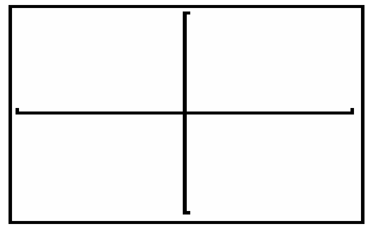
Assignment 24: Vector-Valued Functions (11.1-3)Name______________________
Please provide a handwritten response.

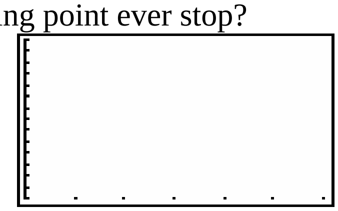
1a. You can graph the vector-valued function $\vec{r}(t) = \langle cos(3t), sin(2t) \rangle, \quad 0 \le t \le 2\pi$ by putting your calculator in parametric mode and letting $\begin{cases} x = cos(3t) \\ y = sin(2t) \end{cases}$. Sketch the resulting curve, known as a "Lissajous curve" on the axes provided below.



$-1 \le x \le 1, -1 \le y \le 1$

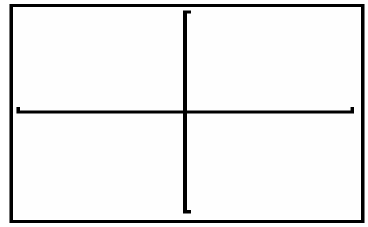
1b. You can easily list the points $\vec{r}(0), \vec{r}\left(\frac{\pi}{4}\right), ..., \vec{r}\left(\frac{n\pi}{4}\right) \quad 0 \le n \le 8$ using the table feature on your calculator. Press (**TABLE**) **TBLST** and set $\Delta Tbl = \frac{\pi}{4}$. Then press **TABLE** to find the points and their corresponding t value. Mark these coordinates on the graph and draw arrows to show the orientation of the curve.

1c. The velocity vector $\vec{v}(t) = \vec{r}\,'(t) = \langle -3\,sin(3t), 2\,cos(2t) \rangle$ is readily found by hand. The speed is given by $\|\vec{v}(t)\| = \sqrt{\vec{v}(t) \cdot \vec{v}(t)}$. Find the speed and record the result below. Sketch the graph of $\|\vec{v}(t)\| = \begin{cases} x = t \\ y = \sqrt{9(sin(3t))^2 + 4(cos(2t))^2} \end{cases}$ over $0 \le t \le 2\pi$ on the axes below. Based on this graph, does the moving point ever stop?



$0 \le x \le 6, .8 \le y \le 3.6$

1d. Define the reparameterization of $\vec{r}_1(t) = \vec{r}(t + 3\,sin\,t)$, $0 \le t \le 2\pi$ and graph the $\vec{r}_1(t)$ over $0 \le t \le 2\pi$. Achieve this by graphing $\begin{cases} X_{2T} = X_{1T}(t + 3\,sin\,t) \\ Y_{2T} = Y_{1T}(t + 3\,sin\,t) \end{cases}$. What is the subtle difference between this graph and that in part **a**? Record the graph on the axes provided.

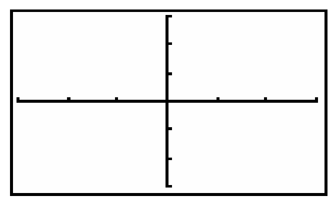


$-1 \le x \le 1, -1 \le y \le 1$
$0 \le t \le 2\pi$

1e. The velocity vector $\vec{v}(t) = \vec{r}\,'_1(t)$ is used to find the speed of a point moving under $\vec{r}_1(t)$. Plot the speed of

$\vec{r}_1(t) = \langle$**-sin(3t+9sin(t))(3+9cos(t)), cos(2t+6sin(t))(2+6cos(t))**$\rangle$ by imitating part **c.**
Note the approximate values of **t** where the speed is zero and use the **Solver** on your calculator setting **speed=0** to find more accurate values. What are the coordinates of these points where $\vec{r}_1(t)$ "stops"? Record your results below.

2a. Sketch the graph of $\vec{r}(t) = \langle 2\cos t + \sin 2t, 2\sin t + \cos 2t \rangle$, $0 \le t \le 2\pi$ on the axes provided below.



$-3 \le x \le 3, -3 \le y \le 3$
$0 \le t \le 2\pi$

2b. Find and mark on the graph any stationary points of $\vec{r}(t)$, as above.

2c. Define the reparameterization of $\vec{r}_1(t) = \vec{r}(t + \sin t)$, $0 \le t \le 2\pi$ and graph $\vec{r}_1(t)$ over $0 \le t \le 2\pi$. Check for stationary points. How do the results compare with part **b?**

2d. Repeat part **c** with $\vec{r}_1(t) = \vec{r}(t^2)$, $0 \le t \le \sqrt{2\pi}$.