## Assignment 8: Derivatives of Explicit Functions (2.1-9) Please provide a handwritten response.

1a. The TI calculators will graph both a function and its derivative. Graph $\boldsymbol{f}(\boldsymbol{x})=\mathbf{3} \boldsymbol{x}^{\mathbf{3}}+\mathbf{2 x} \mathbf{x} \mathbf{1}$ by entering the function as $\mathbf{Y}_{\mathbf{1}}$ and graph the derivative as $\mathbf{Y}_{\mathbf{2}}$. The derivative is entered as follows.

|  | TI-83 Plus/TI-84 Plus | TI-86 |
| :---: | :---: | :---: |
| DERIVATIVE | MATH 8 (nDeriv) <br> Enter $\mathbf{Y}_{\mathbf{2}}=\mathbf{n D e r i v}\left(\mathbf{Y}_{\mathbf{1}}, \mathbf{X}, \mathbf{X}\right)$ in the $\mathbf{Y}=$ MENU. ( $\mathbf{Y}_{\mathbf{1}}$ is found in VARS Y-VARS menu) The resulting graph will be that of the derivative. | 2ND CALC F3 (der1) <br> Enter $\mathbf{y} \mathbf{2}=\operatorname{der} \mathbf{1}(\mathbf{y} 1, \mathbf{x}, \mathbf{x})$ in the $\mathbf{y}(\mathbf{x})=$ MENU. The resulting graph will be that of the derivative. |

Graph $f(x)=3 x^{3}+2 x-1$ and its derivative and record the result below. Use different line styles for the function and its derivative.


1b. The slope $\boldsymbol{m}_{\text {tan }}$ line tangent to the graph of $\boldsymbol{f}$ at, say, $\boldsymbol{x}=\mathbf{1}$ is given by $\mathbf{Y}_{\mathbf{2}}(\mathbf{1})$. Execute $\mathbf{Y}_{\mathbf{2}} \mathbf{( 1 )}$ to see that $\boldsymbol{m}_{\tan }=\mathbf{1 1}$ in this case. Also execute $\mathbf{Y}_{\mathbf{1}} \mathbf{( 1 )}$ to see that $\boldsymbol{y}=\mathbf{4}$ when $\boldsymbol{x}=\mathbf{1}$. The equation of the tangent line at $\boldsymbol{x}=\mathbf{1}$ is $\boldsymbol{y}=\mathbf{1 1}(\boldsymbol{x}-\mathbf{1})+\mathbf{4}=\mathbf{1 1} \boldsymbol{x}-7$. Now, graph both $y_{1}=3 x^{3}+2 x-1$ and $y_{3}=11 x-7$ together on the same set of axes (select $y_{1}$ and $y_{3}$ ). You can also draw the tangent line using the DRAW menu. Does the tangent line really look as though its slope is 11? Why?

|  | TI-83 Plus/TI-84 Plus | TI-86 |
| :---: | :---: | :---: |
| DRAW TANGENT | Graph $Y_{1}=3 x^{\wedge} 3+2 x-1$ | Graph $\mathrm{y} 1=3 x^{\wedge} 3+2 x-1$ |
| LINE TO $f(x)=3 x^{3}+2 x-1$ | 2ND PGRM (DRAW) 5 Tangent( <br> Calculator will return the graph with | GRAPH MORE F2(DRAW) MORE MORE MORE |
| USING THE DRAW MENU | the equation of the tangent line in the upper left hand corner of the screen. Type 1, press ENTER and the calculator will draw the graph of the tangent line to the curve at $\boldsymbol{x}=\mathbf{1}$. | F2(TanLn) Calculator will return TanLn( Type $\mathbf{y 1}, 1$ ) and press enter. The calculator will draw the tangent line to the curve at $\boldsymbol{x}=\mathbf{1}$. |



Graph $\boldsymbol{y}_{1}$ and $\boldsymbol{y}_{3}$


Use DRAW Menu
2. Graph $y=\sin \frac{2 x}{x+1}$ and its first and second derivatives on the axes provided. To find the graph of second derivative $f^{\prime \prime}$ of $y_{1}=f(x)=\sin \frac{2 x}{x+1}, y_{2}=f^{\prime}(x)$ use

|  | TI-83 Plus/TI-84 Plus | TI-86 |
| :---: | :---: | :---: |
| SECOND DERIVATIVES | From MATH menu select 8 (nDeriv( ) and obtain $\mathbf{n D e r i v}\left(\mathbf{Y}_{\mathbf{2}}, \mathbf{X}, \mathbf{X}\right)$. At $\mathbf{x}=\mathbf{1}$ you would enter $\mathbf{n D e r i v}\left(\mathbf{Y}_{\mathbf{2}}, \mathbf{X}, \mathbf{1}\right)$ | From 2ND : (CALC) select F4 (der2) and obtain $\operatorname{der} 2(y 1, x, x)$. At $x=1$ you would enter $\operatorname{der} 2(y 1, x, 1)$ |

Label which is which. Differentiate $y=\boldsymbol{\operatorname { s i n }} \frac{2 x}{x+1}$ by hand and record the results below.


3a. Given $f(x)=x^{2} e^{\sin x}$. What rules would you have to use to differentiate this function by hand? Record your results below.

3b. Plot the first derivative of $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{x}^{2} \boldsymbol{e}^{\sin x}$ on the axes (on the left) provided below (Enter $y_{1}=f(x), y_{2}=f^{\prime}(x)$. Turn $y_{1}$ off.)

3c. According to the definition of derivative, if $\boldsymbol{h}$ is a small fixed number, then the difference quotient $\frac{\boldsymbol{f}(\boldsymbol{x}+\boldsymbol{h})-\boldsymbol{f}(\boldsymbol{x})}{\boldsymbol{h}}$ should be close to $\boldsymbol{f}^{\prime}(\boldsymbol{x})$, and so their graphs should lie close together. For the moment let's choose $\boldsymbol{h}=\mathbf{0 . 5}$. Now plot $\boldsymbol{f}^{\prime}(\boldsymbol{x})$ and the difference quotient on the same set of axes (on the right) below. Enter $\boldsymbol{y}_{1}=\boldsymbol{x}^{2} \boldsymbol{e}^{\sin x}, \boldsymbol{y}_{2}=$ derivative of $\boldsymbol{y}_{1}$, and $y_{3}=\left(y_{1}(x+0.5)-y_{1}\right) /(0.5)$. Do not plot $y_{1}$. Use different line styles for $y_{2}$ and $y_{3}$.


3d. Change the $\mathbf{0 . 5}$ to $\mathbf{0 . 4}$ in the difference quotient in part $\mathbf{c}$. Repeat parts $\mathbf{b}$ and $\mathbf{c}$ again. Are the two graphs closer? Can you still tell them apart?

3e. Experiment with smaller and smaller values of $\boldsymbol{h}$ until the graphs of $\boldsymbol{f}^{\prime}(\boldsymbol{x})$ and the difference quotient over $-\mathbf{4} \leq \boldsymbol{x} \leq \mathbf{4}$ become indistinguishable on your calculator screen. How small does $\boldsymbol{h}$ have to be for this to happen?

