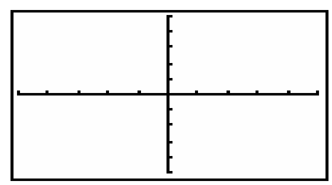
Assignment 20: Fourier Series (8.9) **Name______________________**
Please provide a handwritten response.

1. The signum function $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ can be graphed as follows:

	TI-83 Plus/TI-84 Plus	TI-86
GRAPHING THE SIGNUM FUNCTION	The signum function can be graphed as a piecewise defined function: $Y_1 = (abs(X)/X)$ **or** $Y_1 = (-1)(X < 0) + 1(X > 0)$	The signum function can be graphed as a piecewise defined function: $Y_1 = (abs(X)/X)$ **or** $Y_1 = (-1)(X < 0) + 1(X > 0)$ or as $Y_1 = sign\, x$

Sketch the graph of the signum function on the axes provided below.

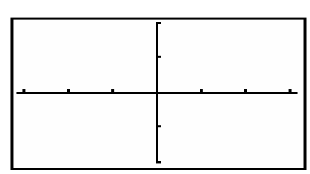


$-5 \le x \le 5, -5 \le y \le 5$

2a. The Fourier coefficients of f, given by the Euler-Fourier formulas as $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$, $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)dx$, $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)dx$ can be computed using **fnInt** on your calculator. Evaluate these coefficients for $k = 1, 2, 3, 4, 5$ and record the results in the table provided. Don't forget to use **2ND ENTRY** to save yourself some typing.

	***k*=1**	***k*=2**	***k*=3**	***k*=4**	***k*=5**
a_k					
b_k					

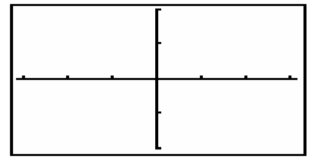
2b. Run the program **FOURIER** using the signum function. Enter the period as π and the number of terms as 5. Enter the signum function as **–1** for $-p \le x < 0$, **1** for $0 \le x < p$. Record your results on the graph provided below.



$-\pi \le x \le \pi, -2 \le y \le 2$

2c. Without clearing the graphing screen at the end of the program, graph $\boldsymbol{Y_4 =}$ **signum x.** Does the approximation seem close over $\boldsymbol{-\pi \le x \le \pi}$?

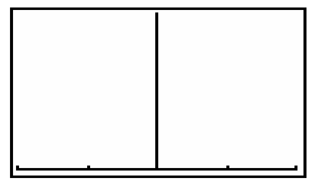
2d. Rerun the program **FOURIER** for the signum function with $\boldsymbol{k = 15}$ and record the result below.



$\boldsymbol{-\pi \le x \le \pi, -2 \le y \le 2}$

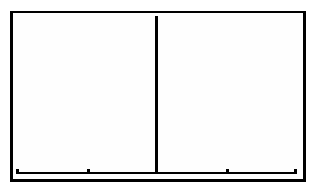
Is the approximation better? Graph the signum function with this approximation to see if it is.

3a. Graph the function $\boldsymbol{y = x - \lfloor x \rfloor}$ over the interval $\boldsymbol{-2 \le x \le 2}$ where $\boldsymbol{y = \lfloor x \rfloor}$ is the **floor function**. Enter $\boldsymbol{y_1 = x - int(x)}$ and sketch the graph below. Do the vertical lines have any significance?



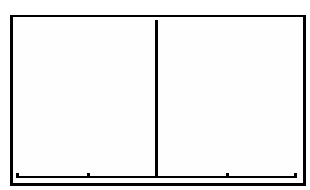
$\boldsymbol{-2 \le x \le 2, 0 \le y \le 1}$

3b. The period P of this function is not $\boldsymbol{2\pi}$. What is it? Formulate equations for $\boldsymbol{y = x - \lfloor x \rfloor}$, $\boldsymbol{-1 \le x < 0}$ (Hint: think of forming the equation of a line through two points) and $\boldsymbol{0 \le x < 1}$. Run the program **FOURIER** with $\boldsymbol{k = 5}$ and sketch the results below.



$\boldsymbol{-2 \le x \le 2, 0 \le y \le 1}$

3c. Now run the program **FOURIER** with $\boldsymbol{k = 15}$. Which graph gives the better approximation of $\boldsymbol{y = x - \lfloor x \rfloor}$?



$\boldsymbol{-2 \le x \le 2, 0 \le y \le 1}$