Annotated Bibliography

This appendix is part bibliography and part acknowledgments. We don't intend to give a comprehensive list of related books that are worth reading. Instead, we wish to give credit to many of the books from which we have derived inspiration. Even this list is nowhere near complete, as we could not possibly list all of the books or thank all of the professors and students from whom we've learned mathematics. We have organized the list by chapter and topic.

CHAPTER 0

Fibonacci sequence: a search on the Internet will turn up several good sites. Two of the many books with interesting discussions of the nautilus shell spiral are:

[1] Huntley, H.E., *The Divine Proportion,* Dover Publications, 1970. (Subtitled *A Study in Mathematical Beauty,* with several interesting connections to art, architecture and break nature.)

[2] Stewart, Ian, *Nature's Numbers,* Basic Books, 1995. (A collection of easy-to-read essays on current topics in mathematics. Stewart is one of the best popular mathematics authors.)

Golf: most of what we discuss at various places in the book comes from one of the two following books.

[3] Cochran, A.J. and M.R. Farrally, eds., *Science and Golf II,* E & FN Spon, 1994. (A collection of research papers on a variety of golf-related topics and at varying degrees of complexity. The proceedings of the 1994 World Scientific Congress of Golf.)

[4] Jorgensen, Theodore, *The Physics of Golf,* AIP Press, 1994. (A technical but practical investigation into all aspects of the golf shot.)

Terminal velocity: this and many other physics and sports topics are presented in Brancazio's excellent book. Most college-level physics books also treat this subject.

[5] Brancazio, Peter, *Sport Science,* Simon and Schuster, 1984. (Brancazio enthusiastically uses sports to illustrate physics concepts and physics to explain sports techniques. Thoroughly enjoyable.)

Graphing calculators: the manuals that come with most graphing calculators are very helpful. They include interesting examples to work out as well as basic keystroke instructions. Keep your manual and use it!

Music and mathematics: a very strong connection between mathematics and music dates back to Pythagoras. The material we develop is adapted from [6] and [7].

[6] Pierce, John R., *The Science of Musical Sound,* W.H. Freeman and Company, 1992. (This is a strong introduction to the science of music. It is fairly technical, but has good explanations and many graphs and diagrams.)

[7] Levenson, Thomas, *Measure to Measure,* Touchstone Books, 1994. (The subtitle, *A Musical History of Science,* describes the author's goal. It is readable and well written.)

[8] Hofstadter, Douglas, *Gödel, Escher, Bach,* Vintage Books, 1980. (A Pulitzer Prize–winning classic of puzzles and explorations of mathematics, art and music.)

Circumference of the earth: the video series *The Ring of Truth* is excellent, highlighting several demonstrations of how knowledge is obtained. The book is a very good companion to the video series.

[9] Morrison, Philip and Phylis, *The Ring of Truth,* Vintage Books, 1987.

Baseball, catching a fly ball: this paper critiques a previous theory of how players judge a fly ball, and introduces a new theory.

[10] McBeath, M.K., Shaffer, D.M. and Kaiser, M.K. "How Baseball Outfielders Determine Where to Run to Catch Fly Balls," *Science,* April 28, 1995.

The number *e*: the long and interesting history of e and the exponential function.

[11] Maor, Eli, *e: The Story of a Number,* Princeton University Press, 1994.

Gateway arch: the Jefferson National Expansion Memorial has an excellent website. Some interesting calculations are done in the module.

[12] Thayer, William, "The St. Louis Arch Problem," UMAP Module 638, COMAP, 1984.

Chaos: one of the fastest-growing research areas in applied mathematics in the 1980s and 1990s. The Gleick book is the best-known general interest chaos reader. The other books cited here are mathematics textbooks.

[13] Gleick, James, *Chaos: The Making of a New Science,* Penguin Books, 1987. (Very popular book, giving a sense of the broad scope of early chaos research.)

[14] Devaney, Robert, *Chaos, Fractals and Dynamics,* Addison-Wesley, 1990. (A nice introduction to the mathematics of chaos. The subtitle *Computer Experiments in Mathematics* is accurate.)

[15] Gulick, Denny, *Encounters with Chaos,* McGraw-Hill, 1992. (A good textbook on the mathematics of chaos theory. Some of the mathematics is at an advanced calculus level.)

[16] Barnsley, Michael, *Fractals Everywhere,* Academic Press, 1988. (Not an easy book to read, but it includes several great images related to the chaos game. Barnsley founded a successful company applying the mathematics of the chaos game to image processing.)

CHAPTER 1

Knuckleball, keep your eye on the ball: these are two topics we return to throughout the book. The original research is presented in a highly readable manner in Watts and Bahill's book, which is the best of several books available on baseball and physics.

[17] Watts, Robert and A. Terry Bahill, *Keep Your Eye on the Ball,* W.H. Freeman, 1990.

Mississippi alligators: one of several examples in Cohen and Stewart's book illustrating the complicated interaction between heredity and environment.

[18] Goodwin, Brian, *How the Leopard Changed Its Spots,* Weidenfeld and Nicolson, 1994.

[19] Cohen, Jack and Ian Stewart, *The Collapse of Chaos,* Viking Penguin, 1994.

History of mathematics: many of our historical notes are adapted from workshop material from Professor V. Frederick Rickey. There are several entertaining history of mathematics books available. More references can be found at the Smith and Minton Calculus website.

[20] Simmons, George F., *Calculus Gems,* McGraw-Hill, 1992.

[21] Smith, David E., *A Source Book in Mathematics,* Dover Publications, 1959.

[22] www-history.dcs.st-and.ac.uk/history.

Today in Mathematics: most of the material for these vignettes comes from [22], personal web pages and Wikipedia.

[23] en.wikipedia.org/wik/Mathematician

Size of pupils: a technical reference for the equation we use is

[24] Crawford, B.H., "The dependence of pupil size upon the external light stimulus under static and variable condition," *Proceedings of the Royal Society Series B,* 121 (1937).

Butterfly effect: the motto of chaos theory (see references [13]–[16]), the name comes from an analogy used by meteorologist and chaos pioneer Edward Lorenz.

[25] Lorenz, Edward, *The Essence of Chaos,* University of Washington Press, 1993.

CHAPTER 2

Speed of runner: an excellent discussion is in Brancazio [5]. Also see

[26] Griffing, David F., *The Dynamics of Sports,* The Dalog Company, 1982.

Desert mirage: this optical illusion is beautifully demonstrated in the video.

[27] Greenler, Robert, "The mirage, the discovery of Greenland and the green flash," Blue Sky Associates, 1990.

EPA gas efficiency: brief discussions of the 55/45 rating can be found at various places within the EPA website.

[28] http://www.epa.gov/otaq/cert/factshts/fefact01.pdf.

Soccer: numerous soccer-related problems are adapted from

[29] Wesson, John, *The Science of Soccer,* Institute of Physics Publishing, 2002. (A popular examination of all aspects of the game, from the physics of the ball to the laws of the game to strategy, with a mathematics appendix.)

Tennis serve: numerous tennis-related problems are adapted from

[30] Brody, Howard, Rod Cross and Crawford Lindsey, *The Physics and Technology of Tennis,* Racquet Tech Publishing, 2002. (A wealth of data, equations and instruction on all aspects of the game, from racket properties to the physics of ball-striking to techniques.)

Keep your eye on the ball: see [17].

Bishop Berkeley, Leibniz: see [20]–[22].

Collisions: see [5] and [26].

Richard Feynman: one of the most influential scientists of the 20th century.

[31] Feynman, Richard, *Surely You're Joking, Mr. Feynman,* Bantam Books, 1986.

Hill functions, allosteric enzyme: these and other interesting mathematical models of biological processes can be found here.

[32] Kaplan, Daniel and Leon Glass, *Understanding Nonlinear Dynamics,* Springer-Verlag, 1995. (Examples are similar to [43] upcoming, but more focused on the mathematics.)

Van der Waals' equation: this can be found in almost any introductory physics or chemistry text.

Chaos theory: see [13]–[15] and [25].

Elliptic curves: a gentle introduction to a difficult but important area of mathematics.

[33] Brown, Ezra, "Three Fermat Trails to Elliptic Curves," *College Mathematics Journal,* May 2000.

CHAPTER 3

Lagrange points: the basics of Lagrange points and some interesting numerical experiments are in this paper presented at the 2003 International Space Olympics in Korolev, Russia.

[34] Minton, Gregory, "The Stability of the L1 Lagrange Point," 2003.

Extinction probability: one of the topics in the genetics section of a book that also explores artificial life and ecology.

[35] Sigmund, Karl, *Games of Life,* Oxford University Press, 1993.

Kepler's wine cask: adapted from

[36] Balk, M.B., "The Secret of the Venerable Wine Cooper," *Quantum,* May 1990.

AIDS epidemiology: a number of preliminary studies have been published about this frightening disease, including

[37] Kirschner, Denise, "Using Mathematics to Understand HIV Immune Dynamics," *Notices of AMS,* February 1996.

Hockey: numerous hockey-related problems are adapted from

[38] Hache, Alain, *The Physics of Hockey,* Johns Hopkins University Press, 2002. (A comprehensive look at hockey, from skating techniques, shooting angles and strategy.)

Soccer: see [29.]

HIV epidemiology: see [37] and [39]

Tree infestation: this problem is discussed in both books:

[39] Murray, J.D., *Mathematical Biology,* Springer-Verlag, 1989. (A wealth of examples in population dynamics, diffusion patterns, neural patterns and epidemiology are presented. Many models use calculus, some are more advanced.)

[40] Strogatz, Steven, *Nonlinear Dynamics and Chaos,* Addison-Wesley, 1994. (Excellent introduction to chaos with great examples throughout.)

Fireflies: see [40] and [49] upcoming.

Flying wing aircraft:

[41] Renz, Peter, "Thoughts on *Innumeracy:* Mathematics versus the World?" *The American Mathematics Monthly,* Mathematical Association of America, October 1993.

[42] Biddle, Wayne, "Skeleton Alleged in the Stealth Bomber's Closet," *Science,* May 12, 1989.

Heart rates: chaos theory has had a profound influence on the study of human physiology in general and cardiology in particular. The interplay between order and disorder is fascinating. See also [30].

[43] Glass, Leon and Michael Mackey, *From Clocks to Chaos,* Princeton University Press, 1988. (Subtitled *Rhythms of Life,* innumerable graphs and some calculus and differential equations are used to explain physiological processes.)

[44] West, Bruce, *Fractal Physiology and Chaos in Medicine,* World Scientific, 1990. (Detailed discussion of differential equation models of heart dynamics, as well as the fractal structure of portions of the cardiovascular system.)

Autocatalytic reactions: a collection of mathematical models of biological and chemical processes.

[45] Thrall, Robert M., ed., *Some Mathematical Models in Biology,* University of Michigan, 1967.

Chemical titrations: covered in this quantitative analysis chemistry book.

[46] Harris, Daniel, *Quantitative Chemical Analysis,* 3rd ed., W.H. Freeman, 1991.

Laser field: see [40].

CHAPTER 4

Space shuttle landing: general projectile motion with some specific equations for the space shuttle can be found in this article.

[47] Long, Lyle and Howard Weiss, "The Velocity Dependence of Aerodynamic Drag: A Primer for Mathematicians," *The American Mathematical Monthly,* Mathematical Association of America, February 1999.

Predator-prey dynamics: a common topic in differential equations texts, the pesticide paradox is in this whimsically named book.

[48] Harte, John, *Consider a Spherical Cow,* University Science Books, 1988.

Synchrony and fire flies: see [40] and this video series.

[49] *The Trials of Life: Talking to Strangers,* Ambrose Video Publishing.

Jaime Escalante: dramatized in the movie *Stand and Deliver,* the life of this outstanding mathematics teacher is well told in this book.

[50] Mathews, Jay, *Escalante,* Henry Holt and Company, 1988.

Integral tables: a CAS such as Mathematica may have better tables than these classic references.

[51] Beyer, W.H., ed., *Standard Mathematical Tables and Formulas,* 29th ed., CRC, 1991.

[52] Jeffrey, Alan, ed., *Gradshteyn and Ryzhik's Tables of Integrals, Series and Products,* 4th ed., Academic Press, 1995.

CHAPTER 5

Brachistochrone and tautochrone: two classic mathematics problems. One of the best descriptions of the mathematics and personalities is in

[53] Dunham, William, *Journey Through Genius,* John Wiley and Sons, 1990.

Juggling: calculus and other areas of mathematics are applied to the patterns and technical details of juggling.

[54] Polster, Burkard, *The Mathematics of Juggling,* Springer-Verlag, 2002.

Baseball forces: Adair's book is a comprehensive discussion of the physics of baseball. See [17], also.

[55] Adair, Robert, *The Physics of Baseball,* Harper and Row, 1990.

Wallaby tendons and human feet: two examples from an excellent discussion of animal movements from crawling and walking to swimming and flying.

[56] Alexander, R. McNeill, *Exploring Biomechanics,* Scientific American Library, 1992.

Projectile motion: many of our best examples come from Brancazio [5] and Alexander [56]. A general but mathematically advanced discussion is in

[57] deMestre, Neville, *The Mathematics of Projectiles in Sports,* Cambridge University Press, 1990.

Air resistance: see [47] and the following well-written module.

[58] Donley, H. Edward, "The Drag Force on a Sphere," UMAP Module 712, COMAP, Inc., 1991.

Knuckleball: see [17].

Weightless training: an entertaining description of the "vomit comet" and the making of the movie *Apollo 13* is

[59] Kluger, Jeffrey, *The Apollo Adventure,* Pocket Books, 1995.

Sweet spot: see [5] and [17].

Model rockets: sophisticated scientific principles go into the design of rockets.

[60] Stine, G. Harry, *Handbook of Model Rocketry,* 6th ed., John Wiley and Sons, 1994.

Pole vault: one of several sports discussed in this overview of Summer Olympics sports.

[61] Mallette, Vincent, *The Science of the Summer Games,* Charles River Media, 1996.

Tennis racket design: brief discussions can be found in [5] and [62], more thorough treatments in [63] and [64].

[62] Schrier, Eric and William Allman, eds., *Newton at the Bat,* Charles Scribner's Sons, 1987. (An interesting collection of essays on a variety of sports.)

[63] Torrey, Lee, *Stretching the Limits,* Dodd, Mead and Company, 1985. (Another interesting collection of essays on a variety of sports.)

[64] Brody, Howard, *Tennis Science for Tennis Players,* University of Pennsylvania Press, 1987. (A thorough and informative look at all aspects of tennis. Numerous graphs.)

Height statistics: thousands of facts about our world.

[65] Brandreth, Gyles, *Your Vital Statistics,* Citadel Press, 1986.

Batting .400: one of several examples of how the variance of a data set can be more meaningful than its mean.

[66] Gould, Stephen Jay, *Full House,* Harmony Books, 1996.

Interspike intervals: see [43] for a variety of mathematical models of brain and heart activity.

CHAPTER 6

Jaime Escalante: see [50]

Brachistochrone: see [53]

Gamma function: numerous details about this amazing function can be found in

[67] Havil, Julian, *Gamma: Exploring Euler's Constant,* Princeton University Press, 2003. (Somewhat technical, but accessible to calculus students.)

CHAPTER 7

Carbon-14 dating: a common example in calculus and differential equations texts, but Braun explains how it's actually done.

[68] Braun, Martin, *Differential Equations and Applications,* Springer-Verlag, 1975.

Lemmings: an exploration of various attempts to explain changes in lemming populations is the focus of this honest portrayal of population ecology. Subtitled *Beautiful Hypotheses and Ugly Facts.*

[69] Chitty, Dennis, *Do Lemmings Commit Suicide?,* Oxford University Press, 1996.

Doomsday model: an interesting if extreme model of population growth.

[70] Forster, Mora and Amiot, "Doomsday: Friday, 13 Nov A.D. 2026," *Science,* November 1960.

Leonhard Euler: a superb biography and accessible translation of the great man's take on calculus.

[71] Dunham, William, *Euler: The Master of Us All,* Mathematical Association of America, 1999.

[72] Euler, Leonhard, translated by John Blanton, *Foundations of Differential Calculus,* Springer-Verlag, 2000.

Critical threshold, spruce budworms: see [39] and [40].

Air resistance: see [47] and [58].

Zebra stripes: see [39] and [40].

Mathematical models: several textbooks apply differential equation models to a variety of physical problems.

[73] Wan, Frederic, *Mathematical Models and Their Analysis,* Harper and Row, 1989.

[74] Giordano, Frank and Maurice Weir, *A First Course in Mathematical Modeling,* Brooks/Cole, 1985.

CHAPTER 8

Music synthesizers: see [6] and [7]. Specific details come from the owner's manual for a Roland Synth-Plus 60 synthesizer.

Fibonacci sequence: a search on the Internet will turn up several good sites. Two of the many books with interesting discussions of the nautilus shell spiral are

[75] Huntley, H.E., *The Divine Proportion,* Dover Publications, 1970. (Subtitled *A Study in Mathematical Beauty,* with several interesting connections to art, architecture and nature.)

[76] Stewart, Ian, *Nature's Numbers,* BasicBooks, 1995. (A collection of easy-to-read essays on current topics in mathematics. Stewart is one of the best popular mathematics authors.)

Inscribed circles, parabolic sequence: the sequence of inscribed circles was suggested by James Albrecht. The circles inscribed in a parabola are from Gregory Minton.

Bouncing ball: a collection of well-developed chaos models with software.

[77] Tufillaro, Nicholas, Tyler Abbott and Jeremiah Reilly, *An Experimental Approach to Nonlinear Dynamics and Chaos,* Addison-Wesley, 1992.

Waiting in line: this is the second in the classic series on probability theory.

[78] Feller, William, *An Introduction to Probability Theory and Its Applications,* volume 2, John Wiley and Sons, 1966.

Riemann-zeta function: an entertaining description of how this function is involved in the Riemann hypothesis, the most famous unsolved problem in mathematics.

[79] Derbyshire, John, *Prime Obsession,* Joseph Henry Press, 2003.

Ramanujan: perhaps the most interesting mathematician in history. The life of this self-trained mathematical genius is told in this excellent book.

[80] Kanigel, Robert, *The Man Who Knew Infinity,* Washington Square Press, 1991.

Unusual fractions: several interesting decimal representations and hints on how to construct your own.

[81] Kreminski, Richard, "Fun Fractions," *Mathematics Teacher,* National Council of Teachers of Mathematics, July 1998.

Image processing: good, brief explanations of medical imaging techniques and other forms of digital processing can be found in this general interest book.

[82] Friedhoff, Richard and William Benzon, *Visualization,* Abrams Publishers, 1989.

Euler's constant: see [67]

Fourier images: programmed in Mathematica with help from Gregory Minton and Emily Wooge.

CHAPTER 9

Sonic boom: many introductory physics books contain good discussions.

[83] Hewitt, Paul, *Conceptual Physics,* 8th ed., Addison-Wesley, 1999.

Faster than the speed of light: suggested by Gregory Minton.

Mandelbrot set: see [13]–[15].

Vector projectile motion: see [57].

Gymnastics: see [5], [26], [62] and [63].

Skiing: physics and chemistry applied to all aspects of skiing.

[84] Lind, David and Scott P. Sanders, *The Physics of Skiing,* AIP Press, 1997.

The Bernoulli brothers: see [72].

Golf putting: The paper "How to Lower Your Putting Score without Improving" by B. Hoadley in [9] focuses on the specific model we use in our book. Pelz provides a helpful, mostly nonmathematical tutorial covering all aspects of putting.

[85] Pelz, Dave, *Putt Like the Pros,* Harper and Row, 1989.

Kepler's laws: a good brief description of the history of planetary analysis, updated to include new discoveries of chaotic orbits.

[86] Peterson, Ivars, *Newton's Clock,* W.H. Freeman, 1993.

CHAPTER 10

Inscribed spheres: a variety of fascinating problems are in

[87] Steele, J. Michael, *Cauchy-Schwartz Master Class,* Cambridge University Press, 2004.

Spinning cube: part of the Demos with Positive Impact project.

[88] Minton, Roland and Gregory Minton, "The Curved Cube," http://astro.ocis.temple.edu/~dhill001/c.

Magnus force: see [5], [17], [26].

CHAPTER 11

RoboCup: the official web site and the home of the Cornell Big Red team are

[89] www.robocup.org

[90] robocup.mae.cornell.edu

Kepler: an enjoyable accounting of Kepler's life and association with Tycho Brahe can be found in

[91] Ferguson, Kitty, *Tycho & Kepler,* Walker and Company, 2002.

Lagrange points: see [34].

CHAPTER 12

Effect of humidity on projectile: see [5].

Nautilus shell: see [1].

Golf ball data: from a Mathematica program written by Geoff Boyer.

Density plots: created using Mathematica.

Tennis graphs: see [64].

Grade average predictions: thanks to Dr. Dan Larsen, Roanoke College.

Digital pictures, Lambert shading: see [82].

Interpretation of mixed partial derivative: adapted from this article.

[92] McCartin, Brian, "What is f_{xy}?" *PRIMUS,* March 1998.

Baseball graphs: see [17].

Golf dimples: see [5] and [26].

Gauge of manufactured metal: thanks to Tom Burns of General Electric.

Mental calculations: one of numerous interesting examples in this exploration of how the human brain does mathematics.

[93] Dehaene, Stanislas, *The Number Sense,* Oxford University Press, 1997.

Football strategy: football is almost as carefully analyzed as baseball.

[94] Carroll, Bob et al., *The Hidden Game of Pro Football,* Warner Books, 1998.

[95] Stern, Hal, "A Statistician Reads the Sports Page," *Chance,* Summer 1998.

Optimal thrust of rocket: a textbook for the calculus of variations, a post-differential-equations course closely related to control theory.

[96] Smith, Donald, *Variational Methods in Optimization,* Prentice-Hall, 1974.

Sailboat steering: the design of racing yachts has become one of the most competitive engineering problems in sports.

[97] Stein, Sherman and Anthony Barcellos, *Calculus,* 5th ed., McGraw-Hill, 1992.

CHAPTER 13

Model rockets: see [60].

Monte Carlo simulation: the interaction between random and nonrandom processes can be surprising.

[98] Ripley, Brian, *Stochastic Simulation,* John Wiley and Sons, 1987.

Moment of inertia: the principal behind many sports phenomena. See [5], [17], [26] and the following general interest science book.

[99] Blanding, Sharon and John Monteleone, *What Makes a Boomerang Come Back,* Longmeadow Press, 1992.

Baseball bats: see [5] and [17].

Direct linear transformation: see the paper "Video Monitoring System to Measure Initial Launch Characteristics of Golf Balls," by Gobush, Pelletier and Days in [3]. The technique itself is derived from

[100] Abdel-Aziz, Y.I. and H.M Karara, "Direct Linear Transformation into Object Space Coordinates in Close-Range Photogrammetry," Symposium on Close-Range Photogrammetry, University, of Illinois at Urbana-Champagne, 1971.

CHAPTER 14

Volkswagen Beetle: the information on the new Beetle came from the Volkswagen website. The older drag coefficients are from this physics book.

[101] Roberson, John and Clayton Crowe, *Engineering Fluid Mechanics,* 3rd edition, Houghton Mifflin, 1985.

Vector calculus: different organizations of the material in this chapter can be found in vector calculus books such as

[102] Schey, H.M., *Div, Grad, Curl and All That,* 3rd ed., W.W. Norton, 1997.

[103] Bressoud, David, *Second Year Calculus,* Springer-Verlag, 1991.

CHAPTER 15

Logistics model with harvesting: includes an important sidebar on how the mathematics of stability affected international fishing policies.

[104] Tauber, Clifford, *Modeling Differential Equations in Biology,* Prentice-Hall, 2001.

Seismometer: one of many differential equations that are animated and illustrated in this excellent interactive software.

[105] CODEE (Consortium for Ordinary Differential Equations Experiments), *ODE Architect,* John Wiley and Sons, 1999.