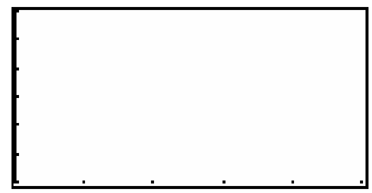
Assignment 7: Limits, Part III (1.7) **Name**____________________

Please provide a handwritten response.

1a. The function $f(x)=\dfrac{(x^3+4)^2-x^6}{x^3}$ can be used to illustrate the dangers of loss of significance errors. Graph $f(x)$ for $10000 \le x \le 100000$. Sketch the result below. Does this graph give any indication of the value of $\lim_{x\to\infty} f(x)$? Explain.



$10000 \le x \le 100000,\ 7.7 \le y \le 8.3$

1b. Now evaluate the function to complete the table below.
Use $y_1(1000)$ to $y_1(10000000)$. You can shorten your typing by using **2ND ENTER (ENTRY)** to repeat what you have typed and edit the number to obtain the number in the table.

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

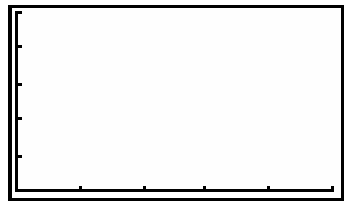
1c. Now evaluate the limit by hand and record the result below. Is it likely that all of these results are correct? Which ones are not?

1d. $f(x)=\dfrac{(x^3+4)^2-x^6}{x^3}$ can be rewritten as $f(x)=\dfrac{8x^3+16}{x^3}$. Enter $f(x)$ in your calculator as y_1 and complete the table below with this new (but equivalent) formula for f. Do you think these new results are more trustworthy?

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

2. Scientific notation is used to write very large or very small numbers in a convenient form; for example, **.000000000002673** would be written in scientific notation as $\mathbf{2.673X10^{-12}}$. Enter $\mathbf{2.673*10\wedge(-12)}$ into your calculator and record the result below.

3a. Find a value of x for which loss of significance occurs in $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$. Graph $y=\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$ on the axes below. Based on this graph, what value would you give for $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$?



$\mathbf{0 \le x \le 100000,\ 0 \le y \le 1}$

3b. Now complete the table below by evaluating $y_1(\mathbf{1.0*10\wedge 8})$, $y_1(\mathbf{1.0*10\wedge 9})$, etc. Where does loss of significance occur?

x	$g(x)$
$1X10^{8}$	
$1X10^{9}$	
$1X10^{10}$	
$1X10^{11}$	
$1X10^{12}$	

3c. We can rewrite y to avoid loss of significance; you can check that multiplying y by $\dfrac{\sqrt{x+4}+\sqrt{x+2}}{\sqrt{x+4}+\sqrt{x+2}}$ gives $y=\dfrac{2\sqrt{x}}{\sqrt{x+4}+\sqrt{x+2}}$. Enter $y=\dfrac{2\sqrt{x}}{\sqrt{x+4}+\sqrt{x+2}}$ as y_1 and complete the table below as in part **b**. Do these results seem more reliable?

x	$g(x)$
$1X10^{8}$	
$1X10^{9}$	
$1X10^{10}$	
$1X10^{11}$	
$1X10^{12}$	

3d. Evaluate $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$ by hand and record the result below. Does it seem to be correct.

3e. Repeat parts **a** and **b** for $\lim\limits_{x\to\infty}x\left(\sqrt{x^3+8}-x^{3/2}\right)$ and record a value of x at which loss of significance occurs.