Assignment 14: Solids of Revolution (5.1-4) **Name______________________**
Please provide a handwritten response.

1. When finding the volume of revolution of a graph like that of $f(x) = \sin x$ over $[0, \pi]$ you can use your calculator to sketch a graph of the area being revolved about the x-axis. Graph this function on the axes provided below.

$0 \le x \le \pi, -1 \le y \le 1$

2. Next sketch the graph of $f(x) = \sqrt{x}$ over $[0, 4]$ which is to be revolved about the x-axis. Sketch this area on the axes provided below.

$0 \le x \le 4, -2 \le y \le 2$

You can think of solids of revolution as using a certain amount of surface area to enclose a certain amount of volume. This leads to the question of what function f over what interval leads to a solid of revolution enclosing as much volume V as possible while using as little surface area S as possible. We can make this precise by studying the ratio $\frac{V}{S^{3/2}}$. It turns out that this fraction never goes above a certain limit regardless of the function f that is used. We can use the calculator to experiment with various possibilities for f to see how much volume a solid of revolution can enclose using a certain amount of area.

3a. The assumption is made that $f(x) \ge 0$ over $a \le x \le b$, and that the graph of f over this interval is being revolved about the x-axis to form a solid of revolution. Since the disk method applies here, the volume is $\pi \int_a^b \left(f(x)\right)^2 dx$. You can now use **πfnInt($Y_1^{\,2}$,x,a,b)** to calculate the volume of the solid. Evaluate the volume of the solid in Question **2** and record the result in the table below.

f(x)	$[a,b]$	V	S	$\frac{V}{S^{3/2}}$
$\sqrt{x}$	$[0,4]$			
$\sin x$	$[0,\pi]$			
$4-x^2$	$[-2,2]$			

3b. To find the total surface area, S, for a given solid, you must include not only the "side" surface area given by $2\pi\int_a^b f(x)\sqrt{1+\left(f'(x)\right)^2}\,dx$ but also the disks, if any, on the ends of the solids whose areas are $\pi\left(f(a)\right)^2$ and $\pi\left(f(b)\right)^2$. (For example, the solid in Question 2 has a disk with area $\pi\left(\sqrt{4}\right)^2 = 4\pi$ on the end where $x = 4$. With $f(x)$ in Y_1 and $f'(x)$ in Y_2 you can use $2\pi\text{fnInt}\left(Y_1\sqrt{1+(Y_2)^2}, x, a, b\right)$ for the "side" surface area of the function in Question **2** and record the **total** surface area in the table above.

3c. Next compute $\frac{V}{S^{3/2}}$ and record the result in the table.

4. Now repeat **3a-c** for the function in question **1**, $f(x) = \sin x$, and record the results in the table. So far, which function gives the larger ratio?

5. Repeat **3a-c** for the function $f(x) = 4 - x^2$ over $-2 \le x \le 2$ and record the results in the table. Examine the graph of $f(x) = 4 - x^2$ to help visualize the volume of revolution and surface area as it is revolved about the x-axis.

6. Invent some functions of your own that you think might be strong contenders and run them through Questions **3a-c** and enter the results in the table above. What do you think the maximum possible value of $\frac{V}{S^{3/2}}$ is, and what shape of curve gives it?