## Assignment 18: Euler's Method (7.3)

Name

## Please provide a handwritten response.

1a. In this assignment you will look at applying Euler's Method to the differential equation $y^{\prime}=\sin y-x^{2}$. If you want to find the value of $y^{\prime}$ at the point $\left(-3, \frac{\pi}{2}\right)$ you can use the SOLVER (see assignment 3) to enter $\mathbf{z}=\boldsymbol{\operatorname { s i n }} \boldsymbol{y}-\boldsymbol{x}^{2}$ (TI-86) or $\mathbf{0}=Z-\sin (\boldsymbol{Y})+X^{2}$ (TI-83 Plus/TI-84 Plus). Assign $x=-3, y=\frac{\pi}{2}$. Solve for $z=y^{\prime}$ by placing the cursor on $\mathbf{z}$ and pressing SOLVE. Find this value for $\boldsymbol{z}=\boldsymbol{y}^{\prime}$ and record your result below.

1b. Your calculator will draw a direction field for this differential equation as follows:

|  | TI-83 Plus/TI-84 Plus | TI-86 |
| :---: | :---: | :---: |
| DRAWING A DIRECTION FIELD | Put $\boldsymbol{Y}^{\prime}$ in $\mathbf{Y}_{\mathbf{1}}$. Specifically put $Y_{1}=\sin (Y)-X^{2}$ <br> Move cursor to on top of = and press enter to deselect it. Set WINDOW values. Here set $0 \leq X \leq 2,1 \leq Y \leq 3$ <br> Run the program FLDPLOT <br> Save the picture. 2ND PRGM (DRAW) STO 1:StorePic 1 ENTER | Set MODE to DifEq and enter the equation using $\mathbf{t}$ for $\mathbf{x}$ and Q1 for $\mathbf{y}$. GRAPH $Q^{\prime}(t)$ <br> Specifically put $Q^{\prime} 1=\sin (Q 1)-t^{2}$ <br> Set FORMAT (GRAPH MORE FORMAT (F1)) to Euler SlpFld Set initial conditions INITC (F3) to 0 <br> Set WINDOW tMin=0,tMax = 2, $\begin{aligned} & \text { tStep }=1, \text { tPlot }=0, \mathrm{xMin}=0, \\ & \mathrm{xMax}=2, \mathrm{xScl}=1, \mathrm{yMin}=1, \\ & \mathrm{yMax}=3, \mathrm{yScl}=1, \mathrm{Estep}=1 \end{aligned}$ <br> GRAPH (F5) |

Roughly sketch the resulting direction field on the axes supplied below.


$$
0 \leq x \leq 2,1 \leq y \leq 3
$$

1c. You can generate a table of ordered pairs using Euler's Method to solve the differential equation $\boldsymbol{y}^{\prime}=\boldsymbol{\operatorname { s i n }} \boldsymbol{y}-\boldsymbol{x}^{2}$ on your calculator.

|  | TI-83 Plus/TI-84 Plus | TI-86 |
| :---: | :---: | :---: |
|  | Put $\boldsymbol{Y}^{\prime}$ in $\mathbf{Y}_{\mathbf{1}} \cdot$ Specifically put | Set MODE to DifEq and enter the |
| GENERATING | $\boldsymbol{Y}_{\mathbf{1}}=\sin (\boldsymbol{Y})-\boldsymbol{X}^{\mathbf{2}}$ | equation using $\mathbf{t}$ for $\mathbf{x}$ and |
| AN | Run the program EULTBL. | Q1 for $\mathbf{y}$. GRAPH $Q^{\prime}(\boldsymbol{t})$ |
| EULER TABLE | Enter an initial value for $\mathbf{X}$ <br> and $\mathbf{Y}$ at the prompts. In | Put $\boldsymbol{Q}^{\prime} \mathbf{1}=\sin (\boldsymbol{Q 1})-\boldsymbol{t}^{2}$ |



According to this table, what is the value of $\boldsymbol{y}(\mathbf{1})$ using this approximation? What is the value of $\boldsymbol{y}(2)$ using this approximation? Record your results below.

1d. This list can be plotted on your calculator as follows.

|  | TI-83 Plus/TI-84 Plus | TI-86 |
| :---: | :---: | :---: |
| $\begin{gathered} \text { GRAPHING } \\ \text { AN } \\ \text { EULER PLOT } \end{gathered}$ | Put $\boldsymbol{Y}^{\prime}$ in $\mathbf{Y}_{\mathbf{1}}$. Specifically put $Y_{1}=\sin (Y)-X^{2}$ <br> Run the program EULGRPH. <br> Enter an initial value for $\mathbf{X}$ and $\mathbf{Y}$ at the prompts. In | Set MODE to DifEq and enter the equation using $\mathbf{t}$ for $\mathbf{x}$ and Q1 for $\mathbf{y}$. GRAPH $Q^{\prime}(t)$ Specifically put $Q^{\prime} 1=\sin (Q 1)-t^{2}$ |
|  | this case enter $\mathbf{X = 0}$ and $\mathbf{Y}=2$ <br> Enter a step size of $\mathbf{. 1}$ at the prompt | $\begin{aligned} & \text { Set FORMAT to Euler FldOff } \\ & \text { GRAPH MORE } \\ & \text { FORMAT (F1) } \\ & \text { Set WINDOW tMin=0,tMax = 2, } \\ & \text { tStep=.1,tPlot=0,xMin=0, } \\ & \text { xMax=2,xScl=1,yMin=1, } \\ & \text { yMax=3,yScl=1,Estep=1 } \\ & \text { GRAPH (F5) } \end{aligned}$ |

Record your results on the graph below.


$$
0 \leq x \leq 2,1 \leq y \leq 3
$$

1e. Repeat 1c. and 1d. using a step of 0.05 and compute the first 40 iterations. Record the values of $\boldsymbol{y}(\mathbf{1})$ and $\boldsymbol{y}(\mathbf{2})$.

1f. Now plot both the field plot and the Euler function together and record your result on the graph in $\mathbf{1 b}$.

|  | TI-83 Plus/TI-84 Plus | TI-86 |
| :---: | :--- | :--- |
| GRAPHING THE EULER <br> FUNCTION ON THE <br> FIELD PLOT | Edit EULGRPH by adding the <br> line RecallPic 1 as line 4 of the <br> program (immediately <br> following ClrDraw) | GRAPH MORE FORMAT <br> and turn on SIpFld. Now press <br> GRAPH (F5). |

