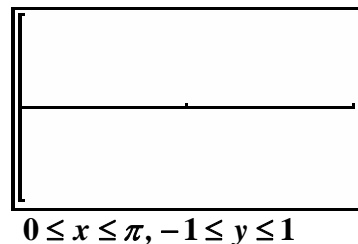


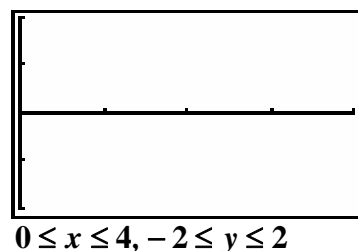
Assignment 14: Solids of Revolution (5.1-4)
Please provide a handwritten response.

Name _____

1. When finding the volume of revolution of a graph like that of $f(x) = \sin x$ over $[0, \pi]$ you can use your calculator to sketch a graph of the area being revolved about the x-axis. Graph this function on the axes provided below.



2. Next sketch the graph of $f(x) = \sqrt{x}$ over $[0, 4]$ which is to be revolved about the x-axis. Sketch this area on the axes provided below.



You can think of solids of revolution as using a certain amount of surface area to enclose a certain amount of volume. This leads to the question of what function f over what interval leads to a solid of revolution enclosing as much volume V as possible while using as little surface area S as possible. We can make this precise by studying the ratio $\frac{V}{S^{3/2}}$. It turns out that this fraction never goes above a certain limit regardless of the function f that is used. We can use the calculator to experiment with various possibilities for f to see how much volume a solid of revolution can enclose using a certain amount of area.

3a. The assumption is made that $f(x) \geq 0$ over $a \leq x \leq b$, and that the graph of f over this interval is being revolved about the x-axis to form a solid of revolution. Since the disk method applies here, the volume is $\pi \int_a^b (f(x))^2 dx$. You can now use $\pi \text{fnInt}(Y_1^2, X, a, b)$ to calculate the volume of the solid. Evaluate the volume of the solid in Question 2 and record the result in the table below.

$f(x)$	$[a, b]$	V	S	$\frac{V}{S^{3/2}}$
\sqrt{x}	$[0, 4]$			
$\sin x$	$[0, \pi]$			
$4 - x^2$	$[-2, 2]$			

3b. To find the total surface area, S , for a given solid, you must include not only the “side” surface area given by $2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} dx$ but also the disks, if any, on the ends of the solids whose areas are $\pi(f(a))^2$ and $\pi(f(b))^2$. (For example, the solid in Question 2 has a disk with area $\pi(\sqrt{4})^2 = 4\pi$ on the end where $x = 4$. With $f(x)$ in Y_1 and $f'(x)$ in Y_2 you can use $2\pi \text{fnInt}(Y_1 \sqrt{1 + (Y_2)^2}, x, a, b)$ for the “side” surface area of the function in Question 2 and record the **total** surface area in the table above.

3c. Next compute $\frac{V}{S^{3/2}}$ and record the result in the table.

4. Now repeat **3a-c** for the function in question 1, $f(x) = \sin x$, and record the results in the table. So far, which function gives the larger ratio?

5. Repeat **3a-c** for the function $f(x) = 4 - x^2$ over $-2 \leq x \leq 2$ and record the results in the table. Examine the graph of $f(x) = 4 - x^2$ to help visualize the volume of revolution and surface area as it is revolved about the x-axis.

6. Invent some functions of your own that you think might be strong contenders and run them through Questions **3a-c** and enter the results in the table above. What do you think the maximum possible value of $\frac{V}{S^{3/2}}$ is, and what shape of curve gives it?