Assignment 10: Newton's Method (3.1)
Name $\qquad$ Please provide a handwritten response.

1a. You can use Newton's method to find the approximate zeros of $f(x)=x-\frac{7}{4} \sin x+\frac{1}{8}$. First graph $f(x)=x-\frac{7}{4} \sin x+\frac{1}{8}$ as $y_{1}$ (watch that you've used enough parentheses) and record your results below.


$$
-3 \leq x \leq 3,-3 \leq y \leq 3
$$

1b. How many zeros does $\boldsymbol{f}$ seem to have over $\mathbf{- 3} \leq \boldsymbol{x} \leq \mathbf{3}$ ? Roughly where are they located?

2a. Suppose Newton's method is applied to $f$ with $\boldsymbol{x}_{\mathbf{0}}=\mathbf{- 1 . 3}$; what would you expect the successive approximations $\boldsymbol{x}_{1}, \boldsymbol{x}_{2}, \ldots$ to do? Why?

2b. Apply $x_{n+1}=x_{n}-\frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}$ to $f(x)=x-\frac{7}{4} \sin x+\frac{1}{8}$ and fill in the chart below by running the program NEWTONS (see appendix) on your calculator. Enter $\boldsymbol{f}(\boldsymbol{x})$ as $\boldsymbol{y}_{\mathbf{1}}$ and $\boldsymbol{f}^{\prime}(\boldsymbol{x})$ as $\boldsymbol{y}_{2}$. Would you say that Newton's method was successful in this case? What is the approximate value of the zero you are looking for?

| $\boldsymbol{n}$ | $\boldsymbol{x}_{\boldsymbol{n}}$ |
| :---: | :---: |
| $\mathbf{0}$ | -1.3 |
| $\mathbf{1}$ |  |
| $\mathbf{2}$ |  |
| $\mathbf{3}$ |  |
| $\mathbf{4}$ |  |
| $\mathbf{5}$ |  |

3a. Now try the program with $\mathbf{x}_{0}=\mathbf{- 1 . 0}$ and complete the table below. (To restart the program after finishing you can press ENTER and the program will run again from the beginning). Did Newton's method succeed in this case?

| $\boldsymbol{n}$ | $\boldsymbol{x}_{\boldsymbol{n}}$ |
| :---: | :---: |
| $\mathbf{0}$ | -1.0 |
| $\mathbf{1}$ |  |
| 2 |  |
| 3 |  |
| $\mathbf{4}$ |  |
| $\mathbf{5}$ |  |

3b. Since using $\boldsymbol{x}_{\mathbf{0}}=\mathbf{- 1 . 0}$ did not lead to the zero you were looking for, perhaps you need to increase $\boldsymbol{x}_{\mathbf{0}}$ a bit more, say to $\mathbf{- 0 . 8}$. Now use your program to fill in the table below. Did Newton's method lead to a zero? Was it the one you were looking for?

| $\boldsymbol{n}$ | $\boldsymbol{x}_{\boldsymbol{n}}$ |
| :---: | :---: |
| $\mathbf{0}$ | -0.8 |
| $\mathbf{1}$ |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

3c. Try it one more time with $\boldsymbol{x}_{\mathbf{0}}=\mathbf{- 0 . 6}$ and complete the table below. Did you finally get the results you expected?

| $\boldsymbol{n}$ | $\boldsymbol{x}_{\boldsymbol{n}}$ |
| :---: | :---: |
| $\mathbf{0}$ | -0.6 |
| $\mathbf{1}$ |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

3d. What do you think is the most difficult aspect of using Newton's method?

4a. In an earlier assignment you used the SOLVER which finds the zeros of functions numerically. Use the SOLVER to solve $f(x)=x-\frac{7}{4} \sin x+\frac{1}{8}$ with $x_{0}=-1.3$. Does the solver reach the same conclusion you did with $\boldsymbol{x}_{0}=\mathbf{- 1 . 3}$ ?

4b. Our attempt to use $\boldsymbol{x}_{\mathbf{0}}=\mathbf{- 1 . 0}$ did not lead to any zero of $\boldsymbol{f}$ at all. Use the SOLVER with $x_{0}=\mathbf{- 1 . 0}$. Did the SOLVER arrive at a zero of $\boldsymbol{f}$ ? Do you think that the SOLVER operates purely by Newton's Method, or does it have other strategies as well?
5. To see another example of sensitivity to the initial guess, execute the SOLVER when $x=.9625$ and $x=.9627$. Record the results below. How far apart are the initial guesses? How far apart are the results?

