Assignment 13: Numerical Integration (4.7) Please provide a handwritten response.
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1. Graph $y=\sqrt[3]{x^{2}+1}$ on the axes provided and estimate the area under the curve. (Be careful about where the origin is!) Record your answer in the space provided below.


2a. Run the program RIEMANN used in Assignment 12 with $\boldsymbol{a}=\mathbf{0}, \boldsymbol{b}=\mathbf{1}, \boldsymbol{n}=\mathbf{1 0}$.

2b. The midpoint of each interval $\left[x_{i-1}, x_{i}\right]$ is given by $\boldsymbol{c}_{i}=\frac{\boldsymbol{x}_{i-1}+\boldsymbol{x}_{i}}{2}$. Find the Midpoint approximation $\sum_{i=1}^{n} f\left(c_{i}\right) \Delta x$ from RIEMANN. Remember, it is result M. Is this result plausible? Enter it in the table below.
3. Calculate the Trapezoidal Rule approximation $\sum_{i=1}^{n} \frac{f\left(x_{i-1}\right)+f\left(x_{i}\right)}{2} \Delta x$ from the program RIEMANN by pressing ENTER and choosing the Trapezoidal (T) option. Enter the result in the table below.
4. Calculate the Simpson's Rule approximation $\sum_{i=1}^{n} \frac{f\left(x_{i-1}\right)+f\left(x_{i}\right)}{2} \Delta x$ from the program RIEMANN by pressing ENTER and choosing the Simpson's Rule (S) option. Enter the result in the table below.

| $\mathbf{n}$ | MIDPOINT | TRAPEZOID | SIMPSON'S |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 0}$ |  |  |  |
| 20 |  |  |  |
| $\mathbf{5 0}$ |  |  |  |

5. Rerun the program with $\boldsymbol{n}=\mathbf{2 0}$ answering questions $\mathbf{2 b} \mathbf{- 4}$ in order. Record your results in the table. Which of the three approximations did not change when $\mathbf{n}$ was increased?
6. Repeat Question $\mathbf{5}$ with $\boldsymbol{n}=50$ and enter the results in the table. Are the three approximations drawing closer together as $\mathbf{n}$ increases?
7. You can use the calculator to accurately calculate $\int_{0}^{1} \sqrt[3]{\boldsymbol{x}^{2}+1} d x$ using
$\operatorname{fnInt}(\boldsymbol{y 1}, \boldsymbol{x}, \mathbf{0}, 1)$ or $\operatorname{fnInt}\left(\left(\boldsymbol{x}^{2}+\mathbf{1}\right) \wedge(\mathbf{1} / \mathbf{3}), \boldsymbol{x}, \mathbf{0}, 1\right)$ and record the result below. Based on this, which of the three approximation methods applied above was the most accurate?

8a. You can almost always take the results of fnInt to be accurate. However, there are some unusual situations that cause trouble for fnInt. For example, let $f(x)=\boldsymbol{\operatorname { s i n }} \frac{\mathbf{1}}{\boldsymbol{x}}$. Sketch the graph (as best you can) over $[\mathbf{0 , 1}]$ on the axes provided below.


$$
0 \leq x \leq 1,-1 \leq y \leq 1
$$

8b. Evaluate $\operatorname{fnInt}(\sin (1 / x), x, .001,1)$ to calculate $\int_{.001}^{1} \sin \frac{1}{x}$ and describe what happens below. Do you think the numerical result is trustworthy?

