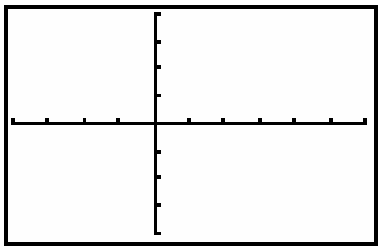
**Assignment 22: Polar Cöordinates (9.4-7)** **Name**_____
**Please provide a handwritten response.**

**1.** Graph the limaçon $r = f(\theta) = 4\cos\theta - 2$ and the ellipse $r = g(\theta) = \dfrac{4}{2+\cos\theta}$ and sketch the results below.

| | **TI-83 Plus/TI-84 Plus** | **TI-86** |
|---|---|---|
| **GRAPHING POLAR FUNCTIONS** | set **MODE** to **Pol**<br>set **FORMAT** to **PolarGC**<br>Press **Y=** and enter $r = r(\theta)$<br>Set $0 \le \theta \le 2\pi, \theta step = \pi / 24$<br>Press **GRAPH** to graph. | set **MODE** to **POLARC** and **POL**<br>Set **FORMAT** (get from **GRAPH MORE F3**) to **PolarGC**<br>From **GRAPH** press $r(\theta) =$ and enter $r = r(\theta)$<br>Set $0 \le \theta \le 2\pi, \theta step = \pi / 24$ |

How many points of intersection do these two curves have?



$-4 \le x \le 6, -4 \le y \le 4$

**2a.** You can use the **Solver** (see assignment 3) on your calculator to find the points of intersection between these two curves. It will be helpful to trace near the points of intersection to identify a value of $\theta$ 'close' to each point of intersection to enter in the **Solver**. Record the values returned by your **Solver** below. Now, find all points of intersection as ordered pairs $(r, \theta)$ corresponding to these values (remember you can evaluate $r_1(\theta)$ to find a corresponding $r$). List these points below.

**2b.** To be absolutely certain you have all points of intersection you will want to solve both $r_1(\theta) = r_2(\theta)$ and $r_1(\theta) = -r_2(\theta + \pi)$. Now solve the second of these equations and record the results below. Also list all points of intersection (from **2a** and **2b**) on the graph above.

**3a**. Write down a sum of definite integrals that gives the area of the region lying both inside the ellipse and outside the limaçon. Also shade this area on the graph above.

**3b.** Use **fnInt** to evaluate these integrals and record the total area below. Based on the graph, is your answer plausible?

**3c.** Repeat parts **a** and **b** for the region lying inside the large loop of the limaçon (ignore the small loop) but outside the ellipse Be sure to keep track of what parts of each curve correspond to which values of $\theta$.

**4a.** The slope of a line tangent to a polar curve at $\theta = a$ is given by the formula: $\left.\frac{dy}{dx}\right|_{\theta=a} = \frac{f'(a)\sin a + f(a)\cos a}{f'(a)\cos a - f(a)\sin a}$. Use this formula with your **Solver** to find all values at which the tangent line to the limaçon is horizontal. Record your results below and mark these points on the graph.

**4b.** The graph suggests that the two curves might be orthogonal at two of their intersection points. Use the formula above to decide whether this is the case. You may also use your calculator to find the required slopes. (Recall that perpendicular lines have negative reciprocal slopes unless they are horizontal and vertical.)

| | **TI-83 Plus/TI-84 Plus** | **TI-86** |
|---|---|---|
| **FINDING THE SLOPE OF A LINE TANGENT TO A POLAR GRAPH** | From the polar graph press **2ND CALC 2** $\frac{dy}{dx}$ Type in values of $\theta$ from **2c** (one at a time). | From the polar graph press **GRAPH MORE F1** to get to **MATH** and choose the **F2** $\frac{dy}{dx}$ Type in values of $\theta$ from **2c** (one at a time). |

At which points of intersection (if any) are the two curves orthogonal? Record your answer below.