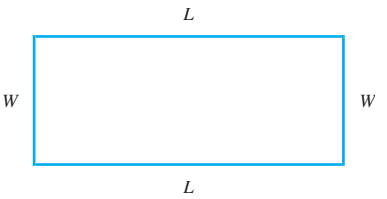
2.6 Introduction to Algebra: Variables and Expressions

2.6 OBJECTIVES

1. Represent addition, subtraction, multiplication, and division by using the symbols of algebra
2. Identify algebraic expressions

In arithmetic, you learned how to do calculations with numbers by using the basic operations of addition, subtraction, multiplication, and division.

In algebra, you will still use numbers and the same four operations. However, you will also use letters to represent numbers. Letters such as x, y, L, or W are called **variables** when they can represent different numerical values. If we need to represent the length and width of *any* rectangle, we can use the variables L and W.



RECALL In arithmetic:
+ denotes addition
− denotes subtraction
· denotes multiplication
÷ denotes division.

You are familiar with the four symbols (+, −, ·, ÷) used to indicate the fundamental operations of arithmetic.

Next, we will look at how these operations are indicated in algebra. We begin by looking at addition.

OBJECTIVE 1

Example 1 Writing Expressions that Indicate Addition

(a) The *sum* of a and 3 is written as $a + 3$.
(b) L plus W is written as $L + W$.
(c) 5 more than m is written as $m + 5$.
(d) x increased by 7 is written as $x + 7$.

CHECK YOURSELF 1

Write, using symbols.

(a) The sum of y and 4
(b) a plus b
(c) 3 more than x
(d) n increased by 6

In Example 2, we look at how subtraction is indicated in algebra.

Example 2 Writing Expressions that Indicate Subtraction

(a) r minus s is written as $r - s$.
(b) The *difference* of m and 5 is written as $m - 5$.
(c) x decreased by 8 is written as $x - 8$.
(d) 4 less than a is written as $a - 4$.

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CHECK YOURSELF 2

Write, using symbols.

(a) w minus z
(b) The difference of a and 7
(c) y decreased by 3
(d) 5 less than b

You have seen that the operations of addition and subtraction are written exactly the same way in algebra as in arithmetic. This is not true in multiplication because the sign $\times$ looks like the letter x. So in algebra we use other symbols to show multiplication to avoid any confusion. Here are some ways to write multiplication.

Definition: Multiplication

A centered dot	$x \cdot y$	These all indicate the *product* of x and y or x times y.
Parentheses	$(x)(y)$	
Writing the letters next to each other	xy	

NOTE x and y are called the **factors** of the product xy.

Example 3 Writing Expressions that Indicate Multiplication

(a) The product of 5 and a is written as $5 \cdot a$, $(5)(a)$, or $5a$. The last expression, $5a$, is the shortest and the most common way of writing the product.
(b) 3 times 7 can be written as $3 \cdot 7$, $(3)(7)$, or $3(7)$.
(c) Twice z is written as $2z$.
(d) The product of 2, s, and t is written as $2st$.
(e) 4 more than the product of 6 and x is written as $6x + 4$.

NOTE You can place letters next to each other or numbers and letters next to each other to show multiplication. But you *cannot* place numbers side by side to show multiplication: 37 means the number "thirty-seven," not 3 times 7.

CHECK YOURSELF 3

Write, using symbols.

(a) m times n
(b) The product of h and b
(c) The product of 8 and 9
(d) The product of 5, w, and y
(e) 3 more than the product of 8 and a

Before we move on to division, we look at how we can combine the symbols we have learned so far.

Definition: Expression

An **expression** is a meaningful collection of numbers, variables, and signs of operation.

NOTE Not every collection of symbols is an expression.

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OBJECTIVE 2

Example 4 Identifying Expressions

(a) $2m + 3$ is an expression. It means that we multiply 2 and m and then add 3.
(b) $x + \cdot + 3$ is not an expression. The three operations in a row have no meaning.
(c) $y = 2x - 1$ is not an expression. The equals sign is not an operation sign.
(d) $3a + 5b - 4c$ is an expression. Its meaning is clear.

CHECK YOURSELF 4

Identify which are expressions and which are not.

(a) $7 - \cdot x$
(b) $6 + y = 9$
(c) $a + b - c$
(d) $3x - 5yz$

To write more complicated products in algebra, we need to use grouping symbols. Parentheses ( ) mean that an expression is to be thought of as a single quantity. Brackets [ ] are used in exactly the same way as parentheses in algebra. Look at Example 5, which shows the use of these signs of grouping.

Example 5 Expressions with More than One Operation

(a) 3 times the sum of a and b is written as

NOTE This can be read as "3 times the quantity *a* plus *b*."

$$3(a + b)$$

The sum of a and b is a single quantity, so it is enclosed in parentheses.

NOTE No parentheses are used in part (b) because the 3 multiplies *only* the *a*.

(b) The sum of 3 times a and b is written as $3a + b$.
(c) 2 times the difference of m and n is written as $2(m - n)$.
(d) The product of s plus t and s minus t is written as $(s + t)(s - t)$.
(e) The product of b and 3 less than b is written as $b(b - 3)$.

CHECK YOURSELF 5

Write, using symbols.

(a) Twice the sum of p and q
(b) The sum of twice p and q
(c) The product of a and the quantity $b - c$
(d) The product of x plus 2 and x minus 2
(e) The product of x and 4 more than x

NOTE In algebra, the fraction form is usually used.

Now we look at the operation of division.

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Example 6 Writing Expressions that Indicate Division

(a) m divided by 3 is written as $\frac{m}{3}$.

(b) The quotient of a plus b and 5 is written as $\frac{a+b}{5}$.

(c) The sum p plus q divided by the difference p minus q is written as $\frac{p+q}{p-q}$.

CHECK YOURSELF 6

Write, using symbols.

(a) r divided by s

(b) The quotient when x minus y is divided by 7

(c) The difference a minus 2 divided by the sum a plus 2

Notice that we can use many different letters to represent variables. In Example 6, the letters m, a, b, p, and q represented different variables. We often choose a letter that reminds us of what it represents, for example, L for *length* or W for *width*.

Example 7 Writing Geometric Expressions

(a) *Length* times *width* is written LW.

(b) One-half of *altitude* times *base* is written $\frac{1}{2}ab$.

(c) *Length* times *width* times *height* is written LWH.

(d) Pi (π) times *diameter* is written πd.

CHECK YOURSELF 7

Write each geometric expression, using symbols.

(a) Two times *length* plus two times *width*

(b) Two times pi (π) times *radius*

READING YOUR TEXT

The following fill-in-the-blank exercises are designed to assure that you understand the key vocabulary used in this section. Each sentence comes directly from the section. You will find the correct answers in Appendix C.

Section 2.6

(a) Letters are called _____ when they can represent different numerical values.

(b) An _____ is a meaningful collection of numbers, variables, and signs of operations.

(c) A raised dot between two letters, x and y, indicates the _____ of x and y.

(d) $y = 2x - 1$ is not an expression because _____ is not an operation sign.

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CHECK YOURSELF ANSWERS

1. **(a)** $y + 4$; **(b)** $a + b$; **(c)** $x + 3$; **(d)** $n + 6$ **2.** **(a)** $w - z$; **(b)** $a - 7$; **(c)** $y - 3$; **(d)** $b - 5$ **3.** **(a)** mn; **(b)** hb; **(c)** $8 \cdot 9$ or $(8)(9)$; **(d)** $5wy$; **(e)** $8a + 3$

4. **(a)** Not an expression; **(b)** not an expression; **(c)** an expression; **(d)** an expression

5. **(a)** $2(p + q)$; **(b)** $2p + q$; **(c)** $a(b - c)$; **(d)** $(x + 2)(x - 2)$; **(e)** $x(x + 4)$

6. **(a)** $\frac{r}{s}$; **(b)** $\frac{x - y}{7}$; **(c)** $\frac{a - 2}{a + 2}$ **7.** **(a)** $2L + 2W$; **(b)** $2\pi r$

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2.6 Exercises

[Objective 1]

Write each of the phrases, using symbols.

1. The sum of c and d

2. a plus 7

3. w plus z

4. The sum of m and n

5. x increased by 2

6. 3 more than b

7. 10 more than y VIDEO

8. m increased by 4

9. a minus b

10. 5 less than s

11. b decreased by 7

12. r minus 3

13. 6 less than r

14. x decreased by 3

15. w times z

16. The product of 3 and c

17. The product of 5 and t

18. 8 times a

19. The product of 8, m, and n

20. The product of 7, r, and s

21. The product of 3 and the quantity p plus q VIDEO

22. The product of 5 and the sum of a and b

23. Twice the sum of x and y

24. 3 times the sum of m and n

25. The sum of twice x and y

26. The sum of 3 times m and n

27. Twice the difference of x and y

28. 3 times the difference of c and d

29. The quantity a plus b times the quantity a minus b

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Name _____

Section _____ Date _____

ANSWERS

1. $c + d$
2. $a + 7$
3. $w + z$
4. $m + n$
5. $x + 2$
6. $b + 3$
7. $y + 10$
8. $m + 4$
9. $a - b$
10. $s - 5$
11. $b - 7$
12. $r - 3$
13. $r - 6$
14. $x - 3$
15. wz
16. $3c$
17. $5t$
18. $8a$
19. $8mn$
20. $7rs$
21. $3(p + q)$
22. $5(a + b)$
23. $2(x + y)$
24. $3(m + n)$
25. $2x + y$
26. $3m + n$
27. $2(x - y)$
28. $3(c - d)$
29. $(a + b)(a - b)$

ANSWERS

30. $(x + y)(x - y)$

31. $m(m - 3)$

32. $a(a + 7)$

33. $\frac{x}{5}$

34. $\frac{b}{8}$

35. $\frac{a + b}{7}$

36. $\frac{x - y}{9}$

37. $\frac{p - q}{4}$

38. $\frac{a + 5}{9}$

39. $\frac{a + 3}{a - 3}$

40. $\frac{m - n}{m + n}$

41. $x + 5$

42. $x + 8$

43. $x - 7$

44. $x - 10$

45. $9x$

46. $2x$

47. $3x + 6$

48. $5x - 10$

49. $2(x + 5)$

50. $3(x - 4)$

30. The product of x plus y and x minus y

31. The product of m and 3 less than m VIDEO

32. The product of a and 7 more than a

33. x divided by 5

34. The quotient when b is divided by 8

35. The quotient of a plus b, and 7

36. The difference x minus y, divided by 9

37. The difference of p and q, divided by 4 VIDEO

38. The sum of a and 5, divided by 9

39. The sum of a and 3, divided by the difference of a and 3

40. The difference of m and n, divided by the sum of m and n

Write each of the phrases, using symbols. Use the variable x to represent the unknown number in each case.

41. 5 more than a number

42. A number increased by 8

43. 7 less than a number

44. A number decreased by 10

45. 9 times a number

46. Twice a number

47. 6 more than 3 times a number

48. 5 times a number, decreased by 10

49. Twice the sum of a number and 5

50. 3 times the difference of a number and 4

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51. The product of 2 more than a number and 2 less than that same number

52. The product of 5 less than a number and 5 more than that same number

53. The quotient of a number and 7

54. A number divided by 3

55. The sum of a number and 5, divided by 8

56. The quotient when 7 less than a number is divided by 3

57. 6 more than a number divided by 6 less than that same number

58. The quotient when 3 less than a number is divided by 3 more than that same number

Write each of the following geometric expressions using symbols.

59. Four times the length of a side (s)

60. $\frac{4}{3}$ times π times the cube of the radius (r)

61. The radius (r) squared times the height (h) times π

62. Twice the length (L) plus twice the width (W)

63. One-half the product of the height (h) and the sum of two unequal sides (b_1 and b_2)

64. Six times the length of a side (s) squared

[Objective 2]

Identify which are expressions and which are not.

65. $2(x + 5)$

66. $4 + (x - 3)$

67. $4 + \div m$

68. $6 + a = 7$

69. $2b = 6$

70. $x(y + 3)$

71. $2a + 5b$

72. $4x + \cdot 7$

ANSWERS

51. $(x + 2)(x - 2)$

52. $(x - 5)(x + 5)$

53. $\frac{x}{7}$

54. $\frac{x}{3}$

55. $\frac{x + 5}{8}$

56. $\frac{x - 7}{3}$

57. $\frac{x + 6}{x - 6}$

58. $\frac{x - 3}{x + 3}$

59. $4s$

60. $\frac{4}{3}\pi r^3$

61. $\pi r^2 h$

62. $2L + 2W$

63. $\frac{1}{2}h(b_1 + b_2)$

64. $6s^2$

65. Expression

66. Expression

67. Not an expression

68. Not an expression

69. Not an expression

70. Expression

71. Expression

72. Not an expression

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ANSWERS

73. $2x$

74. $S - 4{,}000$

75. $I = Prt$

76. $KE = \frac{1}{2}mv^2$

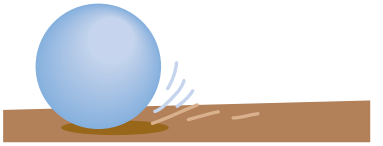
77.

73. Social Science Earth's population has doubled in the last 40 years. If we let x represent Earth's population 40 years ago, what is the population today? ALEKS

74. Science and Medicine It is estimated that Earth is losing 4,000 species of plants and animals every year. If S represents the number of species living last year, how many species are on Earth this year? ALEKS

75. Business and Finance The simple interest (I) earned when a principal (P) is invested at a rate (r) for a time (t) is calculated by multiplying the principal times the rate times the time. Write a formula for the interest earned.

76. Science and Medicine The kinetic energy (KE) of a particle of mass m is found by taking one-half of the product of the mass and the square of the velocity (v). Write a formula for the kinetic energy of a particle.



77. Rewrite each algebraic expression as an English phrase. Exchange papers with another student to edit your writing. Be sure the meaning in English is the same as in algebra. These expressions are not complete sentences, so your English does not have to be in complete sentences. Here is an example.

Algebra: $2(x - 1)$

English: We could write, "One less than a number is doubled." Or we might write, "A number is diminished by one and then multiplied by two."

(a) $n + 3$ **(b)** $\frac{x + 2}{5}$ **(c)** $3(5 + a)$ **(d)** $3 - 4n$ **(e)** $\frac{x + 6}{x - 1}$

Answers

1. $c + d$ **3.** $w + z$ **5.** $x + 2$ **7.** $y + 10$ **9.** $a - b$
11. $b - 7$ **13.** $r - 6$ **15.** wz **17.** $5t$ **19.** $8mn$ **21.** $3(p + q)$
23. $2(x + y)$ **25.** $2x + y$ **27.** $2(x - y)$ **29.** $(a + b)(a - b)$
31. $m(m - 3)$ **33.** $\frac{x}{5}$ **35.** $\frac{a + b}{7}$ **37.** $\frac{p - q}{4}$ **39.** $\frac{a + 3}{a - 3}$
41. $x + 5$ **43.** $x - 7$ **45.** $9x$ **47.** $3x + 6$ **49.** $2(x + 5)$
51. $(x + 2)(x - 2)$ **53.** $\frac{x}{7}$ **55.** $\frac{x + 5}{8}$ **57.** $\frac{x + 6}{x - 6}$
59. $4s$ **61.** $\pi r^2 h$ **63.** $\frac{1}{2}h(b_1 + b_2)$ **65.** Expression
67. Not an expression **69.** Not an expression **71.** Expression
73. $2x$ **75.** $I = Prt$ **77.**

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2.7 Evaluating Algebraic Expressions

2.7 OBJECTIVES

1. Substitute integer values for variables in an expression and evaluate
2. Interpret summation notation

In applying algebra to problem solving, you will often want to find the value of an algebraic expression when you know certain values for the letters (or variables) in the expression. Finding the value of an expression is called *evaluating the expression* and uses the following steps.

Step by Step: To Evaluate an Algebraic Expression

Step 1 Replace each variable by the given number value.

Step 2 Do the necessary arithmetic operations, following the rules for order of operations.

OBJECTIVE 1

Example 1 Evaluating Algebraic Expressions

Suppose that $a = 5$ and $b = 7$.

(a) To evaluate $a + b$, we replace a with 5 and b with 7.

$$a + b = (5) + (7) = 12$$

NOTE In this text, we will usually show such replacement using parentheses to hold the value of the variable.

(b) To evaluate $3ab$, we again replace a with 5 and b with 7.

$$3ab = 3(5)(7) = 105$$

CHECK YOURSELF 1

If $x = 6$ and $y = 7$, evaluate.

(a) $y - x$ **(b)** $5xy$

We are now ready to evaluate algebraic expressions that require following the rules for the order of operations.

Example 2 Evaluating Algebraic Expressions

Evaluate the expressions if $a = 2$, $b = 3$, $c = 4$, and $d = 5$.

(a) $5a + 7b = 5(2) + 7(3)$ Multiply first.

$= 10 + 21 = 31$ Then add.

CAUTION

This is different from
$(3c)^2 = [3 \cdot (4)]^2$
$= 12^2 = 144$

(b) $3c^2 = 3(4)^2$ Evaluate the power.

$= 3 \cdot 16 = 48$ Then multiply.

(c) $7(c + d) = 7[(4) + (5)]$ Add inside the grouping symbols.

$= 7 \cdot 9 = 63$

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(d) $5a^4 - 2d^2 = 5(2)^4 - 2(5)^2$ Evaluate the powers.
$= 5 \cdot 16 - 2 \cdot 25$ Multiply.
$= 80 - 50 = 30$ Subtract.

NOTE The parentheses attach the negative sign to the number **before** it is raised to a power.

(e) $(-a)^2 = (-2)^2$
$= (-2) \cdot (-2)$
$= 4$

(f) $-a^2 = -(2)^2$ Evaluate the powers.
$= -(2 \cdot 2)$
$= -4$

CHECK YOURSELF 2

If $x = 3$, $y = 2$, $z = 4$, and $w = 5$, evaluate the expressions.

(a) $4x^2 + 2$ **(b)** $5(z + w)$ **(c)** $7(z^2 - y^2)$
(d) $-x^2$ **(e)** $(-x)^2$

To evaluate algebraic expressions when a fraction bar is used, do the following: Start by doing all the work in the numerator and then do the work in the denominator. Divide the numerator by the denominator as the last step.

Example 3 Evaluating Algebraic Expressions

If $p = 2$, $q = 3$, and $r = 4$, evaluate:

(a) $\dfrac{8p}{r}$

RECALL In Section 2.5, we mentioned that the fraction bar is a grouping symbol, like parentheses. Work first in the numerator and then in the denominator.

Replace p with 2 and r with 4.

$\dfrac{8p}{r} = \dfrac{8 \cdot (2)}{(4)} = \dfrac{16}{4} = 4$ Divide as the last step.

(b) $\dfrac{7q + r}{p + q} = \dfrac{7(3) + (4)}{(2) + (3)}$ Now evaluate the top and bottom separately.

$= \dfrac{21 + 4}{2 + 3} = \dfrac{25}{5} = 5$

CHECK YOURSELF 3

Evaluate the expressions if $c = 5$, $d = 8$, and $e = 3$.

(a) $\dfrac{6c}{e}$ **(b)** $\dfrac{4d + e}{c}$ **(c)** $\dfrac{10d - e}{d + e}$

Example 4 shows how a scientific calculator can be used to evaluate algebraic expressions.

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Example 4 Using a Calculator to Evaluate Expressions

Use a scientific calculator to evaluate the expressions.

(a) $\dfrac{4x + y}{z}$ if $x = 2$, $y = 1$, and $z = 3$

NOTE You must enclose the entire numerator in parentheses.

Replace x with 2, y with 1, and z with 3:

$$\frac{4x + y}{z} = \frac{4(2) + (1)}{(3)}$$

Now, use the following keystrokes:

(4 × 2 + 1) ÷ 3 =

The display will read 3.

NOTE Both, the numerator and the denominator, must be enclosed in parentheses.

(b) $\dfrac{7x - y}{3z - x}$ if $x = 2$, $y = 6$, and $z = 2$

$$\frac{7x - y}{3z - x} = \frac{7(2) - (6)}{3(2) - (2)}$$

Use the following keystrokes:

(7 × 2 − 6) ÷ (3 × 2 − 2) =

The display will read 2.

CHECK YOURSELF 4

Use a scientific calculator to evaluate the expressions if $x = 2$, $y = 6$, and $z = 5$.

(a) $\dfrac{2x + y}{z}$ **(b)** $\dfrac{4y - 2z}{x}$

Example 5 Evaluating Expressions

Evaluate $5a + 4b$ if $a = -2$ and $b = 3$.

Replace a with -2 and b with 3.

NOTE Remember the rules for the order of operations. Multiply first, then add.

$$\begin{aligned} 5a + 4b &= 5(-2) + 4(3) \\ &= -10 + 12 \\ &= 2 \end{aligned}$$

CHECK YOURSELF 5

Evaluate $3x + 5y$ if $x = -2$ and $y = -5$.

We follow the same rules no matter how many variables are in the expression.

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Example 6 Evaluating Expressions

Evaluate the expressions if $a = -4$, $b = 2$, $c = -5$, and $d = 6$.

(a) $7a - 4c = 7(-4) - 4(-5)$ — This becomes $-(-20)$, or $+20$.

$= -28 + 20$

$= -8$

(b) $7c^2 = 7(-5)^2 = 7 \cdot 25$ — Evaluate the power first, then multiply by 7.

$= 175$

(c) $b^2 - 4ac = (2)^2 - 4(-4)(-5)$

$= 4 - 4(-4)(-5)$

$= 4 - 80$

$= -76$

(d) $b(a + d) = 2[(-4) + (6)]$ — Add inside the brackets first.

$= 2(2)$

$= 4$

CAUTION

When a squared variable is replaced by a negative number, square the negative.

$(-5)^2 = (-5)(-5) = 25$

The exponent applies to -5!

$-5^2 = -(5 \cdot 5) = -25$

The exponent applies only to 5!

CHECK YOURSELF 6

Evaluate the expressions if $p = -4$, $q = 3$, and $r = -2$.

(a) $5p - 3r$ **(b)** $2p^2 + q$ **(c)** $p(q + r)$

(d) $-q^2$ **(e)** $(-q)^2$

As mentioned earlier, the fraction bar is a grouping symbol. Example 7 further illustrates this concept.

Example 7 Evaluating Expressions

Evaluate the expressions if $x = 4$, $y = -5$, $z = 2$, and $w = -3$.

(a) $\dfrac{z - 2y}{x} = \dfrac{(2) - 2(-5)}{(4)}$

$= \dfrac{2 - (-10)}{4}$

$= \dfrac{12}{4} = 3$

(b) $\dfrac{3x - w}{2x + w} = \dfrac{3(4) - (-3)}{2(4) + (-3)} = \dfrac{12 + 3}{8 + (-3)}$

$= \dfrac{15}{5} = 3$

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CHECK YOURSELF 7

Evaluate the expressions if $m = -6$, $n = 4$, and $p = -3$.

(a) $\dfrac{m + 3n}{p}$ **(b)** $\dfrac{4m + n}{m + 4n}$

When an expression is evaluated by a calculator, the same order of operations that we introduced in Section 1.8 is followed.

	Algebraic Notation	Calculator Notation
Addition	$6 + 2$	6 [+] 2
Subtraction	$4 - 8$	4 [−] 8
Multiplication	$(3)(-5)$	3 [×] [(−)] 5 or 3 [×] 5 [+/−]
Division	$\dfrac{8}{6}$	8 [÷] 6
Exponential	3^4	3 [^] 4 or 3 [y^x] 4

In many applications, you need to find the sum of a group of numbers you are working with. In mathematics, the shorthand symbol for "sum of" is the Greek letter Σ (capital sigma, the "S" of the Greek alphabet). The expression Σx, in which x refers to all the numbers in a given group, means the sum of all the numbers in that group.

OBJECTIVE 2

Example 8 Summing a Group of Integers

Find Σx for the group of integers.

$-2, -6, 3, 5, -4$

$$\begin{aligned}\Sigma x &= -2 + (-6) + 3 + 5 + (-4)\\ &= (-8) + 3 + 5 + (-4)\\ &= (-8) + 8 + (-4)\\ &= -4\end{aligned}$$

CHECK YOURSELF 8

Find Σx for each group of integers.

(a) $-3, 4, -7, -9, 8$ **(b)** $-2, 6, -5, -3, 4, 7$

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READING YOUR TEXT

The following fill-in-the-blank exercises are designed to assure that you understand the key vocabulary used in this section. Each sentence comes directly from the section. You will find the correct answers in Appendix C.

Section 2.7

(a) To evaluate an algebraic expression, first replace each _____ by the given number value.

(b) The _____ bar is a grouping symbol.

(c) When a squared variable is replaced with a negative number, square the _____.

(d) In mathematics, we use the _____ letter Σ as shorthand for "sum of."

CHECK YOURSELF ANSWERS

1. **(a)** 1; **(b)** 210 **2.** **(a)** 38; **(b)** 45; **(c)** 84; **(d)** -9; **(e)** 9 **3.** **(a)** 10; **(b)** 7; **(c)** 7
4. **(a)** 2; **(b)** 7 **5.** -31 **6.** **(a)** -14; **(b)** 35; **(c)** -4; **(d)** -9; **(e)** 9
7. **(a)** -2; **(b)** -2 **8.** **(a)** -7; **(b)** 7

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2.7 Exercises

[Objective 1]

Evaluate each of the expressions if $a = -2$, $b = 5$, $c = -4$, and $d = 6$.

1. $3c - 2b$ VIDEO ALEKS
2. $4c - 2b$ ALEKS
3. $8b + 2c$ ALEKS
4. $7a - 2c$ ALEKS
5. $-b^2 + b$ VIDEO
6. $(-b)^2 + b$
7. $3a^2$ VIDEO
8. $6c^2$
9. $c^2 - 2d$
10. $3a^2 + 4c$
11. $2a^2 + 3b^2$
12. $4b^2 - 2c^2$
13. $2(a + b)$
14. $5(b - c)$
15. $4(2a - d)$
16. $6(3c - d)$
17. $a(b + 3c)$ VIDEO
18. $c(3a - d)$
19. $\frac{6d}{c}$ ALEKS
20. $\frac{8b}{5c}$ ALEKS
21. $\frac{3d + 2c}{b}$ ALEKS
22. $\frac{2b + 3d}{2a}$ ALEKS
23. $\frac{2b - 3a}{c + 2d}$ ALEKS
24. $\frac{3d - 2b}{5a + d}$ ALEKS
25. $d^2 - b^2$
26. $c^2 - a^2$
27. $(d - b)^2$
28. $(c - a)^2$
29. $(d - b)(d + b)$
30. $(c - a)(c + a)$
31. $d^3 - b^3$
32. $c^3 + a^3$
33. $(d - b)^3$ VIDEO
34. $(c + a)^3$

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Name _____

Section _____ Date _____

ANSWERS

1. −22
2. −26
3. 32
4. −6
5. −20
6. 30
7. 12
8. 96
9. 4
10. −4
11. 83
12. 68
13. 6
14. 45
15. −40
16. −108
17. 14
18. 48
19. −9
20. −2
21. 2
22. −7
23. 2
24. −2
25. 11
26. 12
27. 1
28. 4
29. 11
30. 12
31. 91
32. −72
33. 1
34. −216

ANSWERS

35. 91

36. −72

37. 29

38. 32

39. 9

40. 64

41. 16

42. 81

43. 41

44. 50

45. 8

46. 23

47. 11

48. 4

49. True

50. False

51. False

52. True

53. 30 in.

35. $(d - b)(d^2 + db + b^2)$

36. $(c + a)(c^2 - ac + a^2)$

37. $b^2 + a^2$

38. $d^2 - a^2$

39. $(b + a)^2$

40. $(d - a)^2$

41. $a^2 + 2ad + d^2$ VIDEO ALEKS

42. $b^2 - 2bc + c^2$ ALEKS

[Objective 2]

For each group of integers, evaluate Σx.

43. 1, 2, 3, 7, 8, 9, 11

44. 2, 4, 5, 6, 10, 11, 12

45. −5, −3, −1, 2, 3, 4, 8

46. −4, −2, −1, 5, 7, 8, 10

47. 3, 2, −1, −4, −3, 8, 6

48. 3, −4, 2, −1, 2, −7, 9

For exercises 49 to 52, decide if the given values make the statement true or false.

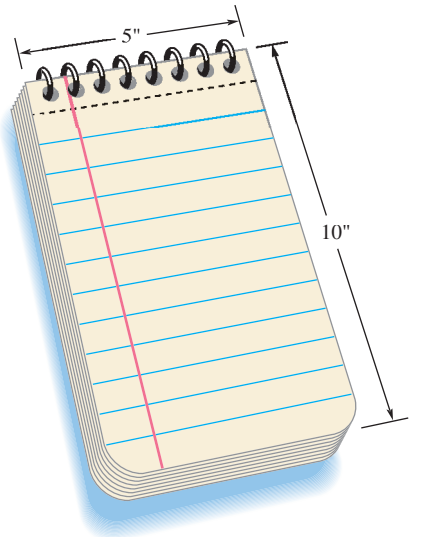
49. $x - 7 = 2y + 5; x = 22, y = 5$

50. $3(x - y) = 6; x = 5, y = -3$

51. $2(x + y) = 2x + y; x = -4, y = -2$

52. $x^2 - y^2 = x - y; x = 4, y = -3$ VIDEO

53. Geometry The perimeter of a rectangle of length L and width W is sometimes given by the formula $P = 2L + 2W$. Find the perimeter when $L = 10$ in. and $W = 5$ in.



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54. Geometry Using the equation given in exercise 53, find the perimeter of a sheet of paper that is 11 in. wide and 14 in. long.

55. Statistics A major-league pitcher throws a ball straight up into the air. The height of the ball (in feet) can be determined by substituting the number of seconds (s) that have passed after the throw into the expression $110s - 16s^2$.

(a) Determine the height after 2 s. (Hint: Replace s with 2 in the expression and then evaluate.)

(b) Find the height after 3 s.

56. Statistics Given the same equation used in exercise 55, find the height of the ball after 4 s. How does this compare to the answer for exercise 55? What has happened?

57. Number Problem Is $a^n + b^n = (a + b)^n$? Try a few numbers and decide if you think this is true for all numbers, for some numbers, or never true. Write an explanation of your findings and give examples.

58. Enjoyment of patterns in art, music, and language is common to all cultures, and many cultures also delight in and draw spiritual significance from patterns in numbers. One such set of patterns is that of the "magic" square. One of these squares appears in a famous etching by Albrecht Dürer, who lived from 1471 to 1528 in Europe. He was one of the first artists in Europe to use geometry to give perspective, a feeling of three dimensions, in his work.

The magic square in his work is this one:

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Why is this square "magic"? It is magic because every row, every column, and both diagonals add to the same number. In this square, there are 16 spaces for the numbers 1 through 16.

Part 1: What number does each row and column add to?

Write the square that you obtain by adding -17 to each number. Is this still a magic square? If so, what number does each column and row add to? If you add 5 to each number in the original magic square, do you still have a magic square? You have been studying the operations of addition, multiplication, subtraction, and division with integers and with rational numbers. What operations can you perform on this magic square and still have a magic square? Try to find something that will not work. Use algebra to help you decide what will work and what won't. Write a description of your work and explain your conclusions.

ANSWERS

54. 50 in.

55. (a) 156 ft (b) 186 ft

56. 184 ft It has begun to fall.

57.

58.

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59. 0°C

60. 100°C

61. 20°C

62. 40°C

Part 2: Here is the oldest published magic square. It is from China, about 250 B.C.E. Legend has it that it was brought from the River Lo by a turtle to the Emperor Yii, who was a hydraulic engineer.

4	9	2
3	5	7
8	1	6

Check to make sure that this is a magic square. Work together to decide what operation might be done to every number in the magic square to make the sum of each row, column, and diagonal the *opposite* of what it is now. What would you do to every number to cause the sum of each row, column, and diagonal to equal zero?

The formula used for converting a Fahrenheit temperature to a Celsius temperature is

$$C = \frac{5}{9}(F - 32)$$

Use this formula to convert each of the following Fahrenheit temperatures to its Celsius equivalent.

59. 32°F

60. 212°F

61. 68°F

62. 104°F

Answers

1. −22 **3.** 32 **5.** −20 **7.** 12 **9.** 4 **11.** 83 **13.** 6
15. −40 **17.** 14 **19.** −9 **21.** 2 **23.** 2 **25.** 11 **27.** 1
29. 11 **31.** 91 **33.** 1 **35.** 91 **37.** 29 **39.** 9 **41.** 16
43. 41 **45.** 8 **47.** 11 **49.** True **51.** False **53.** 30 in.
55. **(a)** 156 ft; **(b)** 186 ft **57.** **59.** 0°C **61.** 20°C

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2.8 Simplifying Algebraic Expressions

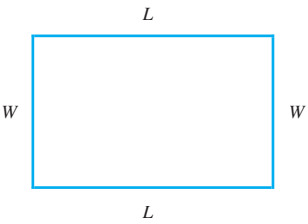
2.8 OBJECTIVES

1. Identify terms, like terms, and numerical coefficients
2. Simplify algebraic expressions by combining like terms

In Section 1.2, we found the perimeter of a rectangle by using the formula

$$P = L + W + L + W$$

There is another version of this formula that we can use. Because we are adding two times the length and two times the width, we can use the language of algebra to write this formula as



Perimeter $= 2L + 2W$

We call $2L + 2W$ an **algebraic expression,** or more simply an **expression.** Recall from Section 2.6 that an expression allows us to write a mathematical idea in symbols. It can be thought of as a meaningful collection of letters, numbers, and operation signs.

Some expressions are

$$5x^2 \qquad 3a + 2b \qquad 4x^3 + (-2y) + 1$$

In algebraic expressions, the addition and subtraction signs break the expressions into smaller parts called **terms.**

Definition: Term

A **term** is a number, or the product of a number and one or more variables, raised to a power.

In an expression, each sign (+ or −) is a part of the term that follows the sign.

OBJECTIVE 1

Example 1 Identifying Terms

(a) $5x^2$ has one term.

(b) $3a + 2b$ has two terms: $3a$ and $2b$.

Term Term

(c) $4x^3 + (-2y) + 1$ has three terms: $4x^3$, $-2y$, and 1.

Term Term Term

NOTE This could also be written as $4x^3 - 2y + 1$

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CHECK YOURSELF 1

List the terms of each expression.

(a) $2b^4$ **(b)** $5m + 3n$ **(c)** $2s^2 - 3t - 6$

Note that a term in an expression may have any number of factors. For instance, $5xy$ is a term. It has factors of 5, x, and y. The number factor of a term is called the **numerical coefficient.** So for the term $5xy$, the numerical coefficient is 5.

Example 2 Identifying the Numerical Coefficient

(a) $4a$ has the numerical coefficient 4.

(b) $6a^3b^4c^2$ has the numerical coefficient 6.

(c) $-7m^2n^3$ has the numerical coefficient -7.

(d) Because $1 \cdot x = x$, the numerical coefficient of x is understood to be 1.

CHECK YOURSELF 2

Give the numerical coefficient for each term.

(a) $8a^2b$ **(b)** $-5m^3n^4$ **(c)** y

If terms contain exactly the *same letters* (or variables) raised to the *same powers,* they are called **like terms.**

Example 3 Identifying Like Terms

(a) The following are like terms.

$6a$ and $7a$
$5b^2$ and b^2
$10x^2y^3z$ and $-6x^2y^3z$
$-3m^2$ and m^2

Each pair of terms has the same letters, with each letter raised to the same power—the numerical coefficients can be any number.

(b) The following are *not* like terms.

Different letters
$6a$ and $7b$

Different exponents
$5b^2$ and b^3

Different exponents
$3x^2y$ and $4xy^2$

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CHECK YOURSELF 3

Circle like terms.

$5a^2b$ ab^2 a^2b $-3a^2$ $4ab$ $3b^2$ $-7a^2b$

Like terms of an expression can always be combined into a single term. Look at the following:

$$\overbrace{x + x}^{2x} + \overbrace{x + x + x + x + x}^{5x} = \overbrace{x + x + x + x + x + x + x}^{7x}$$

Rather than having to write out all those x's, try

RECALL Here we use the distributive property discussed first in Section 1.5.

$$\begin{aligned} 2x + 5x &= (2 + 5)x \\ &= 7x \end{aligned}$$

In the same way,

NOTE You don't have to write all this out—just do it mentally!

$$\begin{aligned} 9b + 6b &= (9 + 6)b \\ &= 15b \end{aligned}$$

and

$$\begin{aligned} 10a - 4a &= 10a + (-4a) \\ &= (10 + (-4))a \\ &= 6a \end{aligned}$$

This leads us to a rule.

Step by Step: To Combine Like Terms

To combine like terms, use steps 1 and 2.

Step 1 Add or subtract the numerical coefficients.

Step 2 Attach the common variables.

OBJECTIVE 2

Example 4 Combining Like Terms

Combine like terms.

NOTE The exponents do not change when combining like terms.

(a) $8m + 5m = (8 + 5)m = 13m$

(b) $5pq^3 - 4pq^3 = 5pq^3 + (-4pq^3) = 1pq^3 = pq^3$

RECALL When any factor is multiplied by 0, the product is 0.

(c) $7a^3b^2 - 7a^3b^2 = 7a^3b^2 + (-7a^3b^2) = 0a^3b^2 = 0$

CHECK YOURSELF 4

Combine like terms.

(a) $6b + 8b$ **(b)** $12x^2 - 3x^2$ **(c)** $8xy^3 - 7xy^3$ **(d)** $9a^2b^4 - 9a^2b^4$

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With expressions involving more than two terms, the idea is just the same.

Example 5 Combining Like Terms

Combine like terms.

(a) $5ab - 2ab + 3ab$

$= 5ab + (-2ab) + 3ab$

$= (5 + (-2) + 3)ab = 6ab$

NOTE The distributive property can be used over any number of like terms.

Only like terms can be combined.

(b) $8x - 2x + 5y$

$(8 + (-2))\,x + 5y$

$= 6x + 5y$

Like terms Like terms

(c) $5m + 8n + 4m - 3n$

$= (5m + 4m) + (8n + (-3n))$

$= 9m + 5n$

Here we have used the associative and commutative properties of addition.

NOTE With practice, you will be doing this mentally, rather than writing out all the steps.

(d) $4x^2 + 2x - 3x^2 + x$

$= (4x^2 + (-3x^2)) + (2x + x)$

$= x^2 + 3x$

As these examples illustrate, combining like terms often means changing the grouping and the order in which the terms are written. Again all this is possible because of the properties of addition that we introduced in Section 1.2.

CHECK YOURSELF 5

Combine like terms.

(a) $4m^2 - 3m^2 + 8m^2$

(b) $9ab + 3a - 5ab$

(c) $4p + 7q + 5p - 3q$

As you have seen in arithmetic, subtraction can be performed directly. As this is the form used for most of mathematics, we will use that form throughout this text. Just remember, by using the additive inverse of numbers, you can always rewrite a subtraction problem as an addition problem.

Example 6 Combining Like Terms

Combine like terms.

(a) $2xy - 3xy + 5xy$

$= (2 - 3 + 5)xy$

$= 4xy$

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(b) $5a - 2b + 7b - 8a$

$= 5a + (-2)b + 7b + (-8)a$

$= 5a + (-8)a + (-2)b + 7b$

$= -3a + 5b$

NOTE Note that x and x^2 are *not* like terms.

(c) $2x^2 + 3x + 4x^2 + x$

$= (2x^2 + 4x^2) + (3x + x)$

$= 6x^2 + 4x$

CHECK YOURSELF 6

Combine like terms.

(a) $4ab + 5ab - 3ab - 7ab$ **(b)** $2x - 7y - 8x - y$

The distributive property studied in Section 1.5 can be used to simplify expressions containing variables. This is illustrated in Example 7.

Example 7 Using the Distributive Property before Combining Like Terms

Use the distributive property to remove the parentheses. Then simplify by combining like terms.

(a) $2(x - 5) - 8x$

$= 2x - 2(5) - 8x$

$= 2x - 10 - 8x$

$= 2x - 8x - 10$

$= -6x - 10$

(b) $3a - 2b + 4(a + 3b)$

$= 3a - 2b + 4a + 4(3b)$

$= 3a - 2b + 4a + 12b$

$= 3a + 4a - 2b + 12b$

$= 7a + 10b$

CHECK YOURSELF 7

Use the distributive property to remove the parentheses. Then simplify by combining like terms.

(a) $7x + 3(x - 1)$ **(b)** $5(m + 2n) + 3(m - n)$

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READING YOUR TEXT

The following fill-in-the-blank exercises are designed to assure that you understand the key vocabulary used in this section. Each sentence comes directly from the section. You will find the correct answers in Appendix C.

Section 2.8

(a) A _____ is a number or the product of a number and one or more variables raised to a power.

(b) The number factor of a term is called the numerical _____.

(c) If terms contain exactly the same variables raised to the same powers, they are called _____ terms.

(d) Like terms in an expression can always be combined into a _____ term.

CHECK YOURSELF ANSWERS

1. **(a)** $2b^4$; **(b)** $5m$, $3n$; **(c)** $2s^2$, $-3t$, -6 **2.** **(a)** 8; **(b)** -5; **(c)** 1

3. The like terms are $5a^2b$, a^2b, and $-7a^2b$ **4.** **(a)** $14b$; **(b)** $9x^2$; **(c)** xy^3; **(d)** 0

5. **(a)** $9m^2$; **(b)** $4ab + 3a$; **(c)** $9p + 4q$ **6.** **(a)** $-ab$; **(b)** $-6x - 8y$

7. **(a)** $10x - 3$; **(b)** $8m + 7n$

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2.8 Exercises

[Objective 1]

List the terms of each expression.

1. $5a + 2$

2. $7a - 4b$

3. $4x^3$

4. $3x^2$

5. $3x^2 + 3x - 7$ VIDEO

6. $2a^3 - a^2 + a$

Identify the like terms in each set of terms.

7. $5ab, 3b, 3a, 4ab$

8. $9m^2, 8mn, 5m^2, 7m$

9. $4xy^2, 2x^2y, 5x^2, -3x^2y, 5y, 6x^2y$

10. $8a^2b, 4a^2, 3ab^2, -5a^2b, 3ab, 5a^2b$

[Objective 2]

Combine like terms.

11. $3m + 7m$ ALEKS

12. $6a^2 + 8a^2$ ALEKS

13. $7b^3 + 10b^3$ VIDEO ALEKS

14. $7rs + 13rs$ ALEKS

15. $21xyz + 7xyz$ ALEKS

16. $4mn^2 + 15mn^2$ ALEKS

17. $9z^2 - 3z^2$ ALEKS

18. $7m - 6m$ ALEKS

19. $5a^3 - 5a^3$ ALEKS

20. $13xy - 9xy$ ALEKS

21. $19n^2 - 18n^2$ VIDEO ALEKS

22. $7cd - 7cd$ ALEKS

23. $21p^2q - 6p^2q$ ALEKS

24. $17r^3s^2 - 8r^3s^2$ ALEKS

25. $10x^2 - 7x^2 + 3x^2$ ALEKS

26. $13uv + 5uv - 12uv$ ALEKS

27. $9a - 7a + 4b$ ALEKS

28. $5m^2 - 3m + 6m^2$ ALEKS

29. $7x + 5y - 4x - 4y$ ALEKS

30. $6a^2 + 11a + 7a^2 - 9a$ ALEKS

31. $4a + 7b + 3 - 2a + 3b - 2$ VIDEO

32. $5p^2 + 2p + 8 + 4p^2 + 5p - 6$

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Name _____

Section _____ Date _____

ANSWERS

1. 5a, 2
2. 7a, −4b
3. $4x^3$
4. $3x^2$
5. $3x^2$, 3x, −7
6. $2a^3$, $-a^2$, a
7. 5ab, 4ab
8. $9m^2$, $5m^2$
9. $2x^2y$, $-3x^2y$, $6x^2y$
10. $8a^2b$, $-5a^2b$, $5a^2b$
11. 10m
12. $14a^2$
13. $17b^3$
14. 20rs
15. 28xyz
16. $19mn^2$
17. $6z^2$
18. m
19. 0
20. 4xy
21. n^2
22. 0
23. $15p^2q$
24. $9r^3s^2$
25. $6x^2$
26. 6uv
27. 2a + 4b
28. $11m^2 - 3m$
29. 3x + y
30. $13a^2 + 2a$
31. 2a + 10b + 1
32. $9p^2 + 7p + 2$

ANSWERS

33. $13a^4$

34. $21p^2$

35. $3a^3$

36. $13m^3$

37. $7x$

38. $4ab$

39. $11mn^2$

40. $14x^2y$

41. $6x + 8$

42. $12z + 6$

43. $42a - 10$

44. $3w - 21$

45. $6s + 12$

46. $9p + 4$

47.

48.

49. $5h - 195$

50. 125 lb

Perform the indicated operations.

33. Find the sum of $5a^4$ and $8a^4$.

34. Find the sum of $9p^2$ and $12p^2$.

35. Subtract $12a^3$ from $15a^3$.

36. Subtract $5m^3$ from $18m^3$.

37. Subtract $4x$ from the sum of $8x$ and $3x$.

38. Subtract $8ab$ from the sum of $7ab$ and $5ab$.

39. Subtract $3mn^2$ from the sum of $9mn^2$ and $5mn^2$.

40. Subtract $4x^2y$ from the sum of $6x^2y$ and $12x^2y$.

Use the distributive property to remove the parentheses in each expression. Then simplify by combining like terms.

41. $2(3x + 2) + 4$

42. $3(4z + 5) - 9$

43. $5(6a - 2) + 12a$

44. $7(4w - 3) - 25w$

45. $4s + 2(s + 4) + 4$

46. $5p + 4(p + 3) - 8$

47. Write a paragraph explaining the difference between n^2 and $2n$.

48. Complete the explanation: "x^3 and $3x$ are not the same because . . ."

49. Allied Health The *ideal body weight*, in pounds, for a woman can be approximated by substituting her height, in inches, into the formula $105 + 5(h - 60)$. Use the distributive property to simplify the ideal body weight formula.

50. Allied Health Use the simplified formula you found in exercise 49 to approximate the ideal body weight of a woman who stands 5 ft 4 in. tall.

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51. Work with another student to complete this exercise. Place $>$, $<$, or $=$ in the blank in these statements.

1^2____2^1

2^3____3^2

3^4____4^3

4^5____5^4

What happens as the table of numbers is extended? Try more examples.

What sign seems to occur the most in your table? $>$, $<$, or $=$?

Write an algebraic statement for the pattern of numbers in this table. Do you think this is a pattern that continues? Add more lines to the table and extend the pattern to the general case by writing the pattern in algebraic notation. Write a short paragraph stating your conjecture.

Answers

1. $5a$, 2 **3.** $4x^3$ **5.** $3x^2$, $3x$, -7 **7.** $5ab$, $4ab$ **9.** $2x^2y$, $-3x^2y$, $6x^2y$
11. $10m$ **13.** $17b^3$ **15.** $28xyz$ **17.** $6z^2$ **19.** 0 **21.** n^2
23. $15p^2q$ **25.** $6x^2$ **27.** $2a + 4b$ **29.** $3x + y$ **31.** $2a + 10b + 1$
33. $13a^4$ **35.** $3a^3$ **37.** $7x$ **39.** $11mn^2$ **41.** $6x + 8$
43. $42a - 10$ **45.** $6s + 12$ **47.** **49.** $5h - 195$ **51.**

ANSWERS

51.

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2.9 Introduction to Linear Equations

2.9 OBJECTIVES

1. Determine whether a given number is a solution for an equation
2. Identify linear equations

In Chapter 1, we introduced one of the most important tools of mathematics, the equation. The ability to recognize and solve various types of equations is probably the most useful algebraic skill you will learn. We continue to build upon the methods of this chapter throughout the remainder of the text. We start by reviewing what we mean by an *equation.*

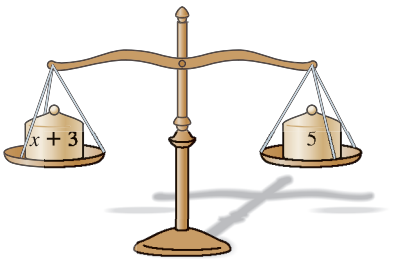
Definition: Equation

An **equation** is a mathematical statement that two expressions are equal.

Some examples are $3 + 4 = 7$, $x + 3 = 5$, and $P = 2L + 2W$.

Note that some of these equations contain letters. In algebra, we use letters to represent numerical values that we don't know or that could change. These letters are called variables. Recall that a *variable* is a letter used to represent a number. In the equation $x + 3 = 5$, x is a variable.

As you can see, an equals sign (=) separates the two equal expressions. These expressions are usually called the **left side** and the **right side** of the equation.



$$\underbrace{x + 3}_{\text{Left side}} \underset{\text{Equals}}{=} \underset{\text{Right side}}{5}$$

NOTE An equation such as $x + 3 = 5$ is called a **conditional equation** because it can be either true or false depending on the value given to the variable.

Just as the balance scale may be in balance or out of balance, an equation may be either true or false. For instance, $3 + 4 = 7$ is true because both sides name the same number. What about an equation such as $x + 3 = 5$ that has a letter or variable on one side? Any number can replace x in the equation. However, in this case, only one number will make this equation a true statement.

$$\text{If } x = \begin{cases} 1 & 1 + 3 = 5 \text{ is false} \\ 2 & 2 + 3 = 5 \text{ is true} \\ 3 & 3 + 3 = 5 \text{ is false} \end{cases}$$

The number 2 is called the **solution** (or **root**) of the equation $x + 3 = 5$ because substituting 2 for x gives a true statement.

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Definition: Solution

A **solution** for an equation is any value for the variable that makes the equation a true statement.

OBJECTIVE 1

Example 1 Verifying a Solution

(a) Is 3 a solution for the equation $2x + 4 = 10$?

To find out, replace x with 3 and evaluate $2x + 4$ on the left.

Left side		*Right side*
$2(3) + 4$	$\stackrel{?}{=}$	10
$6 + 4$	$\stackrel{?}{=}$	10
10	$=$	10

Because $10 = 10$ is a true statement, 3 is a solution of the equation.

(b) Is 5 a solution of the equation $3x - 2 = 2x + 1$?

To find out, replace x with 5 and evaluate each side separately.

RECALL We must follow the rules for the order of operations. Multiply first and then add or subtract.

Left side		*Right side*
$3(5) - 2$	$\stackrel{?}{=}$	$2(5) + 1$
$15 - 2$	$\stackrel{?}{=}$	$10 + 1$
13	$\neq$	11

Because the two sides do not name the same number, we do not have a true statement. Therefore, 5 is not a solution.

CHECK YOURSELF 1

For the equation

$2x - 1 = x + 5$

(a) Is 4 a solution?

(b) Is 6 a solution?

You may be wondering whether an equation can have more than one solution. It certainly can. For instance,

$x^2 = 9$

has two solutions. They are 3 and -3 because

$(3)^2 = 9$ and $(-3)^2 = 9$

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In this chapter, however, we will always work with *linear equations in one variable.* These are equations that can be put into the form

$$ax + b = 0$$

in which the variable is x, a and b are any numbers, and a is not equal to 0. In a linear equation, the variable can appear only to the first power. No other power (x^2, x^3, etc.) can appear. Linear equations are also called **first-degree equations.** The degree of an equation in one variable is the highest power to which the variable appears.

Definition: Linear Equations

Linear equations in one variable are equations that can be written in the form

$ax + b = 0 \qquad a \neq 0$

Every such equation will have exactly one solution.

OBJECTIVE 2

Example 2 Identifying Expressions and Equations

Given the following

$4x + 5$

$2x + 8 = 0$

$3x^2 - 9 = 0$

$5x = 15$

Label each as an expression, a linear equation, or an equation that is not linear.

(a) $4x + 5$ is an expression.

(b) $2x + 8 = 0$ is a linear equation.

(c) $3x^2 - 9 = 0$ is an equation that is not linear.

(d) $5x = 15$ is a linear equation because this can be written as $5x - 15 = 0$.

CHECK YOURSELF 2

Label each as an expression, a linear equation, or an equation that is not linear.

(a) $2x^2 = 8$ **(b)** $2x - 3 = 0$

(c) $5x - 10$ **(d)** $2x + 1 = 7$

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READING YOUR TEXT

The following fill-in-the-blank exercises are designed to assure that you understand the key vocabulary used in this section. Each sentence comes directly from the section. You will find the correct answers in Appendix C.

Section 2.9

(a) An equation is a mathematical statement that two _____ are equal.

(b) A _____ for an equation is any value for the variable that makes the statement true.

(c) _____ equations in one variable are equations that can be written in the form $ax + b = 0$.

(d) Linear equations are also called _____-degree equations.

CHECK YOURSELF ANSWERS

1. **(a)** 4 is not a solution; **(b)** 6 is a solution

2. **(a)** Nonlinear equation; **(b)** linear equation; **(c)** expression; **(d)** linear equation

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Name _____

Section _____ Date _____

ANSWERS

1. Yes
2. No
3. No
4. Yes
5. No
6. Yes
7. Yes
8. No
9. No
10. Yes
11. No
12. Yes
13. Yes
14. No
15. Yes
16. No
17. Yes
18. No
19. Linear equation
20. Expression
21. Expression
22. Linear equation
23. Expression
24. Linear equation
25. Linear equation
26. Expression

2.9 Exercises

[Objective 1]

Is the number shown in parentheses a solution for the given equation?

1. $x + 4 = 9 \quad (5)$

2. $x + 2 = 11 \quad (8)$

3. $x - 15 = 6 \quad (-21)$ VIDEO

4. $x - 11 = 5 \quad (16)$

5. $5 - x = 2 \quad (4)$

6. $10 - x = 7 \quad (3)$

7. $4 - x = 6 \quad (-2)$

8. $5 - x = 6 \quad (-3)$

9. $3x + 4 = 13 \quad (8)$

10. $5x + 6 = 31 \quad (5)$

11. $4x - 5 = 7 \quad (2)$

12. $2x - 5 = 1 \quad (3)$

13. $5 - 2x = 7 \quad (-1)$

14. $4 - 5x = 9 \quad (-2)$

15. $4x - 5 = 2x + 3 \quad (4)$

16. $5x + 4 = 2x + 10 \quad (4)$

17. $x + 3 + 2x = 5 + x + 8 \quad (5)$ VIDEO

18. $5x - 3 + 2x = 3 + x - 12 \quad (-2)$

[Objective 2]

Label each as an expression or a linear equation.

19. $2x + 1 = 9$ VIDEO

20. $7x + 14$

21. $2x - 8$ VIDEO

22. $5x - 3 = 12$

23. $7x + 2x + 8 - 3$

24. $x + 5 = 13$

25. $2x - 8 = 3$

26. $12x - 5x + 2 + 5$

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27. An algebraic equation is a complete sentence. It has a subject, a verb, and a predicate. For example, $x + 2 = 5$ can be written in English as "Two more than a number is five." Or, "A number added to two is five." Write an English version of the following equations. Be sure you write complete sentences and that the sentences express the same idea as the equations. Exchange sentences with another student and see if your interpretation of each other's sentences results in the same equation.

(a) $2x - 5 = x + 1$

(b) $2(x + 2) = 14$

(c) $n + 5 = \frac{n}{2} - 6$

(d) $7 - 3a = 5 + a$

28. Complete the following explanation in your own words: "The difference between $3(x - 1) + 4 - 2x$ and $3(x - 1) + 4 = 2x$ is . . . ."

Answers

1. Yes **3.** No **5.** No **7.** Yes **9.** No **11.** No **13.** Yes
15. Yes **17.** Yes **19.** Linear equation **21.** Expression
23. Expression **25.** Linear equation **27.**

ANSWERS

27.

28.

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2.10 The Addition Property of Equality

2.10 OBJECTIVES

1. Use the addition property to solve an equation
2. Combine like terms while solving an equation
3. Use the distributive property while solving an equation

It is not difficult to find the solution for an equation such as $x + 3 = 8$ by guessing the answer to the question "What plus 3 is 8?" Here the answer to the question is 5, and that is also the solution for the equation. But for more complicated equations, you are going to need something more than guesswork. A better method is to transform the given equation to an *equivalent equation* whose solution can be found by inspection.

Definition: Equivalent Equations

Equations that have the same solution(s) are called **equivalent equations.**

These expressions are all equivalent equations:

$2x + 3 = 5$ $\quad 2x = 2$ $\quad$ and $\quad x = 1$

They all have the same solution, 1. We say that a linear equation is *solved* when it is transformed to an equivalent equation of the form

$x = \square$

The variable is alone on the left side.

The right side is some number, the solution.

NOTE In some cases, we'll write the equation in the form $\square = x$.

The number will be our solution when the equation has the variable isolated on the left or on the right.

The addition property of equality is the first property you will need to transform an equation to an equivalent form.

Property: The Addition Property of Equality

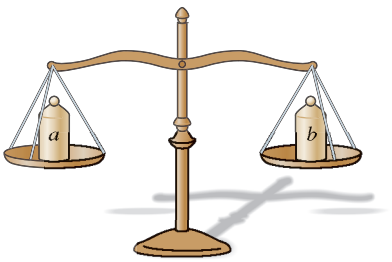
If $a = b$

then $a + c = b + c$

In words, adding the same quantity to both sides of an equation gives an equivalent equation.

RECALL An equation is a statement that the two sides are equal. Adding the same quantity to both sides does not change the equality or "balance."

We said that a true equation was like a scale in balance.



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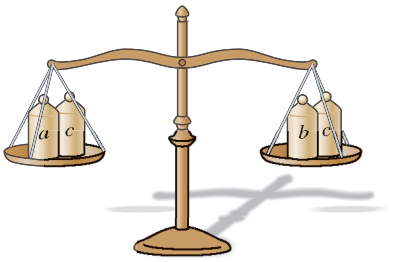
The addition property is equivalent to adding the same weight to both sides of the scale. It remains in balance.



OBJECTIVE 1

Example 1 Using the Addition Property to Solve an Equation

Solve.

$x - 3 = 9$

Remember that our goal is to isolate x on one side of the equation. Because 3 is being subtracted from x, we can add 3 to remove it. We must use the addition property to add 3 to both sides of the equation.

$$\begin{array}{rr} x - 3 = & 9 \\ +3 & +3 \\ \hline x = & 12 \end{array}$$

Adding 3 "undoes" the subtraction and leaves x alone on the left.

Because 12 is the solution for the equivalent equation $x = 12$, it is the solution for our original equation.

NOTE To check, replace x with 12 in the original equation:

$x - 3 = 9$

$(12) - 3 \stackrel{?}{=} 9$

$9 = 9$

Because we have a true statement, 12 is the solution.

CHECK YOURSELF 1

Solve and check.

$x - 5 = 4$

The addition property also allows us to add a negative number to both sides of an equation. This is really the same as subtracting the same quantity from both sides.

Example 2 Using the Addition Property to Solve an Equation

Solve.

$x + 5 = 9$

In this case, 5 is *added* to x on the left. We can use the addition property to add a -5 to both sides. Because $5 + (-5) = 0$, this will "undo" the addition and leave the variable x alone on one side of the equation.

$$\begin{array}{rr} x + 5 = & 9 \\ -5 & -5 \\ \hline x = & 4 \end{array}$$

The solution is 4. To check, replace x with 4 in the original equation.

$(4) + 5 = 9$ (True)

NOTE Recall our comment that we could write an equation in the equivalent forms $x = \square$ or $\square = x$, in which $\square$ represents some number. Suppose we have an equation like

$12 = x + 7$

Adding -7 will isolate x *on the right:*

$$\begin{array}{rr} 12 = & x + 7 \\ -7 & -7 \\ \hline 5 = & x \end{array}$$

and the solution is 5.

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CHECK YOURSELF 2

Solve and check.

$x + 6 = 13$

What if the equation has a variable term on both sides? You can use the addition property to add or subtract a term involving the variable to get the desired result.

OBJECTIVE 2

Example 3 Using the Addition Property to Solve an Equation

Solve.

$5x = 4x + 7$

We start by adding $-4x$ to both sides of the equation. Do you see why? Remember that an equation is solved when we have an equivalent equation of the form $x = \square$.

RECALL Adding $-4x$ is identical to subtracting $4x$.

$$\begin{array}{rr} 5x = & 4x + 7 \\ -4x & -4x \\ \hline x = & 7 \end{array}$$

Adding $-4x$ to both sides *removes* $4x$ from the right.

To check: Because 7 is a solution for the equivalent equation $x = 7$, it should be a solution for the original equation. To find out, replace x with 7:

$$\begin{aligned} 5(7) &\stackrel{?}{=} 4(7) + 7 \\ 35 &\stackrel{?}{=} 28 + 7 \\ 35 &= 35 \quad \text{(True)} \end{aligned}$$

CHECK YOURSELF 3

Solve and check.

$7x = 6x + 3$

You may have to apply the addition property more than once to solve an equation as we see in Example 4.

Example 4 Using the Addition Property to Solve an Equation

Solve.

$7x - 8 = 6x$

We want all the variables on *one* side of the equation. If we choose the left, we add $-6x$ to both sides of the equation. This will remove $6x$ from the right:

$$\begin{array}{rr} 7x - 8 = & 6x \\ -6x & -6x \\ \hline x - 8 = & 0 \end{array}$$

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We want the variable alone, so we add 8 to both sides. This isolates x on the left.

$$\begin{array}{rcr} x - 8 = & & 0 \\ +8 & & +8 \\ \hline x & = & 8 \end{array}$$

The solution is 8. We leave it to you to check this result.

CHECK YOURSELF 4

Solve and check.

$9x + 3 = 8x$

Often an equation has more than one variable term *and* more than one number. In this case, you have to apply the addition property twice in solving these equations.

Example 5 Using the Addition Property to Solve an Equation

Solve.

$5x - 7 = 4x + 3$

We would like the variable terms on the left, so we start by adding $-4x$ to remove the $4x$ term from the right side of the equation:

$$\begin{array}{rcr} 5x - 7 = & & 4x + 3 \\ -4x & & -4x \\ \hline x - 7 = & & 3 \end{array}$$

Now, to isolate the variable, we add 7 to both sides.

$$\begin{array}{rcr} x - 7 = & & 3 \\ +7 & & +7 \\ \hline x & = & 10 \end{array}$$

NOTE You could just as easily have added 7 to both sides and *then* added $-4x$. The result would be the same. In fact, some students prefer to combine the two steps.

The solution is 10. To check, replace x with 10 in the original equation:

$$5(10) - 7 \stackrel{?}{=} 4(10) + 3$$

$$43 = 43 \quad \text{(True)}$$

CHECK YOURSELF 5

Solve and check.

(a) $4x - 5 = 3x + 2$ **(b)** $6x + 2 = 5x - 4$

RECALL By *simplify,* we mean to combine all like terms.

In solving an equation, you should always "simplify" each side as much as possible before using the addition property.

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Example 6 Combining Like Terms and Solving the Equation

Solve.

Like terms Like terms

$$5 + 8x - 2 = 2x - 3 + 5x$$

Because like terms appear on each side of the equation, we start by combining the numbers on the left (5 and -2). Then we combine the like terms ($2x$ and $5x$) on the right. We have

$$3 + 8x = 7x - 3$$

Now we can apply the addition property, as before:

$$\begin{array}{rl} 3 + 8x = & 7x - 3 \\ -7x = & -7x \\ \hline 3 + x = & -3 \\ -3 & -3 \\ \hline x = & -6 \end{array}$$

Add $-7x$.

Add -3.

Isolate x.

The solution is -6. To check, always return to the original equation. That will catch any possible errors in simplifying. Replacing x with -6 gives

$$\begin{aligned} 5 + 8(-6) - 2 &\stackrel{?}{=} 2(-6) - 3 + 5(-6) \\ 5 - 48 - 2 &\stackrel{?}{=} -12 - 3 - 30 \\ -45 &= -45 \quad \text{(True)} \end{aligned}$$

CHECK YOURSELF 6

Solve and check.

(a) $3 + 6x + 4 = 8x - 3 - 3x$ **(b)** $5x + 21 + 3x = 20 + 7x - 2$

We may have to apply some of the properties discussed in Section 1.5 in solving equations. Example 7 illustrates the use of the distributive property to clear an equation of parentheses.

OBJECTIVE 3

Example 7 Using the Distributive Property and Solving Equations

Solve.

$$2(3x + 4) = 5x - 6$$

NOTE $2(3x + 4) = 2(3x) + 2(4) = 6x + 8$.

Applying the distributive property on the left, we have

$$6x + 8 = 5x - 6$$

We can then proceed as before:

$$\begin{array}{rl} 6x + 8 = & 5x - 6 \\ -5x & -5x \\ \hline x + 8 = & -6 \\ -8 & -8 \\ \hline x = & -14 \end{array}$$

Add $-5x$.

Add -8.

NOTE Remember that $x = -14$ and $-14 = x$ are equivalent equations.

The solution is -14. We leave the checking of this result to the reader.

Remember: Always return to the original equation to check.

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CHECK YOURSELF 7

Solve and check each of the equations.

(a) $4(5x - 2) = 19x + 4$ **(b)** $3(5x + 1) = 2(7x - 3) - 4$

Given an expression such as

$-2(x - 5)$

the distributive property can be used to create the equivalent expression.

$-2x + 10$

The distribution of a negative number is used in Example 8.

Example 8 Distributing a Negative Number

Solve each of the equations.

(a) $-2(x - 5) = -3x + 2$

$-2x + 10 = -3x + 2$ Distribute the -2.

$+3x \qquad +3x$ Add $3x$.

$x + 10 = 2$

$-10 = -10$ Add -10.

$x = -8$

(b) $-3(3x + 5) = -5(2x - 2)$

$-9x - 15 = -5(2x - 2)$ Distribute the -3.

$-9x - 15 = -10x + 10$ Distribute the -5.

$+10x \qquad +10x$ Add $10x$.

$x - 15 = 10$

$+15 \qquad +15$ Add 15.

$x = 25$

CHECK YOURSELF 8

Solve each of the equations.

(a) $-2(x - 3) = -x + 5$ **(b)** $-4(2x - 1) = -3(3x + 2)$

When parentheses are preceded only by a negative, or by the minus sign, we say that we have a silent negative one. Example 9 illustrates this case.

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Example 9 Distributing the Silent Negative One

Solve.

$$-(2x + 3) = -3x + 7$$
$$-1(2x + 3) = -3x + 7$$
$$(-1)(2x) + (-1)(3) = -3x + 7$$

$$\begin{array}{rrl} -2x - 3 = & -3x + 7 & \\ +3x \qquad & +3x \qquad & \text{Add } 3x. \\ \hline x - 3 = & 7 & \\ +3 \qquad & +3 \qquad & \text{Add } 3. \\ \hline x = & 10 & \end{array}$$

CHECK YOURSELF 9

Solve.

$-(3x + 2) = -2x - 6$

READING YOUR TEXT

The following fill-in-the-blank exercises are designed to assure that you understand the key vocabulary used in this section. Each sentence comes directly from the section. You will find the correct answers in Appendix C.

Section 2.10

(a) Equations that have the same solution are called _____ equations.

(b) The addition property is equivalent to adding the same _____ to both sides of a scale.

(c) In solving an equation, you should always _____ each side as much as possible before using the addition property.

(d) **Remember:** Always return to the original equation to _____ the result.

CHECK YOURSELF ANSWERS

1. 9 **2.** 7 **3.** 3 **4.** -3 **5.** **(a)** 7; **(b)** -6 **6.** **(a)** -10; **(b)** -3
7. **(a)** 12; **(b)** -13 **8.** **(a)** 1; **(b)** -10 **9.** 4

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2.10 Exercises

[Objectives 1 to 3]

Solve and check each equation.

1. $x + 9 = 11$ ALEKS

2. $x - 4 = 6$ ALEKS

3. $x - 8 = 3$ ALEKS

4. $x + 11 = 15$ ALEKS

5. $x - 8 = -10$ VIDEO ALEKS

6. $x + 5 = 2$ ALEKS

7. $x + 4 = -3$ ALEKS

8. $x - 5 = -4$ ALEKS

9. $11 = x + 5$ ALEKS

10. $x + 7 = 0$ ALEKS

11. $4x = 3x + 4$

12. $7x = 6x - 8$

13. $11x = 10x - 10$

14. $9x = 8x + 5$

15. $6x + 3 = 5x$

16. $12x - 6 = 11x$

17. $8x - 4 = 7x$

18. $9x - 7 = 8x$

19. $2x + 3 = x + 5$ VIDEO

20. $3x - 2 = 2x + 1$

21. $3x - 5 + 2x - 7 + x = 5x + 2$

22. $5x + 8 + 3x - x + 5 = 6x - 3$

23. $3(7x + 2) = 5(4x + 1) + 17$ VIDEO

24. $5(5x + 3) = 3(8x - 2) + 4$

25. Which equation is equivalent to the equation $5x - 7 = 4x - 12$?

(a) $9x = 19$ **(b)** $9x - 7 = -12$ **(c)** $x = -18$ **(d)** $x - 7 = -12$

VIDEO

26. Which equation is equivalent to the equation $12x - 6 = 8x + 14$?

(a) $4x - 6 = 14$ **(b)** $x = 20$ **(c)** $20x = 20$ **(d)** $4x = 8$

27. Which equation is equivalent to the equation $7x + 5 = 12x - 10$?

(a) $5x = -15$ **(b)** $7x - 5 = 12x$ **(c)** $-5 = 5x$ **(d)** $7x + 15 = 12x$

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Name _____

Section _____ Date _____

ANSWERS

1. 2
2. 10
3. 11
4. 4
5. −2
6. −3
7. −7
8. 1
9. 6
10. −7
11. 4
12. −8
13. −10
14. 5
15. −3
16. 6
17. 4
18. 7
19. 2
20. 3
21. 14
22. −16
23. 16
24. −17
25. d
26. a
27. d

ANSWERS

28. True

29. False

30.

True or false?

28. Every linear equation with one variable has exactly one solution.

29. Isolating the variable on the right side of the equation will result in a negative solution.

30. "Surprising Results!" Work with other students to try this experiment. Each person should do the following six steps mentally, not telling anyone else what their calculations are:

(a) Think of a number.
(b) Add 7.
(c) Multiply by 3.
(d) Add 3 more than the original number.
(e) Divide by 4.
(f) Subtract the original number.

What number do you end up with? Compare your answer with everyone else's. Does everyone have the same answer? Make sure that everyone followed the directions accurately. How do you explain the results? Algebra makes the explanation clear. Work together to do the problem again, using a variable for the number. Make up another series of computations that give "surprising results."

Answers

1. 2 **3.** 11 **5.** −2 **7.** −7 **9.** 6 **11.** 4 **13.** −10
15. −3 **17.** 4 **19.** 2 **21.** 14 **23.** 16 **25.** d **27.** d
29. False

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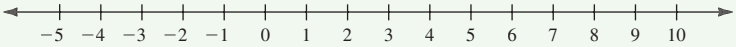
Activity 2: Charting Temperatures

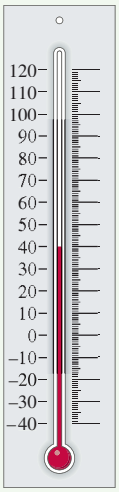
Each chapter in this text includes an activity. The activity is related to the vignette you encountered in the chapter opening. The activity provides you with the opportunity to apply the math you studied in the chapter.

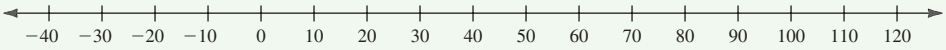
Your instructor will determine how best to use this activity in your class. You may find yourself working in class or outside of class; you may find yourself working alone or in small groups; or you may even be asked to perform research in a library or on the Internet.

In Section 2.1, we looked at a number line. The figure below is the typical representation of the number line.



A thermometer can be thought of as a number line. Compare the thermometer below with the number line that follows.

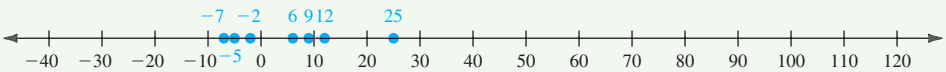




We can use the number line above to locate a series of temperatures. For example, a series of low temperatures for the first seven days of December in Anchorage, Alaska, is presented below.

12°F, −5°F, −7°F, −2°F, 6°F, 9°F, 25°F

Plotting these seven points on the number line, we get



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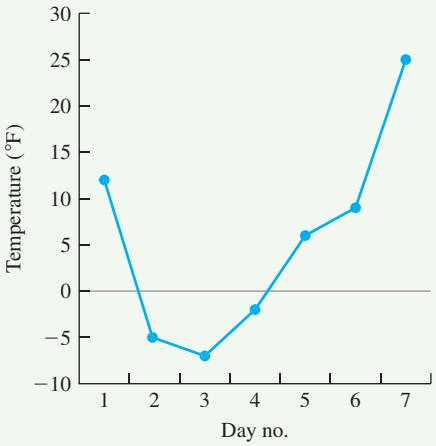
Although this presentation of the data gives us some information about the temperatures, it is really of limited use. Perhaps the most important thing we are looking for when we examine a series of temperatures is a trend. We cannot detect a trend from this number line.

In order to help us see trends, we usually plot temperatures on a time chart. In a time chart, we can look at temperature changes over regular intervals—sometimes hours and sometimes days. The following chart has one point for each of the first seven days in December. Compare the points to the data listed on p. 201.



Do you see the advantages of this kind of presentation? It is called a **line graph** because of the lines that connect the temperature points. The trend in temperatures over the most recent five days is definitely increasing. This guarantees nothing, but it is encouraging!

Make a similar presentation for each of the following data sets.

1. The high temperatures during the first seven days in June of 2004 in Death Valley were

102°F, 98°F, 101°F, 105°F, 106°F, 109°F, 112°F

2. Using the U.S. weather service forecast, plot the predicted high for the next seven days in your area.

3. The record low temperature in the U.S. was in Prospect Camp Creek, Alaska, during Christmas week of 1924. Research the low temperatures in Alaska that week and plot the temperatures over a seven day period.

4. Find a tide chart that gives the high and low tides for some location in the U.S. Use a chart similar to the one used for temperatures to plot the high tides (use a red line) and the low tides (use a blue line) on the same chart.

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