present Value of an Annuity

An *annuity* is a series of equal payments occurring at equal intervals. One example is a series of three annual payments of $100 each. An *ordinary annuity* is defined as equal end-of-period payments at equal intervals. An ordinary annuity of $100 for 3 periods and its present value (p) are illustrated in Exhibit B.6.

Exhibit B.6

Present Value of an Ordinary Annuity Diagram



P3 Apply present value concepts to an annuity by using interest tables.

One way to compute the present value of an ordinary annuity is to find the present value of each payment using our present value formula from Exhibit B.3. We then add each of the three present values. To illustrate, let's look at three $100 payments at the end of each of the next three periods with an interest rate of 15%. Our present value computations are

$$p = \frac{\$100}{(1 + 0.15)^1} + \frac{\$100}{(1 + 0.15)^2} + \frac{\$100}{(1 + 0.15)^3} = \$228.32$$

This computation is identical to computing the present value of each payment (from Table B.1) and taking their sum or, alternatively, adding the values from Table B.1 for each of the three payments and multiplying their sum by the $100 annuity payment.

A more direct way is to use a present value of annuity table. Table B.3 at the end of this appendix is one such table. This table is called a **present value of an annuity of 1 table**. If we look at Table B.3 where $n = 3$ and $i = 15\%$, we see the present value is 2.2832. This means that the present value of an annuity of 1 for three periods, with a 15% interest rate, equals 2.2832.

Decision Insight

Aw-Shucks "I don't have good luck—I'm blessed," proclaimed Andrew "Jack" Whittaker, 55, a sewage treatment contractor, after winning the largest-ever, undivided jackpot in a U.S. lottery. Whittaker had to choose between $315 million in 30 annual installments or $170 million in one lump sum ($112 million after-tax). Says Whittaker, "My biggest problem is to keep my daughter and granddaughter from spending all their money in one week."

A present value of an annuity formula is used to construct Table B.3. It can also be constructed by adding the amounts in a present value of 1 table. To illustrate, we use Tables B.1 and B.3 to confirm this relation for the prior example:

From Table B.1		From Table B.3	
$i = 15\%, n = 1$	0.8696		
$i = 15\%, n = 2$	0.7561		
$i = 15\%, n = 3$	0.6575		
Total	2.2832	$i = 15\%, n = 3$	2.2832

We can also use business calculators or spreadsheet programs to find the present value of an annuity.

Quick Check Answer—p. B-8

3. A company is considering an investment paying $10,000 every six months for three years. The first payment would be received in six months. If this company requires an 8% annual return, what is the maximum amount it is willing to pay for this investment?

Future Value of an Annuity

The future value of an *ordinary annuity* is the accumulated value of each annuity payment with interest as of the date of the final payment. To illustrate, let's consider the earlier annuity of three annual payments of $100. Exhibit B.7 shows the point in time for the future value (f). The first payment is made two periods prior to the point when future value is determined, and the final payment occurs on the future value date.

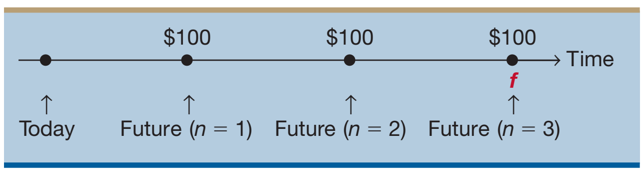


Exhibit B.7

Future Value of an Ordinary Annuity Diagram

One way to compute the future value of an annuity is to use the formula to find the future value of *each* payment and add them. If we assume an interest rate of 15%, our calculation is

$$f = \$100 \times (1 + 0.15)^2 + \$100 \times (1 + 0.15)^1 + \$100 \times (1 + 0.15)^0 = \$347.25$$

This is identical to using Table B.2 and summing the future values of each payment, or by adding the future values of the three payments of 1 and multiplying the sum by $100.

P4 Apply future value concepts to an annuity by using interest tables.

A more direct way is to use a table showing future values of annuities. Such a table is called a **future value of an annuity of 1 table**. Table B.4 at the end of this appendix is one such table. Note that in Table B.4 when $n = 1$, the future values equal 1 ($f = 1$) for all rates of interest. This is so because such an annuity consists of only one payment and the future value is determined on the date of that payment—no time passes between the payment and its future value. The future value of an annuity formula is used to construct Table B.4. We can also construct it by adding the amounts from a future value of 1 table. To illustrate, we use Tables B.2 and B.4 to confirm this relation for the prior example:

From Table B.2		From Table B.4	
i = 15%, n = 0	1.0000		
i = 15%, n = 1	1.1500		
i = 15%, n = 2	1.3225		
Total	3.4725	i = 15%, n = 3	3.4725

Note that the future value in Table B.2 is 1.0000 when $n = 0$, but the future value in Table B.4 is 1.0000 when $n = 1$. Is this a contradiction? No. When $n = 0$ in Table B.2, the future value is determined on the date when a single payment occurs. This means that no interest is earned because no time has passed, and the future value equals the payment. Table B.4 describes annuities with equal payments occurring at the end of each period. When $n = 1$, the annuity has one payment, and its future value equals 1 on the date of its final and only payment. Again, no time passes between the payment and its future value date.

Quick Check Answer—p. B-8

4. A company invests $45,000 per year for five years at 12% annual interest. Compute the value of this annuity investment at the end of five years.

Summary

C1 Describe the earning of interest and the concepts of present and future values. Interest is payment by a borrower to the owner of an asset for its use. Present and future value computations are a way for us to estimate the interest component of holding assets or liabilities over a period of time.

P1 Apply present value concepts to a single amount by using interest tables. The present value of a single amount received at a future date is the amount that can be invested now at the specified interest rate to yield that future value.

P2 Apply future value concepts to a single amount by using interest tables. The future value of a single amount invested at a specified rate of interest is the amount that would accumulate by the future date.

P3 Apply present value concepts to an annuity by using interest tables. The present value of an annuity is the amount that can be invested now at the specified interest rate to yield that series of equal periodic payments.

P4 Apply future value concepts to an annuity by using interest tables. The future value of an annuity invested at a specific rate of interest is the amount that would accumulate by the date of the final payment.

Guidance Answers to Quick Checks

1. $70,000 × 0.6302 = $44,114 (use Table B.1, i = 8%, n = 6).
2. $555,000/$150,000 = 3.7000; Table B.2 shows this value is not achieved until after 17 years at 8% interest.
3. $10,000 × 5.2421 = $52,421 (use Table B.3, i = 4%, n = 6).
4. $45,000 × 6.3528 = $285,876 (use Table B.4, i = 12%, n = 5).

QUICK STUDY

QS B-1
Identifying interest rates in tables
C1

Assume that you must make future value estimates using the *future value of 1 table* (Table B.2). Which interest rate column do you use when working with the following rates?

1. 8% compounded quarterly
2. 12% compounded annually
3. 6% compounded semiannually
4. 12% compounded monthly

QS B-2
Interest rate on an investment P1

Ken Francis is offered the possibility of investing $2,745 today and in return to receive $10,000 after 15 years. What is the annual rate of interest for this investment? (Use Table B.1.)

QS B-3
Number of periods of an investment P1

Megan Brink is offered the possibility of investing $6,651 today at 6% interest per year in a desire to accumulate $10,000. How many years must Brink wait to accumulate $10,000? (Use Table B.1.)

QS B-4
Present value of an amount P1

Flaherty is considering an investment that, if paid for immediately, is expected to return $140,000 five years from now. If Flaherty demands a 9% return, how much is she willing to pay for this investment?

CII, Inc., invests $630,000 in a project expected to earn a 12% annual rate of return. The earnings will be reinvested in the project each year until the entire investment is liquidated 10 years later. What will the cash proceeds be when the project is liquidated?

QS B-5
Future value of an amount P2

Beene Distributing is considering a project that will return $150,000 annually at the end of each year for six years. If Beene demands an annual return of 7% and pays for the project immediately, how much is it willing to pay for the project?

QS B-6
Present value of an annuity P3

Claire Fitch is planning to begin an individual retirement program in which she will invest $1,500 at the end of each year. Fitch plans to retire after making 30 annual investments in the program earning a return of 10%. What is the value of the program on the date of the last payment?

QS B-7
Future value of an annuity P4

EXERCISES

Exercise B-1
Number of periods of an investment P2

Bill Thompson expects to invest $10,000 at 12% and, at the end of a certain period, receive $96,463. How many years will it be before Thompson receives the payment? (Use Table B.2.)

Exercise B-2
Interest rate on an investment P2

Ed Summers expects to invest $10,000 for 25 years, after which he wants to receive $108,347. What rate of interest must Summers earn? (Use Table B.2.)

Exercise B-3
Interest rate on an investment P3

Jones expects an immediate investment of $57,466 to return $10,000 annually for eight years, with the first payment to be received one year from now. What rate of interest must Jones earn? (Use Table B.3.)

Exercise B-4
Number of periods of an investment P3

Keith Riggins expects an investment of $82,014 to return $10,000 annually for several years. If Riggins earns a return of 10%, how many annual payments will he receive? (Use Table B.3.)

Exercise B-5
Interest rate on an investment P4

Algoe expects to invest $1,000 annually for 40 years to yield an accumulated value of $154,762 on the date of the last investment. For this to occur, what rate of interest must Algoe earn? (Use Table B.4.)

Exercise B-6
Number of periods of an investment P4

Kate Beckwith expects to invest $10,000 annually that will earn 8%. How many annual investments must Beckwith make to accumulate $303,243 on the date of the last investment? (Use Table B.4.)

Exercise B-7
Present value of an annuity P3

Sam Weber finances a new automobile by paying $6,500 cash and agreeing to make 40 monthly payments of $500 each, the first payment to be made one month after the purchase. The loan bears interest at an annual rate of 12%. What is the cost of the automobile?

Exercise B-8
Present value of bonds P1 P3

Spiller Corp. plans to issue 10%, 15-year, $500,000 par value bonds payable that pay interest semiannually on June 30 and December 31. The bonds are dated December 31, 2008, and are issued on that date. If the market rate of interest for the bonds is 8% on the date of issue, what will be the total cash proceeds from the bond issue?

Exercise B-9
Present value of an amount P1

McAdams Company expects to earn 10% per year on an investment that will pay $606,773 six years from now. Use Table B.1 to compute the present value of this investment.

Exercise B-10
Present value of an amount and of an annuity P1 P3

Compute the amount that can be borrowed under each of the following circumstances:

1. A promise to repay $90,000 seven years from now at an interest rate of 6%.
2. An agreement made on February 1, 2008, to make three separate payments of $20,000 on February 1 of 2009, 2010, and 2011. The annual interest rate is 10%.

Exercise B-11
Present value of an amount P1

On January 1, 2008, a company agrees to pay $20,000 in three years. If the annual interest rate is 10%, determine how much cash the company can borrow with this agreement.

Exercise B-12
Present value of an amount P1

Find the amount of money that can be borrowed today with each of the following separate debt agreements *a* through *f*:

Case	Single Future Payment	Number of Periods	Interest Rate
a.	$40,000	3	4%
b.	75,000	7	8
c.	52,000	9	10
d.	18,000	2	4
e.	63,000	8	6
f.	89,000	5	2

Exercise B-13
Present values of annuities
P3

C&H Ski Club recently borrowed money and agrees to pay it back with a series of six annual payments of $5,000 each. C&H subsequently borrows more money and agrees to pay it back with a series of four annual payments of $7,500 each. The annual interest rate for both loans is 6%.

1. Use Table B.1 to find the present value of these two separate annuities. (Round amounts to the nearest dollar.)
2. Use Table B.3 to find the present value of these two separate annuities.

Exercise B-14
Present value with semiannual compounding
C1 P3

Otto Co. borrows money on April 30, 2008, by promising to make four payments of $13,000 each on November 1, 2008; May 1, 2009; November 1, 2009; and May 1, 2010.

1. How much money is Otto able to borrow if the interest rate is 8%, compounded semiannually?
2. How much money is Otto able to borrow if the interest rate is 12%, compounded semiannually?
3. How much money is Otto able to borrow if the interest rate is 16%, compounded semiannually?

Exercise B-15
Future value of an amount P2

Mark Welsch deposits $7,200 in an account that earns interest at an annual rate of 8%, compounded quarterly. The $7,200 plus earned interest must remain in the account 10 years before it can be withdrawn. How much money will be in the account at the end of 10 years?

Exercise B-16
Future value of an annuity P4

Kelly Malone plans to have $50 withheld from her monthly paycheck and deposited in a savings account that earns 12% annually, compounded monthly. If Malone continues with her plan for two and one-half years, how much will be accumulated in the account on the date of the last deposit?

Exercise B-17
Future value of an amount plus an annuity P2 P4

Starr Company decides to establish a fund that it will use 10 years from now to replace an aging production facility. The company will make a $100,000 initial contribution to the fund and plans to make quarterly contributions of $50,000 beginning in three months. The fund earns 12%, compounded quarterly. What will be the value of the fund 10 years from now?

Exercise B-18
Future value of an amount P2

Catten, Inc., invests $163,170 today earning 7% per year for nine years. Use Table B.2 to compute the future value of the investment nine years from now.

Exercise B-19
Using present and future value tables
C1 P1 P2 P3 P4

For each of the following situations, identify (1) the case as either (*a*) a present or a future value and (*b*) a single amount or an annuity, (2) the table you would use in your computations (but do not solve the problem), and (3) the interest rate and time periods you would use.

a. You need to accumulate $10,000 for a trip you wish to take in four years. You are able to earn 8% compounded semiannually on your savings. You plan to make only one deposit and let the money accumulate for four years. How would you determine the amount of the one-time deposit?

b. Assume the same facts as in part (*a*) except that you will make semiannual deposits to your savings account.

c. You want to retire after working 40 years with savings in excess of $1,000,000. You expect to save $4,000 a year for 40 years and earn an annual rate of interest of 8%. Will you be able to retire with more than $1,000,000 in 40 years? Explain.

d. A sweepstakes agency names you a grand prize winner. You can take $225,000 immediately or elect to receive annual installments of $30,000 for 20 years. You can earn 10% annually on any investments you make. Which prize do you choose to receive?

Table B.1

Present Value of 1

$$p = 1/(1 + i)^n$$

Periods	Rate 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.8929	0.8696
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.7972	0.7561
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7118	0.6575
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6355	0.5718
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5674	0.4972
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5066	0.4323
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4523	0.3759
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4039	0.3269
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3606	0.2843
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3220	0.2472
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.2875	0.2149
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2567	0.1869
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2292	0.1625
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2046	0.1413
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.1827	0.1229
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1631	0.1069
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1456	0.0929
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1300	0.0808
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1161	0.0703
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1037	0.0611
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0588	0.0304
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0334	0.0151
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356	0.0189	0.0075
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0107	0.0037

Table B.2

Future Value of 1

$$f = (1 + i)^n$$

Periods	Rate 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1200	1.1500
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2544	1.3225
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.4049	1.5209
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5735	1.7490
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.7623	2.0114
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.9738	2.3131
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.2107	2.6600
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.4760	3.0590
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.7731	3.5179
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	3.1058	4.0456
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.4785	4.6524
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.8960	5.3503
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	4.3635	6.1528
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.8871	7.0757
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	5.4736	8.1371
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	6.1304	9.3576
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	6.8660	10.7613
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	7.6900	12.3755
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	8.6128	14.2318
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	9.6463	16.3665
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347	17.0001	32.9190
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494	29.9599	66.2118
35	1.4166	1.9999	2.8139	3.9461	5.5160	7.6861	10.6766	14.7853	20.4140	28.1024	52.7996	133.1755
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.2857	14.9745	21.7245	31.4094	45.2593	93.0510	267.8635

$$p = \left[1 - \frac{1}{(1+i)^n}\right] / i$$

Table B.3

Present Value of an Annuity of 1

Periods	Rate 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.8929	0.8696
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.6901	1.6257
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4018	2.2832
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.0373	2.8550
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6048	3.3522
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.1114	3.7845
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.5638	4.1604
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	4.9676	4.4873
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.3282	4.7716
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.6502	5.0188
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	5.9377	5.2337
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.1944	5.4206
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.4235	5.5831
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.6282	5.7245
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	6.8109	5.8474
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	6.9740	5.9542
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.1196	6.0472
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.2497	6.1280
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.3658	6.1982
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.4694	6.2593
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770	7.8431	6.4641
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269	8.0552	6.5660
35	29.4086	24.9986	21.4872	18.6646	16.3742	14.4982	12.9477	11.6546	10.5668	9.6442	8.1755	6.6166
40	32.8347	27.3555	23.1148	19.7928	17.1591	15.0463	13.3317	11.9246	10.7574	9.7791	8.2438	6.6418

$$f = [(1+i)^n - 1] / i$$

Table B.4

Future Value of an Annuity of 1

Periods	Rate 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	15%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1200	2.1500
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3744	3.4725
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7793	4.9934
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.3528	6.7424
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	8.1152	8.7537
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	10.0890	11.0668
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	12.2997	13.7268
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.7757	16.7858
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	17.5487	20.3037
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	20.6546	24.3493
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	24.1331	29.0017
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	28.0291	34.3519
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	32.3926	40.5047
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	37.2797	47.5804
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	42.7533	55.7175
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	48.8837	65.0751
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	55.7497	75.8364
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	63.4397	88.2118
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	72.0524	102.4436
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471	133.3339	212.7930
30	34.7849	40.5681	47.5754	56.0849	66.4388	79.0582	94.4608	113.2832	136.3075	164.4940	241.3327	434.7451
35	41.6603	49.9945	60.4621	73.6522	90.3203	111.4348	138.2369	172.3168	215.7108	271.0244	431.6635	881.1702
40	48.8864	60.4020	75.4013	95.0255	120.7998	154.7620	199.6351	259.0565	337.8824	442.5926	767.0914	1,779.0903