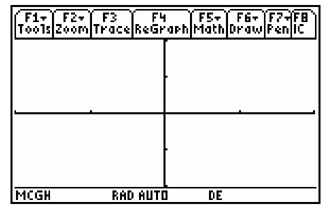
**Assignment 21: Fourier Series (8.9)** **Name**______________________
**Please provide a handwritten response.**

**1.** The signum function $f(x)=\begin{cases} \mathbf{1}, & x>0 \\ \mathbf{0}, & x=0 \\ \mathbf{-1}, & x<0 \end{cases}$ can be graphed as $y = sign(x)$

Sketch the graph of the signum function on the axes provided below.




$-5 \le x \le 5, -5 \le y \le 5$

**2a**. The Fourier coefficients of $f$, given by the Euler-Fourier formulas as $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$,

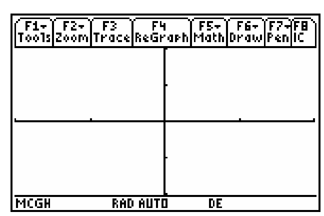
$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)dx$, $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)dx$ can be computed on your calculator. Evaluate these coefficients for $k = 1,2,3,4,5$ on your calculator by evaluating

$(1/\pi)\int(y1(x)*\cos(k*x),x,-\pi,\pi)\,|\,k=\{1,2,3,4,5\}$ for $a_k$ and

$(1/\pi)\int(y1(x)*\sin(k*x),x,-\pi,\pi)\,|\,k=\{1,2,3,4,5\}$ for $b_k$. Record the results in the table below.

| | ***k*=1** | ***k*=2** | ***k*=3** | ***k*=4** | ***k*=5** |
|---|---|---|---|---|---|
| $a_k$ | | | | | |
| $b_k$ | | | | | |

**2b**. Graph the partial sum for the first three terms of the Fourier series $F5 = \frac{a_0}{2} + \sum_{m=1}^{5}\left(a_k(m)\cos(mx) + b_k(m)\sin(mx)\right)$ by running the program **fourier( )**[1]**.** Set your window for $-\pi \le x \le \pi, -2 \le y \le 2, xres = 5$. From **MODE** set **Display Digits** to **FLOAT 2**. At the program prompts enter $p = \pi, k = 3,\ f(x) = -1$ for $-p \le x \le 0,$ and $f(x) = 1$ for $0 \le x \le p$.

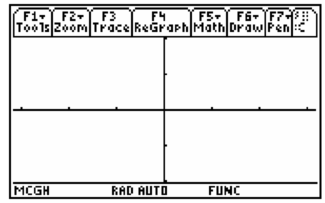



$-\pi \le x \le \pi, -2 \le y \le 2$

**2c.** Press **ENTER** at the **PAUSE** to add the graph of the Signum function by entering $y4 = Sign(x)$ at the prompt. Does the approximation seem close over $-\pi \le x \le \pi$?

[1] This program is **EXTREMELY SLOW!** Go relax while it runs.

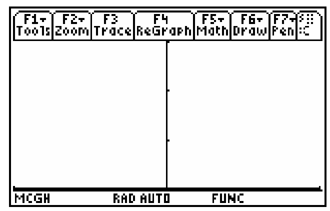
**2d.** Rerun the program **fourier( )** for the signum function with $\boldsymbol{k = 5}$ and record the result below.



$\boldsymbol{-\pi \le x \le \pi, -2 \le y \le 2}$

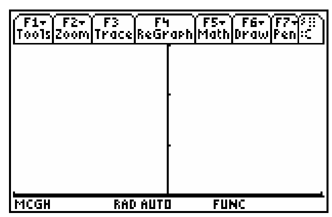
Is the approximation better? Graph the signum function with this approximation by pressing **ENTER** at the **PAUSE** in the program to see if it is. Would adding more terms make the approximation any better?

**3a.** Graph the function $\boldsymbol{y = x - \lfloor x \rfloor}$ over the interval $\boldsymbol{-2 \le x \le 2}$ where $\boldsymbol{y = \lfloor x \rfloor}$ is the **floor function**. Enter $\boldsymbol{y_1 = x - int(x)}$ and sketch the graph below. Do the vertical lines have any significance?



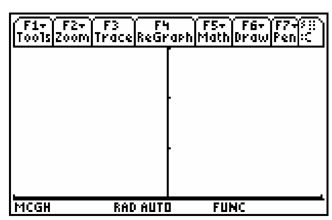
$\boldsymbol{-1 \le x \le 1, 0 \le y \le 3}$

**3b.** The period P of this function is not $\boldsymbol{2\pi}$. What is it? Formulate equations for $\boldsymbol{y = x - \lfloor x \rfloor}$, for $\boldsymbol{-1 \le x < 0}$ (try $\boldsymbol{y = x + 1}$) and for $\boldsymbol{0 \le x < 1}$ (try $\boldsymbol{y = x}$). Set the indicated window with ***xres*=8** and **Display Digits** at **FLOAT 3.** Run the program **fourier( )** with $\boldsymbol{f(x) = x + 1 \text{ for } \text{-}p \le x \le 0}$ and $\boldsymbol{f(x) = x \text{ for } 0 \le x \le p}$, $\boldsymbol{k = 3}$ and sketch the results below.



$\boldsymbol{-1 \le x \le 1, 0 \le y \le 3}$

**3c.** Now run the program **fourier( )** with $\boldsymbol{k = 5}$. Which graph gives the better approximation of $\boldsymbol{y = x - \lfloor x \rfloor}$?



$\boldsymbol{-1 \le x \le 1, 0 \le y \le 3}$